

YUMI DEADLY CENTRE School of Curriculum

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Marine Studies: Coxswain Certificate Mathematics behind Handling Small Boats and Ships

Booklet VM1: Angle, Distance, Direction, and Navigation



YuMi Deadly Maths Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at <u>http://ydc.qut.edu.au</u> and staff can be contacted at <u>ydc@qut.edu.au</u>.

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CRICOS No. 00213J

This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

Queensland University of Technology

DEADLY MATHS VET

Marine Studies: Coxswain Certificate MATHEMATICS BEHIND HANDLING SMALL BOATS AND SHIPS

BOOKLET VM1 ANGLE, DISTANCE, DIRECTION, AND NAVIGATION 08/05/09

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THIS BOOKLET

This booklet (Booklet VM1) was the first booklet produced as material to support Indigenous students completing certificates associated with marine studies/coxswain certificate at the Thursday Island campus of Tagai Secondary College. It may also be applicable to the Coxswain Certificate at Queensland TAFEs, most notably at the Tropical North Queensland TAFE campus at Thursday Island. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

- VB1: Mathematics behind whole-number place value and operations Booklet 1: Using bundling sticks, MAB and money
- VB2: Mathematics behind whole-number numeration and operations Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure
- VC1: Mathematics behind dome constructions using Earthbags Booklet 1: Circles, area, volume and domes
- VC2: Mathematics behind dome constructions using Earthbags Booklet 2: Rate, ratio, speed and mixes
- VC3: Mathematics behind construction in Horticulture Booklet 3: Angle, area, shape and optimisation
- VE1: Mathematics behind small engine repair and maintenance Booklet 1: Number systems, metric and Imperial units, and formulae
- VE2: Mathematics behind small engine repair and maintenance Booklet 2: Rate, ratio, time, fuel, gearing and compression
- VE3: Mathematics behind metal fabrication Booklet 3: Division, angle, shape, formulae and optimisation
- VM1: Mathematics behind handling small boats/ships Booklet 1: Angle, distance, direction and navigation
- VM2: Mathematics behind handling small boats/ships Booklet 2: Rate, ratio, speed, fuel and tides
- VM3: Mathematics behind modelling marine environments Booklet 3: Percentage, coverage and box models
- VR1: Mathematics behind handling money Booklet 1: Whole-number and decimal numeration, operations and computation

CONTENTS

Page

OV	ERVIEW	1
1.	ANGLE AND DIRECTION	
2.	 USING COMPASSES ON BOATS/SHIPS 2.1 Taking account of magnetic variation/boat deviation 2.2 Working out compass bearings 	
3.	DISTANCE AND SPEED3.1 Base 60 Numbers3.2 Position, Distance and Speed	

OVERVIEW

This Booklet

The Coxswain certificate is an important vocational course for Indigenous students who live on the coast or on Islands (e.g. Torres Strait Islands). In their situations, boats are widely used and often are the main form of transportation. Coxswain Certification is often necessary if the boats are large or used for commercial activity.

Mathematically, there are two particular difficulties in the Coxswain Certificate:

- (1) <u>Navigation</u>: This is based on position, direction and distance that are both based on angle (in degrees, minutes and 10ths of minutes). It requires understanding compass bearings in practice on boats and in planning on maps. It also requires changing the course to take into account magnetic variation of the location and further changing this to take account of boat variations.
- (2) <u>Travel and Tides:</u> Rate and ratio are important in planning travel. The amount of fuel needed is determined by speed (knots which are nautical miles/hour) and consumption (litres per hour), both of which are rates. The time at which a boat can enter and leave a mooring is determined by relationships between tides, time and draughts of boats, all of which are based on ratio/proportion.

Thus, this book VM1 is based on navigation and the next, VM2, is based on travel and tides. The focus of this booklet is on the following mathematics:

- (i) the notion of angle and the base 60 system on which it is based;
- (ii) how angle is used for position and direction;
- (iii) how to take account of magnetic variations and boat deviations; and developing nautical mile in relation to angle and the circumference of Earth at its equator;
- (iv) developing nautical mile in relation to angle and the circumference of the earth at the equator; and
- (v) utilising Geographic Positioning Systems (GPS) to find your way.

