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Minjerribah Maths Project

MAST Additive-Unknown Lessons

Booklet S.3: creating, using and solving unknowns in Addition and Subtraction stories

Minjerribah Maths Project
Secondary Booklet 3

MAST ADDITIVE-UNKNOWN LESSONS

Maths as Story Telling lessons on creating, using and solving unknowns in Addition and Subtraction stories

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YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

MINJERRIBAH MATHS PROJECT

MAST

MAST ADDITIVE-UNKNOWNS LESSONS

BOOKLET S.3

CREATING, USING AND SOLVING UNKNOWNS IN ADDITION AND SUBTRACTION STORIES

VERSION 1: 2007

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BACKGROUND

Minjerribah Maths Project

The Minjerribah Maths Project was a research study to find effective ways to teach Indigenous students mathematics, and for Indigenous students to learn mathematics. It realised that effective mathematics teaching is crucial for Indigenous students' futures as mathematics performance can determine employment and life chances. It endeavoured to find ways that will encourage and enable more Indigenous students to undertake mathematics subjects past Year 10 that lead to mathematics-based jobs.

Some educators argue that Indigenous students learn mathematics best through concrete "hands-on" tasks, others by visual and spatial methods rather than verbal, and still others by observation and non-verbal communication. However, these findings may be an artefact of Indigenous students being taught in Standard Australian English with which they may not have the words to describe many mathematical ideas and the words they have may be ambiguous. It is important to recognise that Indigenous students come from a diverse social and cultural background and investigations into Indigenous education should take this into consideration. Indigenous people also have common experiences, which can be reflected upon to suggest ways forward.

There is evidence that school programs can dramatically improve Indigenous learning outcomes if they reinforce pride in Indigenous identity and culture, encourage attendance, highlight the capacity of Indigenous students to succeed in mathematics, challenge and expect students to perform, provide a relevant educational context in which there is Indigenous leadership, and contextualising instruction into Indigenous culture. However, the majority of teachers of Indigenous students are non-Indigenous with little understanding of Indigenous culture and these non-Indigenous teachers can have difficulties with contextualisation and reject it in favour of familiar Eurocentric approaches. Thus, there is a need to build productive partnerships between non-Indigenous teachers and the Indigenous teacher assistants employed from the community to assist them and the Indigenous community itself.

There is also some evidence that Indigenous students tend to be holistic, learners, a learning style that appreciates overviews of subjects and conscious linking of ideas and should appreciate algebraic structure. Thus, algebra could be the basis for Indigenous mathematics learning. This position is interesting because algebra is the basis of many high status professions. It is also based on generalising pattern and structure, skills with which Indigenous students may have an affinity because their culture contains components (e.g., kinship systems) that are pattern-based and which may lead to strong abilities to see pattern and structure. Finally, algebra was the vehicle which enabled the first Indigenous PhD in mathematics to understand mathematics. As he reminisced:

> When reflecting back on my education, my interest in mathematics started when I began to learn about algebra in my first year of high school. … For me, algebra made mathematics simple because I could see the pattern and structure or the generalisation of algebra much clearer than the detail of arithmetic.

Therefore, the Minjerribah Maths Project was set up to answer the following questions. *Can we improve achievement and retention in Indigenous mathematics by refocusing mathematics teaching onto the pattern and structure that underlies algebra? In doing this, are there Indigenous perspectives and knowledges we can use? Can we at the same time provide a positive self-image of Indigenous students?*

The project's focus was to put Indigenous contexts into mathematics teaching and learning (making Indigenous peoples and their culture visible in mathematics instruction) and to integrate the teaching of arithmetic and algebra (developing the reasoning behind the rules of arithmetic, while teaching arithmetic, so that these can be extended to the rules of algebra). The overall aim is to improve Indigenous students' mathematics education so they can achieve in formal abstract algebra and move into high status mathematics subjects.

