



YUMI DEADLY CENTRE  
School of Curriculum

Enquiries: +61 7 3138 0035

Email: [ydc@qut.edu.au](mailto:ydc@qut.edu.au)

<http://ydc.qut.edu.au>

# Marine Studies: Coxswain Certificate Mathematics behind Modelling Marine Environments

## Booklet VM3: Percentage, Coverage, and Box Models



**DEADLY MATHS VET**

**Tagai Secondary College & Thursday  
Island TAFE**

**Marine Studies – Coxswains Certificate**

**MATHEMATICS BEHIND  
MODELLING MARINE  
ENVIRONMENTS**

**BOOKLET VM3: PERCENTAGE,  
COVERAGE, & BOX MODELS  
VERSION 1**

**Deadly Maths Consortium**

**Tom J Cooper & Chris J Matthews & Annette R Baturu  
with**

**Matthew T Michaelson, Elizabeth A Duus, & Kaitlin M Moore**

**Deadly Maths Consortium**

**School of Mathematics, Science and Technology Education  
Faculty of Education, QUT**

This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths  
Past Project Resource

## Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

## YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called “Deadly Maths”.

“YuMi” is a Torres Strait Islander word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at <http://ydc.qut.edu.au> and staff can be contacted at [ydc@qut.edu.au](mailto:ydc@qut.edu.au).

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School of Curriculum  
QUT Faculty of Education  
S Block, Room S404, Victoria Park Road  
Kelvin Grove Qld 4059  
Phone: +61 7 3138 0035  
Fax: + 61 7 3138 3985  
Email: [ydc@qut.edu.au](mailto:ydc@qut.edu.au)  
Website: <http://ydc.qut.edu.au>

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Queensland University of Technology

**DEADLY MATHS VET**

**Marine Studies: Coxswain Certificate**

**MATHEMATICS BEHIND MODELLING  
MARINE ENVIRONMENTS**

**BOOKLET VM3**

**PERCENTAGE, COVERAGE,  
AND BOX MODELS**

**08/05/09**

Research Team:

Tom J Cooper

Chris J Matthews

Annette R Baturu

with

Matthew T Michaelson

Elizabeth A Duus

Kaitlin M Moore

Deadly Maths Group

School of Mathematics, Science and Technology Education,  
Faculty of Education, QUT

## THIS BOOKLET

This booklet (VM3) was the third booklet produced as material to support Indigenous students completing certificates associated with Marine Studies at the TAFE campus on Thursday Island run by Tagai College. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

- VB1: Mathematics behind whole-number place value and operations  
Booklet 1: Using bundling sticks, MAB and money
- VB2: Mathematics behind whole-number numeration and operations  
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure
- VC1: Mathematics behind dome constructions using Earthbags  
Booklet 1: Circles, area, volume and domes
- VC2: Mathematics behind dome constructions using Earthbags  
Booklet 2: Rate, ratio, speed and mixes
- VC3: Mathematics behind construction in Horticulture  
Booklet 3: Angle, area, shape and optimisation
- VE1: Mathematics behind small engine repair and maintenance  
Booklet 1: Number systems, metric and Imperial units, and formulae
- VE2: Mathematics behind small engine repair and maintenance  
Booklet 2: Rate, ratio, time, fuel, gearing and compression
- VE3: Mathematics behind metal fabrication  
Booklet 3: Division, angle, shape, formulae and optimisation
- VM1: Mathematics behind handling small boats/ships  
Booklet 1: Angle, distance, direction and navigation
- VM2: Mathematics behind handling small boats/ships  
Booklet 2: Rate, ratio, speed, fuel and tides
- VM3: Mathematics behind modelling marine environments  
Booklet 3: Percentage, coverage and box models
- VR1: Mathematics behind handling money  
Booklet 1: Whole-number and decimal numeration, operations and computation

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## OVERVIEW

### Preamble

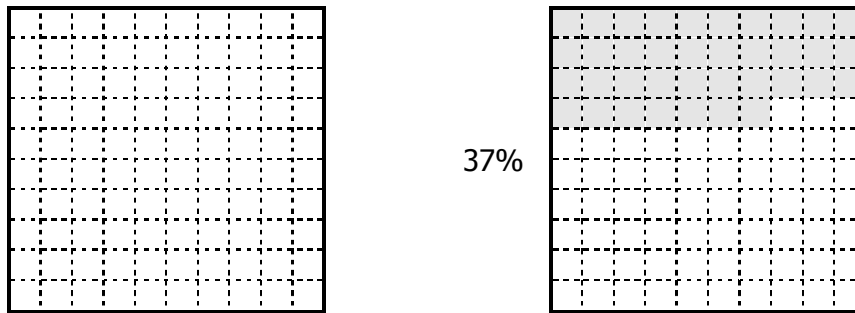
In previous books we have looked at the mathematics behind the Coxswain Certificate - angle and distance, navigation, tides, and rates (speed, fuel consumption) and so on. In this booklet, we look at another aspect of marine, the environment. Students at Tagai Secondary Campus on Thursday Island are part of a project to look at seagrass coverage, an important component of the ecosystem of the sea around the Torres Strait Islands.

The students estimate seagrass average from photographs. This combines two things – the area covered and the thickness or density of the coverage. Thus percentage covers with even covered and density into the thickness of the coverage.

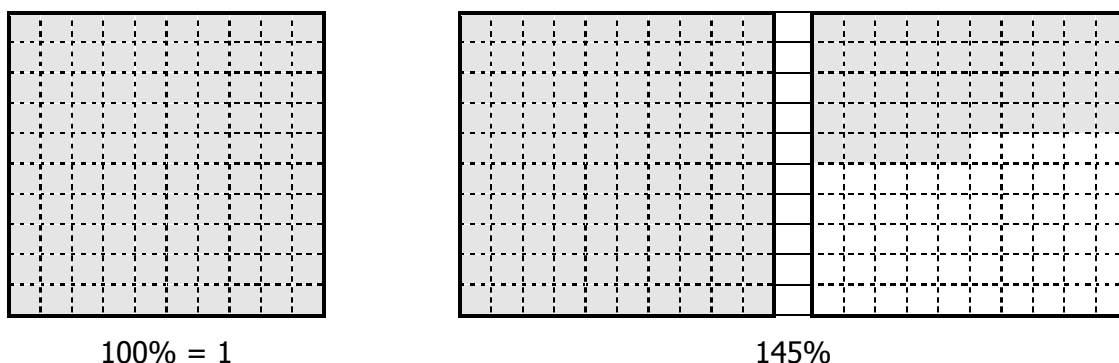
Since seagrass is part of the ecosystem, there is a opportunity to introduce a simple modelling method for ecological study, the box model.

### Percent

Percent as per-cent means per hundred that in a common fraction. Thus 37% really means 37 hundredths or 37 parts per hundred. The best way to think of this in using a 10x10 grid as below:



This means that 100% is 100 hundredths, one whole, and that percentages over 100 are greater than 1, for example:



In terms of decimal factors, this means that percent are when the decimal points moves to the hundredth (making this position the one).

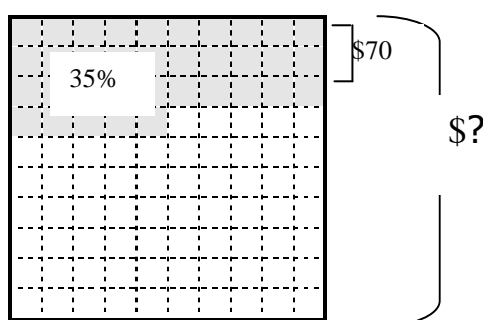
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			3	6	7	4

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			3	6	7	4

$$3.674 = 367.4\%$$

Percent problems can be 3 types = (a) find 35% of \$70, (b) find whole is 35% of \$70, and (c) \$24 is what percent of \$70. There are three ways of solving these problems,

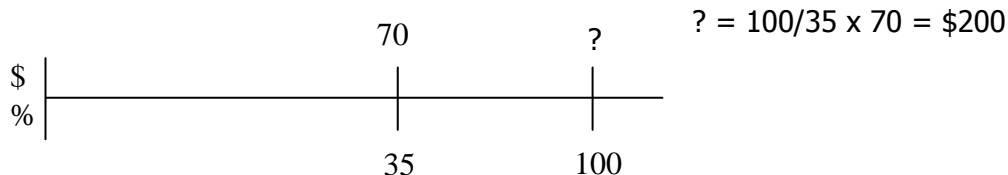
- (1) Picture (area model)



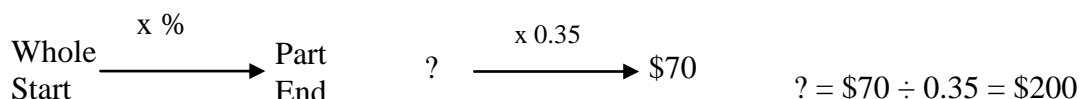
$$1\% = \$70/35 = \$2$$

$$\text{Thus, } 100\% = \$200$$

- (2) Double Number Line



- (3) Change



### **Density**

Density is normally a rate mass/volume – it gives an idea of the amount of material per given volume. For example: lead has a higher density than water so it sinks, while wood has a lower density so it floats. To get this normal form of density, divide mass by volume, for example,  $1.4\text{kg/m}^3$ .