Mathematics behind navigation

(1) Navigation is based on *angle* which is based on *turn*. To turn is to change from one direction to a second, and angle is the amount of turn, e.g.,



- (2) Angle was developed by the Babylonians who believed that the numbers 6 and 60 were of great religious significance. They were impressed that six equilateral triangles fitted together to show a complete turn. They gave each triangle 60 degrees which made the full turn 360 degrees.
- (3) Angle was used to determine direction and elevation. With regard to direction, a complete turn of 360 degrees was divided into 4 quarters (at 90, 180, 270 and 360 or 0 degrees) which were designated North (N), East (E), South (S) and West (W) as on right.



(4) If wish to travel from A to B, a line can be drawn on a map between A and B.

If the map has North marked, a protractor can be used to determine the angle between the line from A to B and the North direction. This becomes the <u>true</u> direction.

- (5) A compass shows the angle to magnetic north. Because magnetic and true north are different, there is a <u>variation</u> of a few degrees between magnetic directions and true directions which depends on the part of the world you are in. If the variation is to the East, it has to be subtracted from the true North to give the magnetic direction that will get you from A to B. If the variation is West, it has to be added.
- (6) All boats have a magnetic field. This affects the compass. Therefore, each boat effects a <u>deviation</u> on its compass. Once again, to determine the direction on the compass that will get from A to B, the deviation is subtracted if it is East and added if it is West.
- (7) This subtracting/adding of variations and deviations onto the true north bearing found from the map will give the magnetic compass bearing on the boat's compass that will enable the boat to reach the map reference for which the original bearing was found. In other words, following the bearing from the map will not get you to your destination. Variation and deviation have to be accounted for before you can get the boat compass will get you to your destination.
- (8) The Babylonians believed that the number 60 was so important that they based their number system on it.

 Therefore a Babylonian
 4
 35
 12
 was:

 12 ones + 35 sixes + 4
 60x60s or
 4x3600
 14400

 35x60
 2100
 2100

 12x1
 $\frac{12}{16512}$ 12

(9) The Babylonians also used 60 for fractions. So their number system was

3600s	60s	1s	1/60ths	1/3600ths

They partitioned ones into 60 pieces called minutes and then further partitioned the 60ths into 60 pieces called seconds. Minutes were denoted by a single inverted comma and seconds by an ordinary double inverted comma. This means 14° 16' 35" is 14 plus 16/60 plus 35/3600. **Note:** The Babylonians also developed time (hours, minutes, seconds) also based on 60.

(10) Position on the earth is determined by angle from the cent the earth as a sphere). Angle up and down gives latitude v longiitude. In practice latitude and longitude are given in te and tenths of minutes (seconds were considered too small) position is given by numbers that represent two bases (60t and 10ths for minutes ←→ tenths of minutes). The equato and down 90 degrees (from 90°N to 90°S). Greenwich me It goes from 180°E to 180°W.



(11) Distance is measured in terms of the length that is one minute of angle at the equator. This is called a <u>nautical mile</u> (nm is its notation). Speed is measured in nautical miles per hour and one nm/h is called a <u>knot</u>.

Pedagogy

- (1) Place all mathematics teaching possible within the context of coxswain activity and within the context of the students (Island living and Torres Strait Islander culture).
- (2) Focus mathematics teaching on mathematics as a whole structure, its pattern and interactions, using the learning style advocated by Uncle Ernie Grant. This means that mathematics teaching needs to focus on understanding how mathematics is related, sequenced and integrated, that is, on the connections between mathematics ideas.
- (3) Where possible begin all teaching using whole body of students (kinaesthetic) then move to using materials, computers and pictures (modelling) and then to having the student think of a "picture in the mind" (imagining), that is, use a variety of representations that follow this sequence:



Kinaesthetic → Modelling → Imagining

- (4) Relate real world situations (within the context of the students) to models of the mathematics and then to language and finally to symbols (the Payne-Rathmell triangle as on right)
- (5) Use the generic pedagogies of generalising, reversing and being flexible, that is, ensuring that the most general understanding possible is developed, that activities move in

both directions (model to symbol and symbol to model), and that students have flexible understandings of concepts and processes (including language).