This project was undertaken through action-research collaboration with Dunwich State School teachers by putting into practice processes to improve and sustain these enhanced Indigenous mathematics outcomes. The research is qualitative and interpretive and aims to address Indigenous mathematics-education questions in ways that give sustained beneficial outcomes for Indigenous people. It is based on the following analysis of the present situation with regard to Indigenous mathematics teaching and learning.

The Minjerribah Maths Project was a collaboration between Griffith University and Queensland University of Technology with Dr Chris Matthews from Griffith University as coordinator. The researchers involved in the project were:

- Dr Chris Matthews, Coordinator and Principal Researcher, Centre for Environmental Systems Research, Griffith University;
- Professor Tom Cooper, Researcher, Centre for Learning Innovation, QUT;
- Ms Margaret Grenfell, Research Assistant, Centre for Environmental Systems Research, Griffith University;
- Ms Tiara Cassady, Research Assistant, Centre for Learning Innovation, QUT.
- Mr Todd Phillips, Research Assistant, Centre for Environmental Systems Research, Griffith University.
- Ms Ashlee Surha, Research Assistant, Centre for Environmental Systems Research, Griffith University.

MAST (Maths as Story Telling) pedagogy

MAST is the first product developed for the Minjerribah Maths Project. It is an attempt to work from the storytelling world of the Indigenous student through to the formal world of algebra by experiences with the creation of symbols that have personal meaning. The storytelling starts with simple arithmetic but moves quickly to algebraic thinking. It enables Indigenous students to bring their everyday world of symbols into mathematics.

It is an answer to the dilemma of contextualising the teaching and learning of algebra. It focuses on representing mathematical equations as stories which leads to contextualising of mathematical symbols. It is an approach to symbolisation based on students creating and using their own symbols, drawn from their socio-cultural background, to describe these stories as a precursor to working with the accepted mathematics symbols. It utilises the current knowledge of the Indigenous student, which is drawn from their world, such as art, dancing, sport or driving, as a starting point for building understanding of arithmetic symbolism in a way that can be easily extended to algebraic symbolism. The approach has five steps. These steps are explained for addition. Obviously, the other three operations could be similarly undertaken.

Step 1**.** Students explore the meaning of *symbols* and how symbols can be assembled to tell and create a story. This is initially done by looking at symbols in Indigenous situations (e.g., exploring and understanding symbols in paintings) and then creating and interpreting symbols for simple actions (e.g. walking to and sitting in a desk).

Step 2**.** Students explore *simple addition story* by acting it out as a story (e.g. two groups of people joining each other). A discussion is then generated to identify the story elements such as the different groups of people and the action (the joining of the two groups) and the consequences of the action (the result of the joining).

Step 3**.** Students *create their own symbols* to represent the story. This step could be done in a freestyle manner; however, we have opted to take a more structured approach by using concrete materials (which are familiar to the students) to represent the objects (or people) in the story. The story is then created by allowing the students to construct the two groups of people with the concrete materials and construct their own symbol for "joining two groups" and lay this out to represent the action (or history) of the story. In a similar fashion, the students then construct their own symbol for "resulting in" or "same as" to tell the story of what happens after this action has taken place. Figure 1 gives an example of an addition story that was constructed by a student in Year 2.

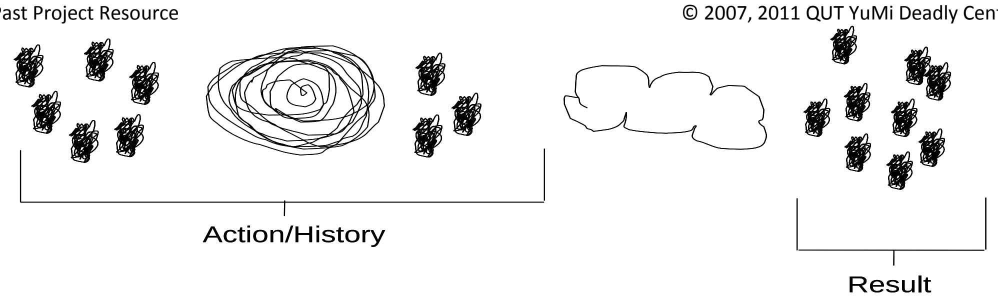


Figure 1. A Year 2 student's representation of the addition story 6 + 3 = 9.