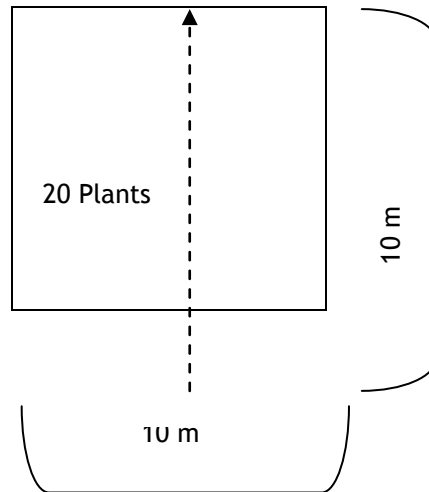
Density can also be used for area and length, but here it usually refers to how much mass, volume or objects per area or per length are rather than how much mass per volume (which is a 3D notion). Two examples here are farming (e.g., number of tonnes of fertilizer or litres of pesticide per square kilometre or numbers of sheep per hectare) or traffic (e.g., number of cars per kilometre).

Two particular examples are pertinent to this booklet:



- (1) density of decimals – number of numbers per given length (this can be seen as infinite and independent of the length of the section in which density is being determined); and
- (2) density of vegetation – number of seagrass plants per square metre.

For seagrass, it would take a long time to count all the plants per given area, so the answer is to count how many is a straight line. Then, area formulae will give a good estimate of the number of plants in an area.

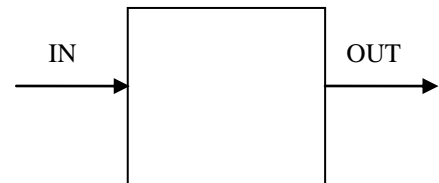


Twenty plants in a 10m line would mean that the number of plants in a 10m x 10m area would be  $20 \times 20$  or 400. This would mean that the density is 400 plants divide area of  $10 \times 10$  or 100 square metres. This gives a density of  $400/100 = 4$  plants per  $\text{m}^2$ .

### ***Box Model***

A box model is a useful to model an environment. It is a box or square with an input and an output.

The easiest thing to use the box model is a tank or a lake – here the activity can be reduced to water in/water out.



However, it can also be used for more complex systems, e.g., populations of animals. Here, input is new animals being born or arriving from outside, and output is animals dying or leaving.

The box model allows students to represent change in terms of algebraic equations as the input and output can vary like a variable. One way to introduce the box model and the equations is to allow students to create their own symbols for the equations before adopting normal algebra letters. This enables creativity and better ownership of the model.

For Indigenous students, the modelling activity can be based on a “care for country” initiative that eventually will focus on an environmental problems.

### ***Prior knowledge for box modelling***

- Understand simple arithmetic (addition and subtraction only)
- Understand notion of unit in measurement
- Know metric volume units for fluids
- Able to calculate volume and surface area
- Use Excel spreadsheet (including graphs)

### ***Suggested sequence for learning the box model***

- (1) Develop a mathematical model that simulates the water level in a water tank attached to a building within the school.
  - (2) Develop the model on paper and then execute within Excel. Excel will then be used to explore the behaviour of the model and to start ask questions like, "Does the model reflect reality?"
  - (3) Continue with the modelling, moving from tanks to more complex water systems, e.g., lakes.
  - (4) Design modelling activities such that, ultimately, students will get an understanding of "what mathematical modelling is" and experience the modelling process.
  - (5) Use activities that monitor levels within a system that has inputs and outputs. Students will learn what this model is, how and when to use it, and its advantages and limitations.
- (1) Extend the box model will be extended to other issues than water – e.g., population.

#### Definition of a box model

A box model is a way to consider real world situations by placing the world into a "box" which has three characteristics: (1) an amount of an attribute within the box that is able to vary; (2) an input that adds attribute; and (3) an output that removes attribute. Although appearing to be too simple, the box model is capable of modelling complex environmental situations – as a single box or as a collection of interaction boxes, where, e.g., the output of one box becomes the input of the next box.

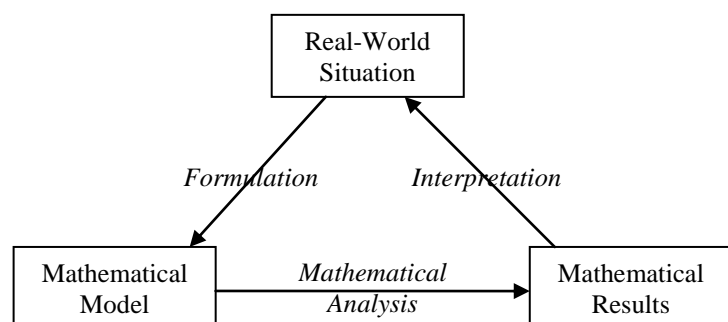
#### Development of box model mathematics

The box model will be developed though 4 stages.

- (1) Conceptualisation – three sub-stages: (a) free discussion – discussing the real world situation that will be modelled; conceptual model – developing a conceptual model of how this reality can "sit in a box"; and mathematical model - building symbols and relationships that mathematically show how the "box" works (this step uses that MAST approach of allowing students to construct their own symbol systems).
- (2) Generalisation – showing how the mathematics iterates around a time interval (called a time step), and how findings can be generalised to algebraic relationships.
- (3) Computer model – transferring the iterations to Excel spreadsheet so that ,many iterations can be calculated quickly.
- (4) Validation – using the Excel spreadsheet to apply the model in a real world situation to see if it works in a "common sense" manner.

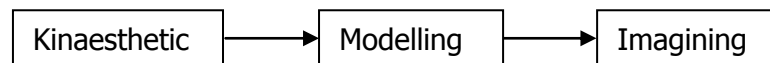
#### Box modelling process

The process of modelling from real world problem to real world solution is as on right:

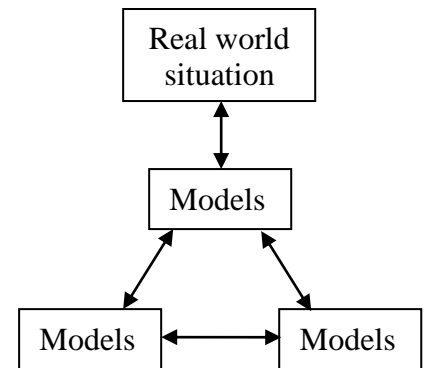


## ***Pedagogy***

- (1) Place all mathematics teaching possible within the context of marine activity within the context of the students (Island living and Torres Strait Islander culture).
- (2) Focus mathematics teaching on mathematics as a whole structure, its pattern and interactions. This means that mathematics teaching needs to focus on understanding how mathematics is related, sequenced and integrated, that is, on the connections between mathematics.
- (3) There possible, begin all teaching using whole body of students (kinaesthetic) then move to using material, computers and pictures (modelling) and then to having student think of a "picture in the mind" (imagining), that is, use a variety of representation that follow this sequence:



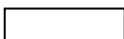
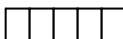
- (4) Relate real world situations (within the context of the students) to models of the mathematics and then to language and finally to symbols (the Payne-Rathmell triangle as on the right)
- (5) Use the generic pedagogies of generalising, reversing and being flexible, that is, ensuring that the most general understanding possible is developed, that activities move in both directions (model to symbol and symbol to model), and that students have flexible understandings of concepts and processes (including language).



## 1. PERCENTAGE

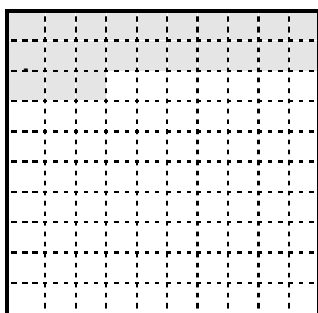
### 1.1 Meaning of Percent

When a whole is divided into equal parts, each part is a fraction and is named by the number of parts.

For example, a whole  divided into 5 equal parts  means that each part is one-fifth.

If a whole is divided into 100 equal parts, each part is one-hundredth. A special name for hundredth is percent (means "per centum" or "per hundred").