1. ANGLE AND DIRECTION

1.1 Turn and angle

(1) *Angle as turn.* Stand and pick two directions. Place arms together in front of body and aim at the first direction. Turn body (holding arms in same position in front of body) to second position.





Relate the change from one direction to another to the drawing of angle – stress that angle is the turn between the two directions.

(2) *Comparing angles.* Build an *angle wheel* out of a square of plastic with a circle drawn on it and a radius line and a circle of plastic with a radius line as on right. Pin the two pieces together through the centre of both circles so that the circle will turn on top of the square (and as it turns, the two radius lines will turn apart showing an angle).

Use the wheel to compare size of angles (the bigger angle has the most turn) as below – turn the wheel so that it equals the first angle, then move wheel to second angle and check whether the wheel has to be turned more (first angle is smaller) or less (first angle is larger)?



- (3) Informal units. Apply number to angle (and introduce the notion of unit) using the angle wheel by marking steps along the circle drawn on the square. This can then be placed over an angle to measure the angle in terms of the number of steps between the two lines as on right (measure = 2 units). Measure angles with this angle measurer and compare angles in terms of number of units.
- (4) *Formal units (degrees)*. Introduce the formal units of *degrees*. Discuss the history of how the units were chosen (6 equilateral triangles, each with 60 degrees). Discuss how small the degrees are. Show and discuss the protractor. Measure and compare angles with a protractor. Include angles greater than 180 degrees.

1.2 Compass bearings

(1) *North, East, South, West*. Take a piece of paper, tear off the corners, and fold the paper into "quarters" as on right. Put two of these angles together and show they

make a straight line. State that the angle which is half a straight line has a special name (*right angle*). Show on a protractor that the right angle is 90 degrees while the straight line angle is 180 degrees. Find things in the everyday world that are right angles (e.g. tables, doors, windows, etc.).



Open paper and place on floor. Stand along one of the folds and place hands out in front, and turn a quarter turn clockwise four times, as turning placing feet along the next fold line. Λ



Discuss how these 4 quarter turns are used in compass bearings – say they represent North, East, South and West. Repeat this but saying "North, East, South and West" as you turn.

(2) Compass bearings. Obtain compasses and discuss how presented. Note the 360 degrees but with North as 0 degrees, East as 90 degrees, South as 180 degrees and West as 270 degrees. Note that angle is measured in the opposite direction to a protractor – compass is from North to East (clockwise) where a protractor is to West (anti-clockwise). Repeat the paper turns clockwise starting from North and saying as turn, "North 0 degrees, East 90 degrees, South 180 degrees, and West 270 degrees". Compare this with protractor where turn anti-clockwise (actually get students to turn bodies).

Discuss that how can have different ways to give a bearing, e.g., 350 degrees, which means from North to East clockwise, can be also stated as 10 degrees West. Thus with small angles from North, it is common to say 15 degrees East or 20 degrees West.

Look at different directions (e.g., 35 degrees, 145 degrees, 205 degrees, 35 degrees West and so on) and work out where they are on the compass and where this is with respect to North, East, South and West.

(3) Finding and following bearings. Go outside and show how to find a bearing to something in the distance and how to find where a bearing is pointing: (a) point the compass at a distant tree, dial in North and read the direction off the compass (e.g. 125 degrees); and (b) dial in a direction, align North and see where the compass is pointing.

Do some small scale orienteering and other compass bearing activities with students to find and follow compass bearings. For example:

- (i) <u>Find Dollar</u>: Put a dollar on the ground. Using a compass, walk forward 10 paces, turn 120°, take another ten paces, turn 120°, and take another ten paces. Now look down, can you see your dollar?
- (ii) <u>Solve this Riddle:</u> Get a starting point. Set up directions and distances from start to hiding places for words. Find a riddle and use words as

'things to find'. Once students find all words they can re-form the riddle and solve it. (Note: For distance, get students to find how many of their paces is 10m and give distances in metres.)