Thus 23 percent is 23 hundredths as below. It has a special notation, e.g., 23%. It can be represented as shown on a 10x10 grid paper (which divided into hundredths).



In terms of fractions, twenty-three hundredths is  $23/100$  and in terms of decimals twenty-three hundredths is 0.23. This means that 23% is the same as  $23/100$  and 0.23. It also means that 100% is the same as  $100/100$  or 1.

In terms of place values, percent (%) are numbers where the decimal point is at the hundredths, for example:

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		2•	3	4	

Normal positioning of decimal point gives 2.34

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		2	3	4•	

Decimal point at hundredths gives 234%

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			1•	6	8	4

This enables part percentages: Normal 1.684

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			1	6	8•	4

Percentage = 168.4%

**Percent activities**

(1) Shade the following on the attached page

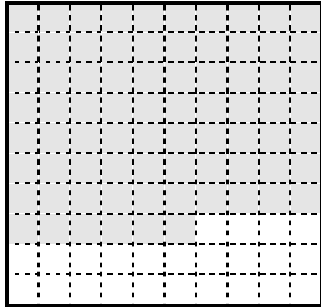
(a) 36%

(b) 217%

(c) 0.78%

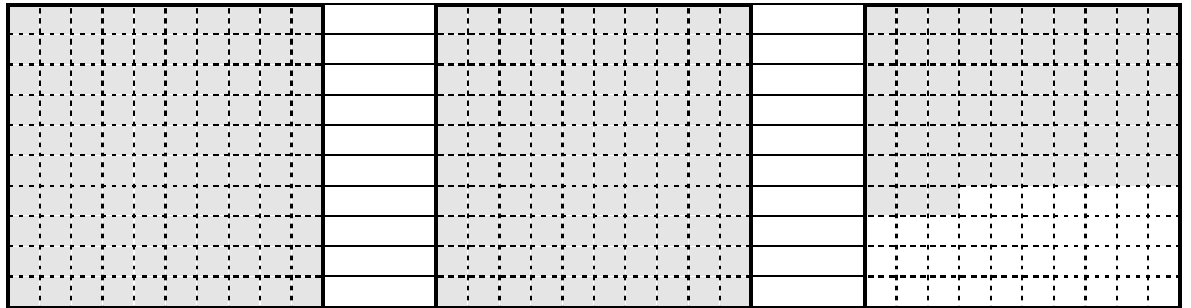
(d) 182/100

(2) Write the shading as shown (as a percent % and as a decimal)



Percent = \_\_\_\_\_.

Decimal = \_\_\_\_\_.



Percent = \_\_\_\_\_.

Decimal = \_\_\_\_\_.

(3) Write these numbers on the place value chart:

(a) 34%

(b) 371%

(c) 42.6%

(d) 3200%

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths

(4) Write these numbers as decimals and percentages

	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths		Dec.	%
(a)				1	4					
(b)					2	9				
(c)				3	6	4	2			
(d)				4	0	1	4			

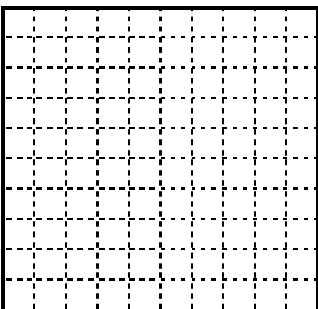
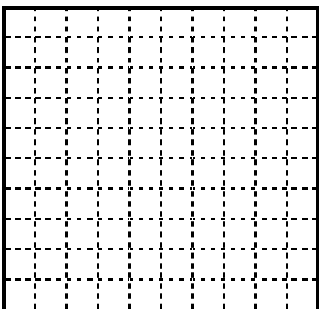
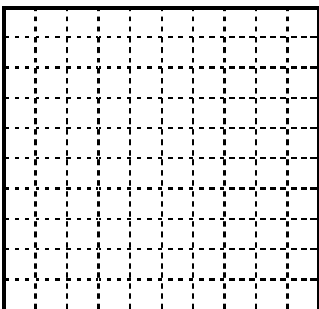
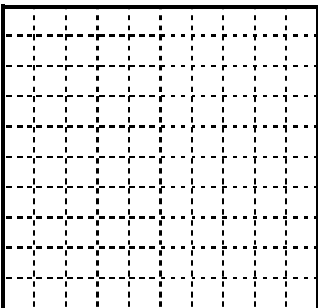
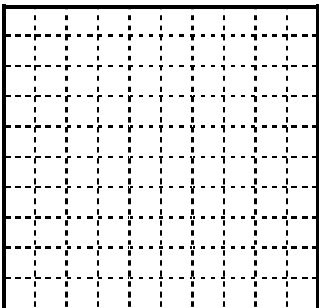
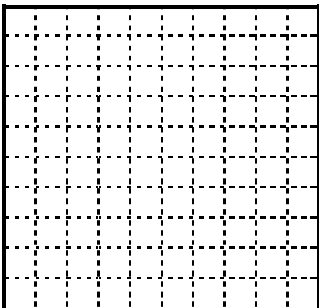
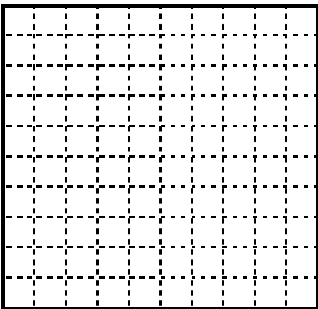
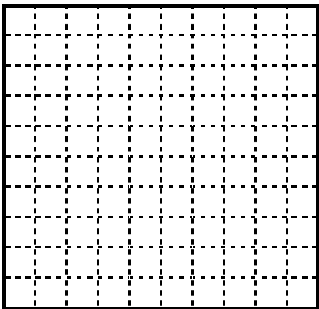
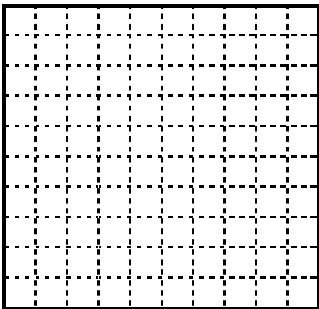
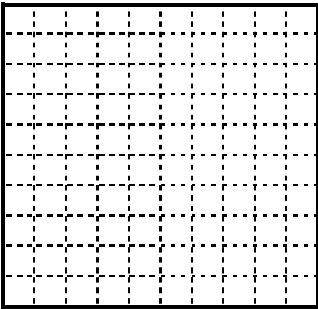
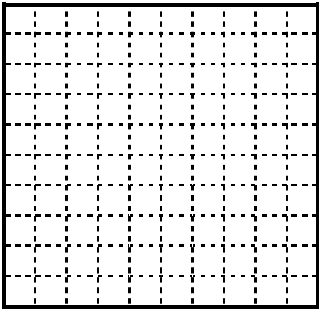
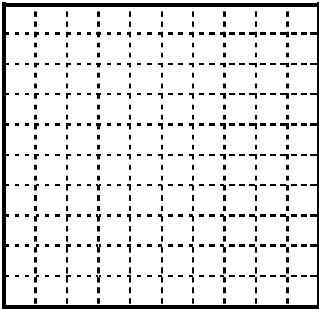
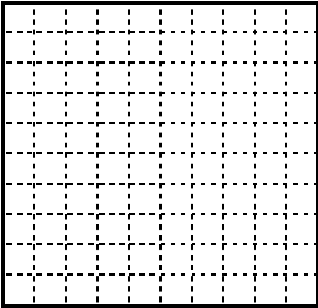
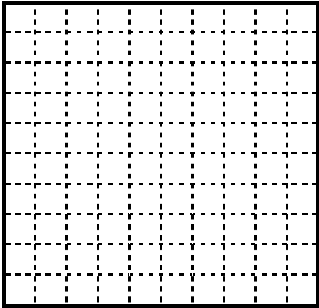
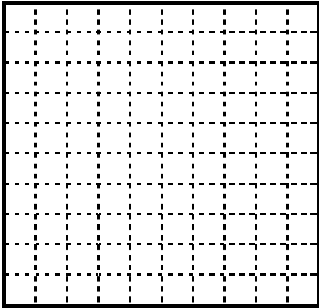
(5) Complete the following table - fill in all cells (the first one has been done for you).

%	37%	48%					7%
Fraction	37/100		64/100			86/100	
Decimal	0.37			0.28	0.91		

(6) Percent (%) can be more than 1, complete the following- fill in all cells (the first one has been done for you).

%	343%			436%			382%
Fraction	343/100		561/100		111/100		
Decimal	3.43	1.28				2.06	

**10x10 grids for % activities**



## Percent Calculations

There are three types of percent calculations

- Type 1: Percentage unknown* Find 85% is \$68. This is straight forward multiplication (e.g.,  $0.85 \times 68 = \$57.80$ )
- Type 2: Total unknown* 85% is \$68, what is the total? This is division (e.g.,  $68 \div 0.85 = \$80$ )
- Type 3: Percent unknown* What % of \$68 is \$51? This is also division (e.g., % is  $51/68 \times 100/1 = 75\%$ )

There are three ways to solve the problems:

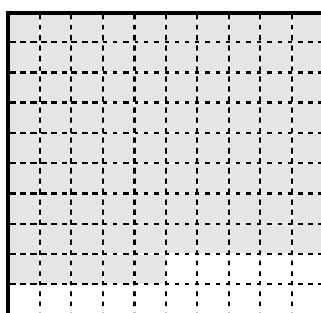
(1) Use a diagram

Type 1

Drawing

Working

What is  
85% of \$68?



\$68

?

$$100\% = \$68$$

$$1\% = 68 / 100$$

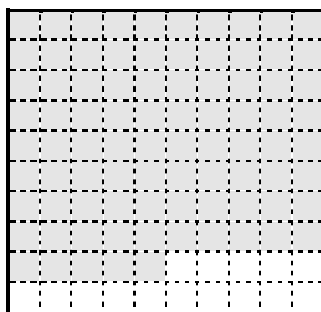
$$85\% = 85 \times 68 / 100$$

Type 2

Drawing

Working

85% is \$68,  
what is the  
total?



\$68

?

$$85\% = \$68$$

$$1\% = 68 / 85$$

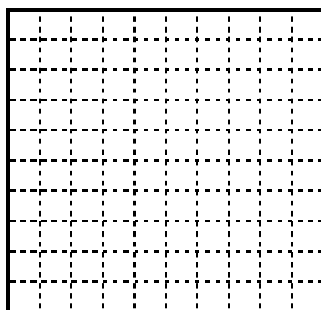
$$100\% = 100 \times 68 / 85$$

Type 3

Drawing

Working

What % of \$68  
Is \$51



\$68

~

$$\$68 = 100\%$$

$$\$1 = 100 / 68$$

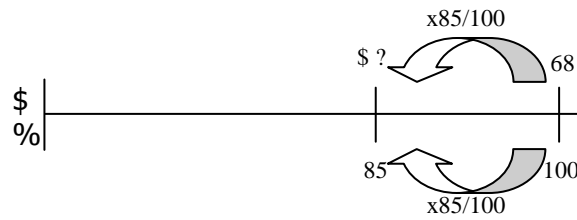
$$\$51 = 51 \times 100 / 68$$



## (2) Double number line

Type 1:Drawing:Working:

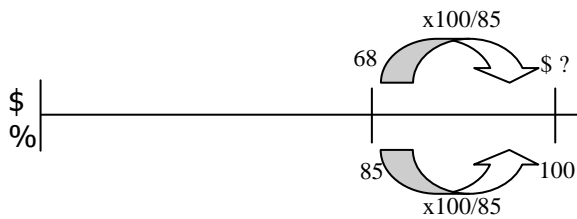
What is  
85% of  
\$68?



$$\$? = \$68 \times 85/100$$

Type 2:Drawing:Working:

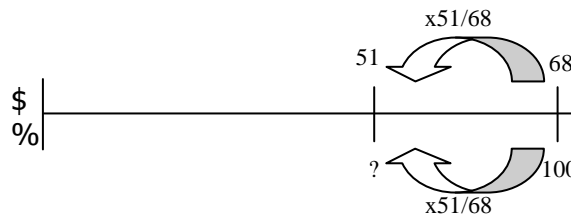
\$68 is 85%,  
what is the  
total?



$$\$? = 68 \times 100/85$$

Type 3:Drawing:Working:

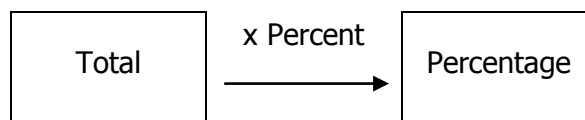
What % of  
\$68 is \$51?



$$\$? = 100 \times 51/68$$

## (3) Multiplication change diagram

The change diagram is:



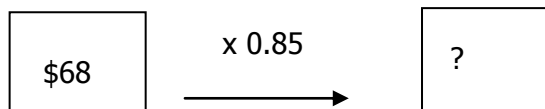
Multiply to find percentage, divide to find total or percent.

Problem

Drawing

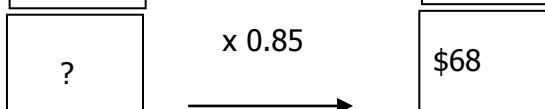
Working

Type 1:



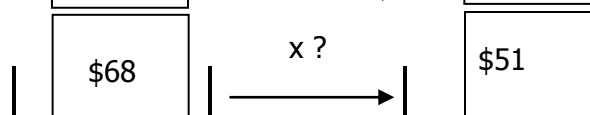
$$? = \$68 \times 0.85$$

Type 2:



$$? = 68 / 0.85$$

Type 3:



$$? = 51 / 68$$

***Percent calculation activities***

Solve the following (use a calculator) by all 3 methods.

Problem

10x10 grid Drawing

Double number line

Change diagram

- (1) John paid 65% of the cost of a TV. The TV cost \$185. How much did John pay?
  
  
  
  
  
  
  
  
  
  
- (2) June paid \$125. This was 45% of the cost of the TV. How much did the TV cost?
  
  
  
  
  
  
  
  
  
  
- (3) Fred paid \$250 towards a \$375 TV. What percentage (%) did Fred pay?

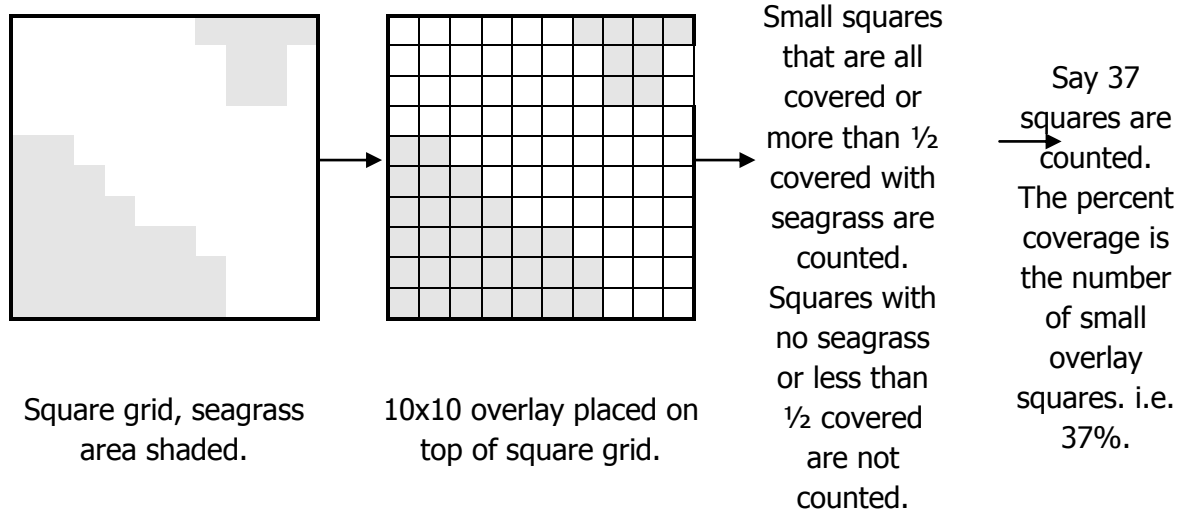
## 2. COVERAGE AND DENSITY

### 2.1 Coverage

When studying marine environments, it is useful to be able to record the extent to which an animal or fish is present in the ocean or the extent to which a plant (e.g. seaweed or seagrass) covers the ocean floor. In this way we can measure the health of the sea or determine the effect on the number of animals (fish from hunting or fishing). There are a lot of situations; we will look at some of them.