- (iii) <u>Follow the track:</u> Set up a track from point to point so that at each point, students are given, or have to find, directions and distances to the next point. This is actually small distance orienteering.
- (4) Determining true bearings from a map. Start with a map. Propose a trip (from A to B). Join A to B with an arrow. Find direction North on the map and draw an arrow with this direction from A. Measure angle clockwise at A from North direction to B direction. This is called the <u>true</u> bearing.

If a map has a rose (a drawing of a compass) use parallel lines to relate direction A to B to the rose (as on right). The parallel line has to go through the centre of the rose. Then the angle can be read off the rose. Parallel rulers as below can assist here.







2. USING COMPASSES ON BOATS/SHIPS

2.1 Taking account of magnetic variation/boat deviation

- (1) *Effect on direction*. Make North vertically upward. Draw an arrow from a point in North direction. Draw two further arrows from that point which are 10 degrees W and 15 degrees E from North. Discuss the difference in direction. Discuss what effect this will have on compass bearings?
- (2) Effect on travel. Draw an arrow from a point in another direction. Use a protractor to measure the true compass bearing. Obtain a map of region and get the magnetic variation for the area. Choose a reasonably long trip between two points on the map, say C and D. Join C to D with an arrow. Construct another arrow from C which is different by the magnetic variation. Do we miss D? What would happen if the magnetic variation halved? Doubled? Or was in the opposite direction?

Discuss how the bearing would have to change so that the magnetic compass was to take you in the true direction?

(3) *Taking account of magnetic variation.* Consider that the true direction (from the map) is 70 degrees. Assume that the magnetic variation is 10 degrees E. To understand how we have to change the magnetic direction, make up the following teaching aid. Construct two cardboard compasses with Magnetic able to sit inside the True with a pin through the centre so that the two compasses can turn independently. A pointer is added to show direction.



Now if the true direction we have to navigate is 70 degrees, we can show this on the joint compasses as in (A). A magnetic variation of 10 degrees E would mean that North for the magnetic points at 10 degrees E true. We turn the magnetic compass on top of the true compass so N on magnetic is 10 on true as in (B). The magnetic direction is now 60 degrees; thus the 10 degrees has to be subtracted as it is East (or clockwise). Repeating this for other examples will show that <u>East variations always have to be subtracted</u>.



This process with East variations can be repeated for West variations, e.g., true direction 120 degrees and variation 15 degrees W. This will show that, for this example, the magnetic direction is 135 and, for many examples, that <u>West variations</u> <u>always have to be added</u>.

(4) *Taking account of boat variation.* In the same way true and magnetic compasses can be integrated, magnetic and boat compasses can be put together to help understand what happens with boat variation. Construct two cardboard compasses with Boat able to sit inside Magnetic with a pin through the centre so that the two compasses can turn independently. A pointer is again added to show direction.

If the magnetic direction we have to navigate is 150 degrees, we can show this on joint compasses as in (X). A deviation of 20°W would mean that North for the boat compass points at 20°W magnetic. We turn the boat compass on top of the magnetic compass so N on boat is 20°W (or 340 degrees) on magnetic as in (Y). The boat direction is now 170 degrees, thus the 20°W has been added. Repeating this for other examples (including degrees W) will show the same law for magnetic-boat as for true-magnetic, that <u>East/clockwise deviations always have to be</u> <u>subtracted and West/anti-clockwise deviations</u> <u>always have to be added</u>.



2.2 Working out compass bearings

- (1) When faced with the problem of working out the boat compass direction to use when magnetic variation and boat deviation are known, we have to follow the steps below:
 - <u>Step 1</u> Use a protractor on the map to work out true direction.
 - <u>Step 2</u> Change true to magnetic direction by subtracting/adding magnetic variation depending on whether it is E/W respectively.
 - <u>Step 3</u> Change magnetic to boat direction by subtracting/adding boat deviation depending on whether it is E/W respectively.
- (2) These three steps can be understood by building a three part compass model. Construct three cardboard compasses with Boat able to sit inside Magnetic which in turn sits inside True with a pin through the centre so that the three compasses can turn independently. A pointer is again added to show direction (as on right). The model on the right shows that the outer true compass has a direction of 150 degrees, the magnetic which has a 10 degrees E variation is 140, while the boat which has a 20 degrees W deviation is 160 degrees, that is, boat direction is 150 - 10 + 20 = 160 degrees.
- (3) Magnetic variation is fixed for a position but boat deviation depends on the boat and what it carries. It is often necessary to check the deviation by "swinging the compass". This is done by aligning the boat with two Islands or other outcrops for which the map gives accurate direction and then checking the amount the boat compass varies from this true reading taking into account magnetic variation. The ship may have to be turned in different directions as these may give different deviations.