(1) Photographic (e.g., plane, satellite) records:

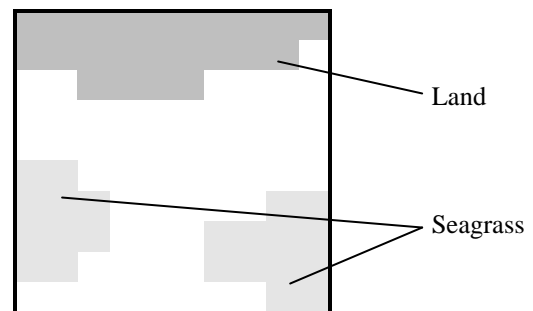
These often divide the land or sea into square grids. The picture shows where they, say, grass covers the sea bed. Using a plastic 10x10 overlay to enable the percentage coverage to be determined. For example:

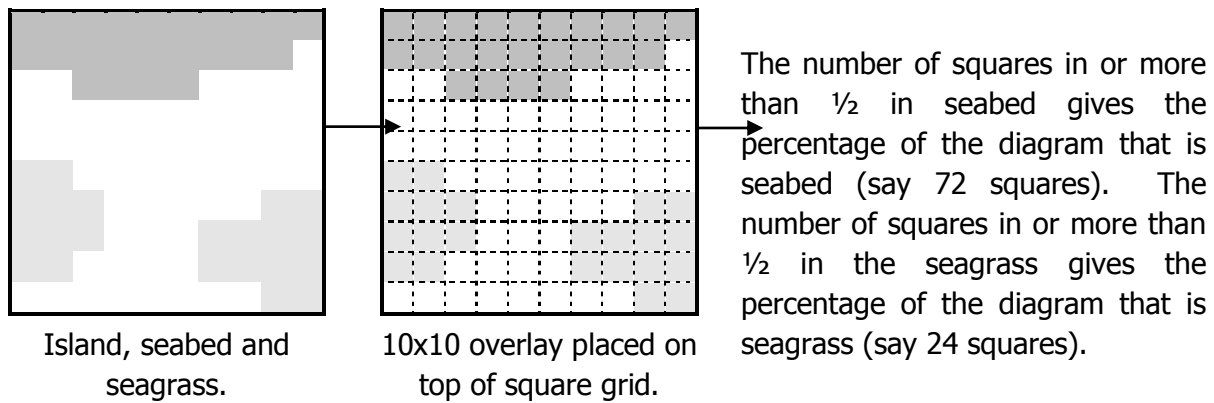


(2) Photographic records – all square, not seabed

These are again square grids. But now the picture shows land, seabed and seagrass. For example: see diagram on right.

Once again a 10x10 overlay can be used, as in the example below (but there is now a complication):





If we are looking a coverage of seagrass across sea and land, then 24% is the answer (24 squares out of 100 squares). However, if we want the coverage of seagrass in relation to the sea only, we have to take into account that the seabed is only 72 squares of the diagram and the 24 squares of seagrass has to be looked at in relation to this 72 squares to find the %.

There are many ways to calculate this percent.

- (i) To make the seagrass a percent, requires the seabed to be 100%. Thus the 72 squares has to change to 100, that is  $\times 100/72$ . Then the seagrass must change accordingly, thus the percent coverage is:

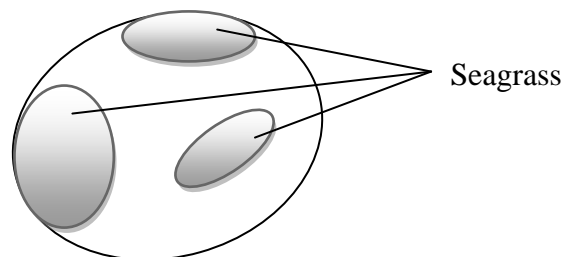
$$24 \times 100/72 = 33.33\%$$

- (ii) Consider the seagrass as a factor of the seabed. This would be:

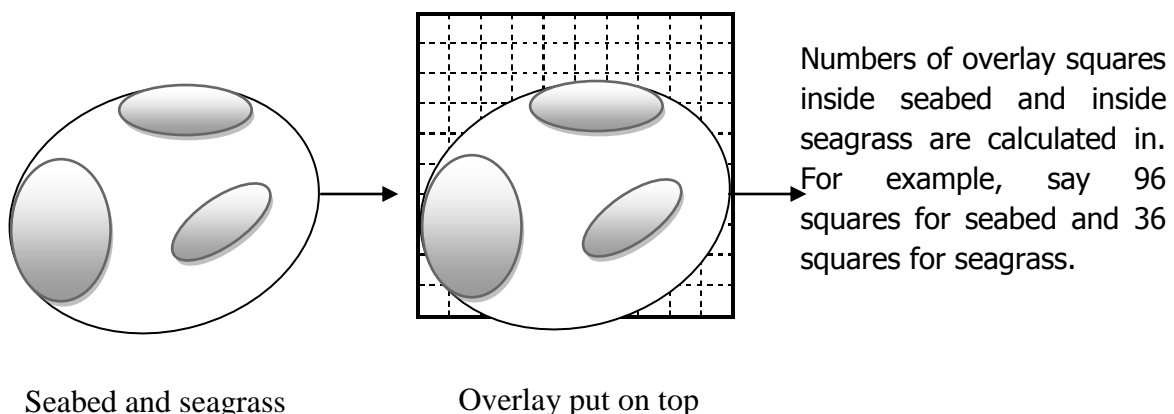
$$\text{Seagrass/Seabed} = 24/72 = 1/3 \text{ which is } 33.33\%$$

### (3) Real situations

Here a seabed is drawn that is to be considered with the positions of seagrass marked on it. For example:



Here an overlay just of squares is used (does not have to be 100).

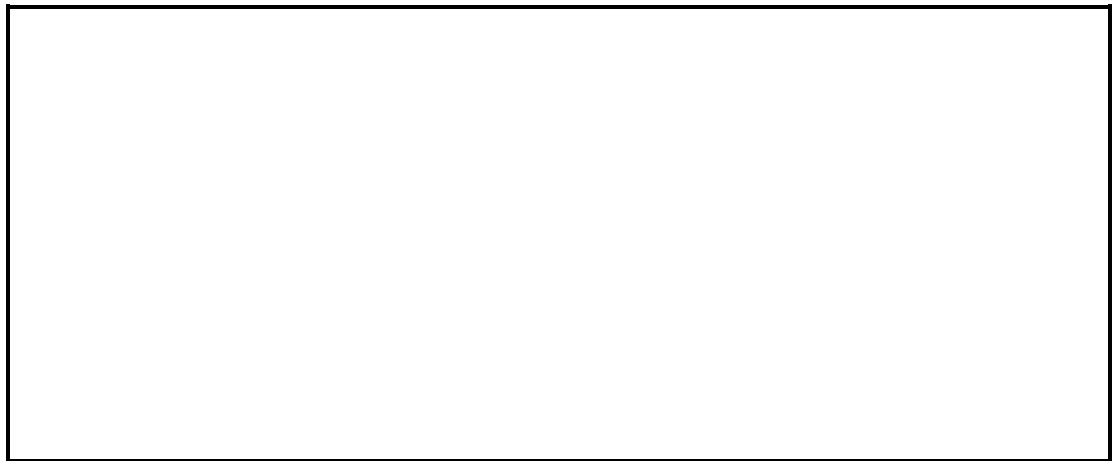
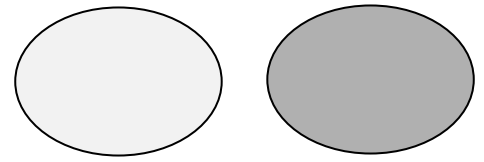


The percent of seagrass is calculated using method (b) above. For example:

$$\text{Seagrass/seabed} = 36/96 = 3/8 \text{ } 0.375 \text{ which is } 37.5\%$$

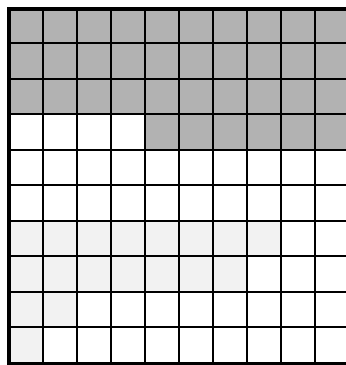
### ***Coverage Exercises***

Trace diagrams onto overlay (or 10x10 grid) and work out the percent coverage of seagrass on the seabed. Estimate first, look at the difference between the estimate and correct answer, and try to do a better estimate for the next example.

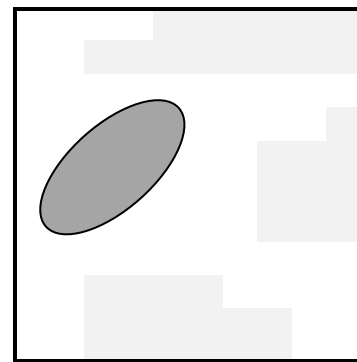


Overlay

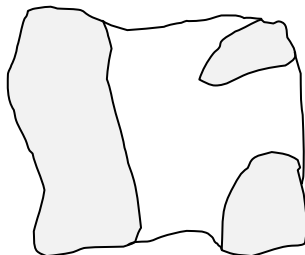
(1)



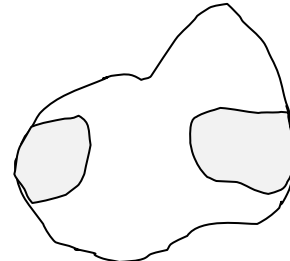
(2)



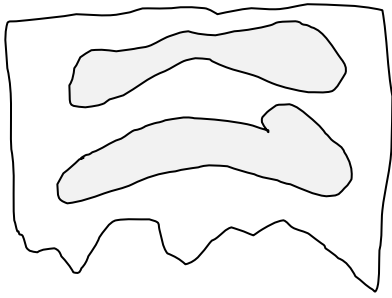
(3)



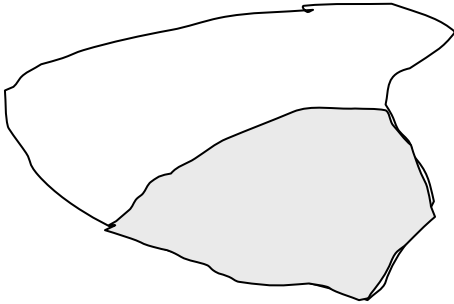
(4)



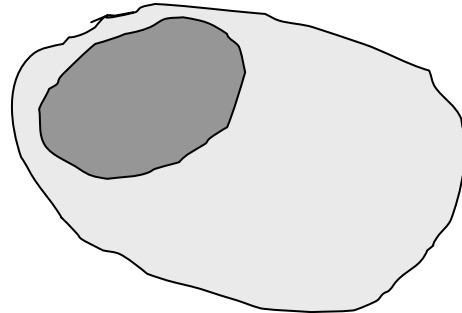
(5)



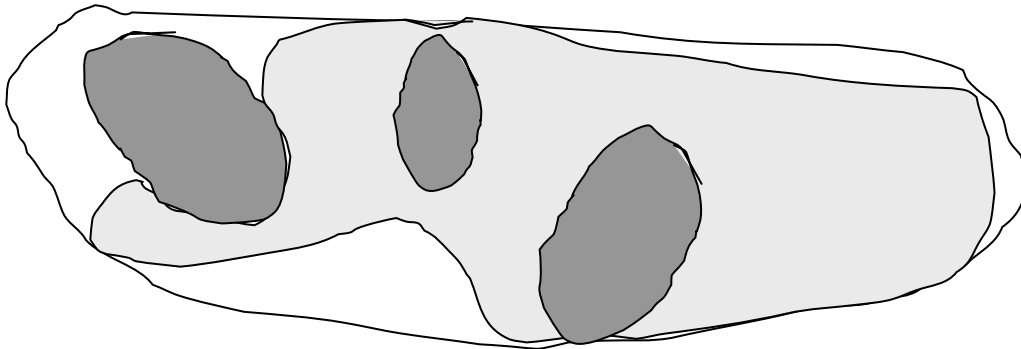
(6)



(7)



(8)



## 2.2 Density and coverage

Of course, seagrass being something that is eaten by fish has attributes other than coverage. One of these is density. How many plants in a given area? A low density coverage in a large area may be less effective in terms of fish food than a high density coverage of a smaller area.

### **Density**

Density can be worked out in more than one way. We will look at two of them.

#### (1) By inspection

The number of plants can be simply counted. One way to do this is to follow a line across the seagrass counting plants. This will give the number of plants per metre. This is more accurate if more than one count in more than one direction is made and over a reasonably long distance.

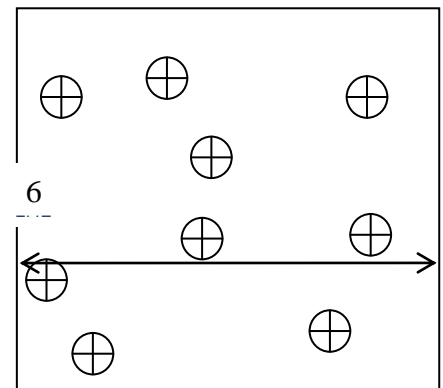
If there is an average of 3 plants per metre, then the average number of plants per square metre is  $3 \times 3$  or 9 plants as in the example on right.

If the best possible density is known (say this is 12 plants per square metre), then a percent density can be found as follows:

Density of plants is actual number of plants/m divided by maximum number of plants/m:

$$= 9/12 = 3/5 = 0.75$$

or 75%



#### (2) By photographs

Different density seagrass crops can give different colours on a picture. With experience, a density could be given from the picture. It may only be a general indication, that is, only 25%, 50%, 70% and 100% may be possible to ascertain.

### **Total coverage.**

To find the total coverage, we have to combine density and coverage. If the percent coverage of seagrass is 35% and the seagrass has a density of 75%, then the percent of seagrass available is 35% of 75% which can be determined two ways:

(a)  $35\% \times 75\% = 35/100 \times 75/100 = 2625/10000 = 26.25/100 = 26.25\%$

(b)  $0.35 \times 0.75 = 0.2625$  which is 26.25%


### **Density and coverage activities:**


#### (3) Calculate densities for the following inspection:


- a. Seagrass was 24 plants in 10 metres; greatest possible density of plants is 8.8 per meter?
- b. Seagrass was 36 plants in 20 metres; greatest possible density was 5.4 plants per metre<sup>2</sup>?


- c. Seagrass was 16 plants in 5 metres; greatest possible density was 14.2 plants per metre<sup>2</sup>?

- (4) Densities from photographs were:

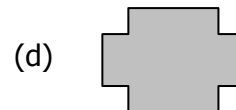
25% 

50% 

75% 

100% 

Estimate densities for these pictures



- (5) Calculate total coverage for the following:

- coverage 35% and density 50%
- coverage 62% and density 26%
- coverage 72% and density 75% for  $\frac{1}{4}$  and 50% for  $\frac{3}{4}$ .

- (6) Calculate total coverage for the following:

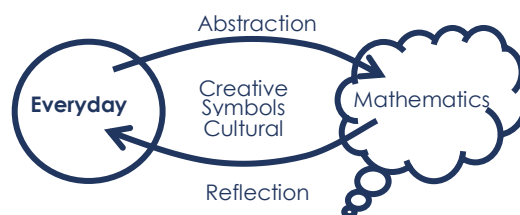
- 135 plants in 20 m, maximum coverage 200 plants/m<sup>2</sup>, coverage 86% of seabed
- 185 plants in 50m, maximum coverage 15.6 plants/m<sup>2</sup>, coverage 43% of seabed
- Seabed in 73% of area, two areas of seagrass - first area is 27% of area and is 42 plants for 10m and the second area is 24% of area and is 27 plants in 10m – maximum coverage is 26.8 plants/m<sup>2</sup>.



### 3. BOX MODEL

#### 3.1 Conceptualisation

The building of a box model follows the normal framework for modelling – a mathematical model is developed from real life, used to solve problems and then the solution is interpreted in the real world. It has two stages as follows. These stages will be described for a box model of a rainwater tank.



A box model is simple – it represents a situation with an input and output.

##### (1) Free discussion

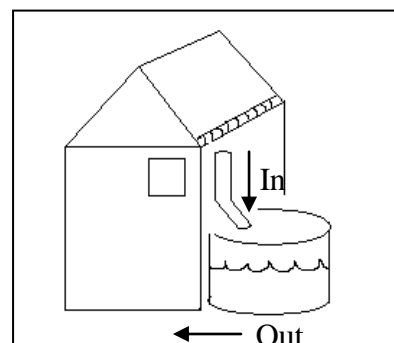
The development begins with free discussion. Students are asked to think about the story of the water tank and answer questions regarding the tank, for example, (a) what is the function of the water tank, (b) where does the water come from, (c) where does the water go, and (d) is the water used up in any way?



\*Teacher note: At this stage, we need not worry about the overflow part of the tank. If students raise the issue, reinforce that it is a good suggestion and we will come back to this later.

##### (2) Conceptual to mathematical model

Here the students conceive of the model by looking at components and developing symbols. The steps are as follows.

- The students free draw the water tank connected to a building (as on right).
- The drawing (and the students' conceptual understanding of the model) should clearly show the water flows into and out of the tank;
- The students create symbols for the input and output, i.e., the water flowing into the tank and the water flowing out. These symbols can be compound and reflect where the water comes from and where it goes, e.g.,



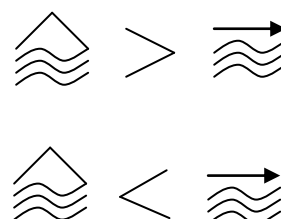
Input:  roof water; Output  leaving water

Note: The compound symbols, such as for roof water, are appropriate because they represent where the water comes from and goes to, but other symbols will do, even algebraic ones, if they have meaning to the students.