3. DISTANCE AND SPEED

3.1 Base 60 Numbers

(1) *Number Systems*- Number systems are built around the one. Ones are grouped into larger numbers according to the base are partitioned into equal parts to make small numbers by the same base. Construct the following cards:



Give these to students and get the other students to organise them into the right order left to right. Give another student the card:



Ask this student to move left and right along the place value positions, asking the class what the number becomes. Give each student a slide rule- see below.



The slide rule slips in front of the place value chart. Thus, the 6 in the slide rule can be slid along the place values as the student with the 6 moves along front of the students with the place values. Students can record the numbers on a calculator. This requires introducing the decimal after the ones and introducing the zero as a place holder. More numbers can be added – extra students with extra cards and different slides. Students can be asked to multiply and divide by 10, 100, etc. on their calculators as the numbers move/slide. This is to ensure students see the number system as a structure as follows



(2) *Base 60 systems*- Repeat (1) above for base 60 (cards, slide rules, calculators)



Introduce new notation (where ° is degrees, ' is minutes, and " is seconds).



Discuss some examples:

3 28 41° 14′ 38″

Degrees would be $3x3600 + 28x60 + 41 = 12521^{\circ}$. Minutes would be 14/60 and seconds 38/3600. Relate this to time.

(3) Mix up the systems- Discuss what happens if we only we only use

Hundreds	Tens	Ones	Minutes (60ths)	Tenths of Minutes	Hundredths of Minutes

as our system. Look at example: 43° 16.56′ This would be 43 degrees, 16/60ths, 5/10 of a 60th, and 6/100 of a 60th. The system would operate as follows:



The decimal point would be after the minutes. Look at maps so students can see how it is read.

3.2 Position, Distance and Speed

(1) Angle in Spheres- Look at a sphere as on right.



Cut it vertically as for A. The angles P and V gives the distance North and South from the equator.



Now cut it horizontally as for B. The angles n and s gives the distance East and West from a starting point (which is a line through Greenwich).



Put the two together as for C. Position d is angle x East from Greenwich and angle y North from the equator.



Discuss this with students. How big can angle x be? How big can angle y be? How could we make the sphere to show position. Does there have to be another line somewhere like Greenwich? Investigate how the sun and time/date are needed to find latitude and longitude?

- (2) *Latitude and longitude-* Show on a sphere of the earth how it can be broken into horizontal hoops and verticals lines to show latitude and longitude. Discuss how this translates to a map. Discuss how since in reality it is round and the map is flat, this leads to straight lines for latitude and curved lines for longitude.
- (3) Nautical Miles- Distance on water is determines by angle. The earth is cut (imagining) at equator as below (A). An angle of one minute (remember 1'=1/60°) is made at the centre. This is really a small angle, but by the time it gets to the equator it is reasonably wide. This is the unit 1' of latitude at the equator or 1' of longitude anywhere as below (B). It is called a nautical mile (or nm). It is about 1852m as surface of the earth. Investigate:
 - a) What angle gives 60 nautical miles?
 - b) How many nm for all the way around the earth?
 - c) How many Km's?





(4) *Knots*- Speed is therefore in nautical miles per hour. These are called knots – i.e., 1 knot is one minute of angle each hour. Investigate:

a) How many knots to get around the world in 80 days?

b) How many knots to get around the world in 1 day?

c) How many Km/Hr in 10 knots? How many knots is 10Km/Hr?

d) Why do planes have to go faster than boats to cover the same distance in knots?

Think/imagine the plane and boat going around the world. Act this out on a globe.