- Discuss with students the effect of input and output on the model (what changes?) and identify a name for this (water level)
- Discuss what happens to the water level in the tank, e.g.,

They use their symbols to show the water level going up

or the water level going down it could go down



- Discuss with the students whether it is important to know the water level over time (e.g., water conservation issues, economic and environment consideration).

### ***Conceptualisation activities***

Build a box model for each of the scenarios below. For each model, ensure you develop the following:

- a drawing of their model for the situation;
- relevant symbols for input and output (note – can have more than one type of input and output);
- the effect of changes in input and output the name of the result of this effect; and
- equations that show the changes in the the effect.


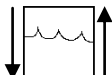
- (1) A lake which is filled by rain and emptied by evaporation.
- (2) A dam which is filled by rain and water pumped out of a bore and emptied by evaporation and irrigation of a crop.
- (3) A small country town – make up your own inputs and outputs.

## **3.2 Generalisation**

To use the box model to help understand reality, we have to understand how it allows us to understand what happens across time. This requires the model to be generalised and this uses the idea of iteration (cycles of regular activity) and patterns. Again we use the water tank as an example, and we follow these four stages.

### **(1) Time steps and relationships**

Discuss with the students what we are modelling until they focus on the crucial feature, that is, the effect of the inputs and outputs, the water level in the water tank. Introduce the idea of a starting point in time (the technical term for this is *initial condition*)'. Ask questions such as: What is the initial condition of the tank? Is it half full? Is it empty? And so on. Stress the need for a starting level in the tank. Go through these steps:

- Create a symbol for the initial water level in the tank, e.g., the symbols on right. 
- Introduce to the students the idea of a *time step* as a period of time across which we watch for change. For the tank, discuss possibilities, e.g., 1 day.
- Ask what happens after 1 day (or one time step), i.e., we will have a new water level.
- Create a symbol for the new water level at the end of the time step, e.g., as on right 

Discuss relationships within the model – look at how water levels on one day are affected by the levels on the previous day, the input from the roof and the output to the house. Discuss how we could “tell the story” of a day in the life of the tank using our created symbols and the symbols for equals, addition and subtraction. Use the following steps:


- construct it in words first, “the water level at the end of the day (or the start of the new day) is the water level at the start of the previous day plus inputs and subtract outputs”;
- write this in “shorthand”, e.g., new level = old level plus input subtract output;
- replace with symbols, writing down how we can calculate the new water level, e.g.,

$$\begin{array}{|c|} \hline \downarrow \uparrow \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} + \begin{array}{c} \triangle \\ \text{---} \\ \text{---} \\ \text{---} \end{array} - \begin{array}{c} \rightarrow \\ \text{---} \\ \text{---} \\ \text{---} \end{array} ; \text{ and}$$

- read these symbols and tell the story they represent.


## (2) Units of measurement


Discuss how we can make this model more real. Focus on measurement. Ask questions such as: How is water measured when it is in tanks? Then:


- guide the students in a discussion about volume and how it is measured;
- decide on a unit, e.g., litres, and discuss the need to maintain this unit throughout the model;
- discuss what would be a normal size tank for a school and how we could find out;
- discuss what would be the normal initial water level (  for a new tank and for an existing tank; and
- get the students to convert their decision on into the chosen measurement, e.g., litres.


Note that the initial level or the initial condition in a box model can be chosen by the students, e.g., ½ full, empty, ¼ full.

## (3) Calculation

Look at how water comes in (inputs) and goes out (outputs)? Start with inputs? How do we calculate the water contribution from the roof (  )? Guide the students through a discussion on rainwater measurements, surface area of the roof. Ask questions concerning:




- what units should we use, that is, the same as the water level; and
- and how we convert  into the chosen measurement, e.g., litres.

Discuss how we calculate the amount of water coming out from the tank (  ). Use the example of a known tank (e.g., a tank at a building that all/most students know):

- discuss water uses in of that tank;
- estimate a daily usage from the above discussion; and
- convert  into the chosen units, e.g., litres.

Calculate the new water level for the chosen time step, one day – remember the formula:

$$\begin{array}{|c|} \hline \downarrow \uparrow \\ \hline \end{array} = \begin{array}{|c|} \hline \\ \hline \end{array} + \begin{array}{c} \triangle \\ \text{---} \\ \text{---} \\ \text{---} \end{array} - \begin{array}{c} \rightarrow \\ \text{---} \\ \text{---} \\ \text{---} \end{array}$$

- put your calculations for ,  and  into the equation;
- discuss what happens; and
- explain why the water level goes up (or goes down) and by how much.

#### (4) Iteration and generalisation

We know look at how we can go beyond one step. However, each time step/day has its own equation. Writing the equation twice could be confusing. So we need a way to differentiate between the days? This is how it can be done:

- get students to look at a timeline of three days and consider how we can differentiate between the symbols for day 1, day 2 and day 3;



3

- encourage students to consider subscripts, for example, give everything between 0 and 1 in day 1 a subscript 0, everything between 1 and 2 in day 2 a subscript 1, and everything between 1 and 2 in day 2 a subscript 1 (and so on), and look again at the timeline (Note – giving the first day 0 is a convention of modelling); and



3

- use the formula to calculate the end of day water level for each day (do the first two for the students and then ask them to complete the remaining).

$$\text{day 1 } \downarrow \uparrow_0 = \text{water level}_0 + \text{rainfall}_0 - \text{water usage}_0$$

$$\text{day 2 } \downarrow \uparrow_1 = \text{water level}_1 + \text{rainfall}_1 - \text{water usage}_1$$

$$\text{day 3 } \downarrow \uparrow_2 =$$



$$\text{day 4 } \downarrow \uparrow_3 =$$

$$\text{day 5 } \downarrow \uparrow_4 =$$

Discuss what changed for each of the days and why, for example:

- will be different because rainfall is not the same each day;
- will be different because water usage is usually different for each day;
- will be different because water level will be different; and
- will be different because the final water level will be different because it is based on the other three things.

Discuss what is the same between the days, looking particularly at and . Discuss how and are related. Argue how the water level at the end of day 1 is the same as the water level at the beginning of day 2. Therefore = .

Replace  at the end of each day with  at the start of the next day, as is shown in the timeline:



Redo the calculation for each day (once again, do the first two and get the students to complete the next three):

$$\text{day 1} \quad \boxed{\text{wavy line}}_1 = \boxed{\text{wavy line}}_0 + \begin{array}{c} \triangle \\ \text{wavy line} \end{array}_0 - \begin{array}{c} \rightarrow \\ \text{wavy line} \end{array}_0$$

$$\text{day 2} \quad \boxed{\text{wavy line}}_2 = \boxed{\text{wavy line}}_1 + \begin{array}{c} \triangle \\ \text{wavy line} \end{array}_1 - \begin{array}{c} \rightarrow \\ \text{wavy line} \end{array}_1$$

$$\text{day 3} \quad \boxed{\text{wavy line}}_3 =$$

$$\text{day 4} \quad \boxed{\text{wavy line}}_4 =$$

$$\text{day 5} \quad = \boxed{\text{wavy line}}_5$$

Discuss generalising the "rule" for the calculation. Do this in three steps.

- specific examples, e.g., work out the calculation for day 7, day 11, day 45 and day 173;

$$\text{day 7} \quad \boxed{\text{wavy line}}_7 = \boxed{\text{wavy line}}_6 + \begin{array}{c} \triangle \\ \text{wavy line} \end{array}_6 - \begin{array}{c} \rightarrow \\ \text{wavy line} \end{array}_6$$

$$\text{day 11} \quad \boxed{\text{wavy line}}_{11} =$$

$$\text{day 45} \quad \boxed{\text{wavy line}}_{45} =$$

$$\text{day 173} \quad \boxed{\text{wavy line}}_{173} =$$

- language, state the calculation in words for any day; and
- variable, determine the calculation when n is let stand for any day

$$\text{day } n \quad \boxed{\text{wavy line}}_n =$$

Note: The problem in generalising to n is that we have to work out how to write 1 less than n. which is (n-1).

### **Generalising activities**

- (1) Complete all the unfinished sections of the above activities.

- (2) Develop the generalisation for the three situations from the Conceptualisation activities, namely, the lake situation, the dam situation, and the small country town situation:
- determining time steps and relationships appropriate for each situation;
  - determining appropriate units of measurement (if relevant) for each situation;
  - undertake the necessary calculations for each situation; and
  - set up the iterations and subscripts and determine the generalisation for each situation.

## 4. MODELLING ENVIRONMENTS

### 4.1 Computer models

To model real environments requires the use of computers, notably spreadsheets, to follow the iterations and translate the formulae into reality (and numbers). Again this will be explored using the example of the water tank. Three stages are provided.

#### (1) Initial days

Suggest to the students that you wish to model the water level in the tank for five days. Discuss what has to be worked out across the 5 days as follows.

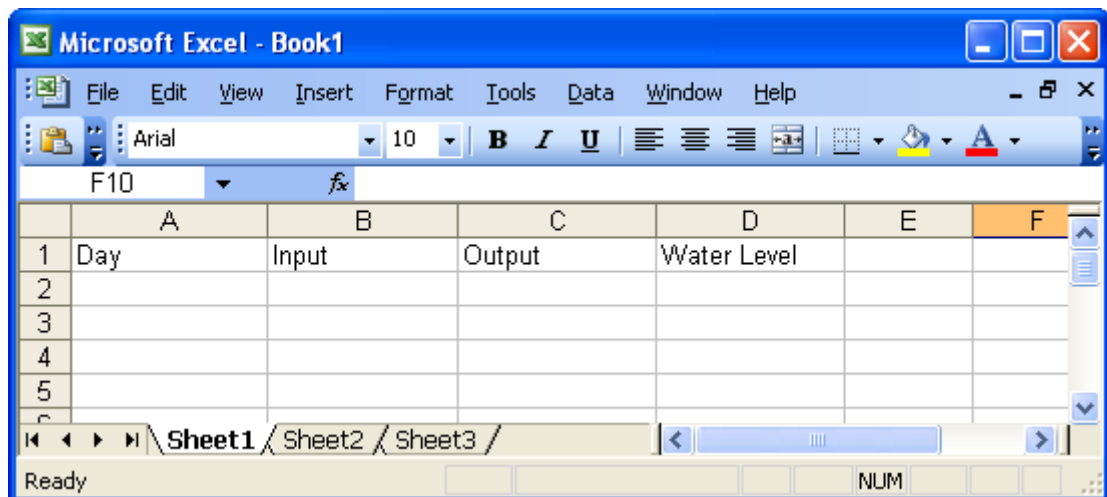
- use data from the end of step 3 to work out values for 5 days –  $\square_0$ ,  $\triangle_0$  &  $\rightarrow_0$  for day 1;  $\triangle_1$  &  $\rightarrow_1$  for day 2;  $\triangle_2$  &  $\rightarrow_2$  for day 3; and so on for 5 days;
- with this information, water levels can be calculated at the end of each day, e.g.,

$$\begin{aligned} \text{day 1 } \square_1 &= \square_0 + \triangle_0 - \rightarrow_0 \\ \text{day 2 } \square_2 &= \square_1 + \triangle_1 - \rightarrow_1 \quad \text{and so on for 5 days;} \end{aligned}$$

- using pen and paper (and a calculator), calculate water levels at the end of the first 5 days by putting in values and calculating  $\square_1$  to  $\square_5$ .

#### (2) Computers

Discuss how we could do the calculations for 20 or 30 days. Discuss implementing the model in Excel. Start again with 5 days. Construct a table in Excel (see below), starting in cell "A<sub>1</sub>", i.e., column A, row 1, and using the following headings:



Column A represents our day counter - the same as the subscript in the equations. So start your counter from zero in column A and end on five (look back to your equations to understand why).

To build the excel worksheet, reflect back on what else is required. Encourage students to see that they need to put in the starting point (the 'initial condition'). Consider the equations and generate a discussion re where the Initial Condition could be put (and why it should be put there).

Include the initial condition in column D, row 2 (under water level), i.e., D<sub>2</sub> cell. Also place the words "Initial condition" in the cell E<sub>2</sub> to remind us what this number represents.

Add in the information available in terms of Input and Output. Then, your final spreadsheet should look like below.

	A	B	C	D	E
1	Day	Input	Output	Water Level	
2	0	10	5	19	Initial Condition
3	1	2	8		
4	2	5	11		
5	3	7	8		
6	4	0	3		
7	5				
8					

### (3) Excel calculation

The next step is to set up Excel so that it will calculate the water level at the end of day 1 – this calculation will be placed in cell D<sub>3</sub>. To do this:

- look back at the equations for water level – consider the one that calculates;

$$\text{day 1} \quad \boxed{\text{Lake}}_1 = \boxed{\text{Lake}}_0 + \text{Input}_0 - \text{Output}_0$$

- Identify which cells relate to these symbols, e.g.:

$\boxed{\text{Lake}}_0$  relates to cell D2 (Initial condition);

$\text{Input}_0$  relates to cell B2; and

$\text{Output}_0$  relates to cell C2;

(note that these symbols have zero subscripts and that their day counter is also zero);

- write the above equation in terms of cell numbers, i.e., D3 = D2 + B2 – C2
- enter the right-hand side of the equation into the cell D3 for Excel (to do this move your cursor to D3 and type the following: = D2 + B2 – C2); and
- hit the enter button and check your answer against your hand calculation.

### Computer models activities

Set up an excel spreadsheet for the three situations from Section 4.1:

- (1) A lake which is filled by rain and emptied by evaporation.
- (2) A dam which is filled by rain and water pumped out of a bore and emptied by evaporation and irrigation of a crop.



- (3) A small country town – make up your own inputs and outputs.

## 4.2 Validation

The last step in the computer model is to set up Excel so that it can undertake all calculations at once. These calculations will validate the model as well as being an application of the model. There are three stages.

### (1) Pattern

The first stage is to recognise the pattern in the Excel cell equations:

- look at days 1 and 2, the time steps to the end of days 1 and 2, and remember that the equations are:

$$\begin{array}{lcl} \text{day 1} & \boxed{\text{wavy}}_1 & = \boxed{\text{wavy}}_0 + \begin{array}{c} \triangle \\ \text{wavy} \end{array}_0 - \begin{array}{c} \rightarrow \\ \text{wavy} \end{array}_0 \\ \text{day 2} & \boxed{\text{wavy}}_2 & = \boxed{\text{wavy}}_1 + \begin{array}{c} \triangle \\ \text{wavy} \end{array}_1 - \begin{array}{c} \rightarrow \\ \text{wavy} \end{array}_1 \end{array}$$

- identify which cells relate to these symbols and put the cell equation as follows:

$$\text{day 1} \quad D4 = +D3 + B3 - C3 ; \text{ and}$$

- reflect back to the equation and point out the iterative pattern:

$$\text{day 2} \quad D5 = +D4 + B4 - C4 ; \text{ and}$$

$$\text{day 3} \quad D6 = +D5 + B5 - C5 .$$

### (2) Copy and paste

The next stage is to complete the days that are required by copying and pasting. Once you have a certain equation structure, excel can be set up to repeat that pattern. Therefore, you can cut and paste the equation for day 1 into the cell for day 2 (i.e., D5) and the equation should have the right structure. This is how it works:

- move your cursor to cell D4, click *copy* in the Edit menu, move your cursor to cell D5, and click *paste* in the Edit menu.
- check the cell equation in D5 to ensure that the cell has  $+D4 + B4 - C3$ ; and
- continue cutting and pasting down the column.

Note: A copy of the cell equation can be pasted into many cells at once by using *paste special* from the Edit menu. Discuss how this is done.

### (3) Model reality

Look at the results from the Excel calculations. Try different Initial Conditions and different inputs and outputs. For example, what would happen if inputs and outputs were always the same? Do the results make sense?. Try some other scenarios, e.g., start with a full tank, always take out more than put in. What happens here? Does this new scenario make sense.

If the Excel tank acts like a real tank, i.e., the scenarios make sense, there is a good chance that the model is working.

**Validation and application activities**

- (1) Run various scenarios for the water tank using Excel and check for sensibleness.
- (2) Run various scenarios using Excel for the three Section 4.1 situations:
  - A lake which is filled by rain and emptied by evaporation.
  - A dam which is filled by rain and water pumped out of a bore and emptied by evaporation and irrigation of a crop.
  - A small country town – make up your own inputs and outputs.
- (3) Use the box model and excel to run a model of a marine environment that is relevant for your context. For example, you could look at fish or animal stocks, pollution, and so on.

**Special Seagrass activity**

- (1) Build a model for seagrass – the amount of seagrass is the “water level”
  - the input is things that increase the amount of seagrass (e.g., things that stimulate growth of existing seagrass and/or spread of seagrass to new plants, decrease in fish that eat seagrass, good growing conditions, etc.), and
  - the output is things that decrease the amount of seagrass (e.g., increase in fish eating the grass, condition that cause grass to die such as pollution, etc.).
- (2) Transfer the mathematics of the model to an Excel spreadsheet.
- (3) Adjust inputs and outputs and run the spreadsheet to get the same results as are happening in the real world.
- (4) Discuss reasons for seagrass changes.