Step 4. Students share their symbol systems with the group and any addition meanings their symbols may have. For example, in Figure 1, the student's "joining" symbol was a vortex that sucked the two groups together. The teacher then selects one of the symbol systems for all the students to use to represent a new addition story. This step is important to accustom students to writing within different symbol systems and to develop a standard classroom symbol system.

Step 5. Students *modify the story* (a key step in introducing algebraic ideas) under direction of the teacher. For example, the teacher takes an object from the action part of the story (see Figure 1), asks whether the story still makes sense (normally elicits a resounding "No"), and then challenges the students by asking them to find different strategies for the story to make sense again. There are four possibilities: (1) putting the object back in its original group, (2) putting the object in the other group on the action side, (3) adding another action (plus 1) to the action side, and (4) taking an object away from the result side. The first three strategies introduce the notion of compensation and equivalence of expression, while the fourth strategy introduces the balance rule (equivalence of equations). At this step, students should be encouraged to play with the story, guided by the teacher, to reinforce these algebraic notions.

Step 6. Students explore the meaning of *unknown* under direction of the teacher. For example, the teacher sets an example with an unknown (e.g. John bought a pie for $3 and an ice cream and he spent $7). The teacher asks the students to represent this without working out the value of the ice cream. Students invent a symbol for unknown and use it in stories with unknowns. Then the students are challenged to solve for unknowns using the balance rule. They have to first determine the operations to leave the unknown on its own. Thus, begins solutions to unknowns in linear equations.

Mathematics behind MAST

The MAST pedagogy is a way of introducing concepts, principles and unknowns for the four operations. The mathematics behind the activities in the booklets is now discussed.

Symbols and concepts (Booklets 1 and 4). The symbols for the four operations and equals are + addition, - subtraction, x multiplication, ÷ division, and = equals. Numbers and these symbols make up expressions (a number sentence without equals, such as 3+4 or 6x7-3) and equations (a number sentence that has an equals sign, such as 3+4=7 or 40-1=6x7-3). The concepts of the operations are complex and cover many situations. The best meanings for the four operations are as follows.

(1) Addition and subtraction are when situations involve joining to make a total or separating a total into parts – addition is when the parts are known and the total is unknown, and subtraction is when the total and one unknown is known and the other part is unknown. For example, in the story *I went to the bank and took out $7.983; this left $5,205 in the bank; how much did I have to start from?*, the operation is addition because $7,983 and $5,205 are parts and the amount at the start is the total and is the unknown (even though the action and the language is "take-away").

(2) Multiplication and division are when situations involve combining equal groups to make a total and separating the total into equal groups – multiplication is when the number of groups and the number in the group is known and the total is unknown, and division is when the total is known and one of the number of groups or number in each group is unknown. For example, in the story *There are 8 times as many oranges as apples; there are 56 oranges; how many apples?*, the operation is division because 8 is the number of groups, 56 is the total and the number of apples is an unknown group (even though the action and language is "times").

However, for these booklets, the following initial simpler (and incomplete) meanings are used:

Addition - joining Subtraction - take-away,

Multiplication - combining equal groups Division - sharing or breaking into groups equally.

The idea in Booklets 1 and 4 (addition and subtraction in 1, and multiplication and division in 4) is that mathematics symbols are a way of telling stories of everyday life with these meanings.

Principles (Booklets 2 and 5). An important component of algebra is understanding when these two things do not change (i.e., are equivalent or invariant) because in this case we can construct arguments where we replace the expression or equation with an equivalent one and find answers and solution to unknowns, such as 6x7-3 = 42-3 = 39 and 6*x*-3 = 39 is the same as 6*x*-3+3 = 39+3 is the same as 6*x* = 42 is the same as 6*x*/6 = 42/7 which is the same as *x*=7. Thus it is important in both arithmetic and algebra to teach when expressions and equations are equivalent.

Two important principles or rules for equivalence are compensation for expressions and balance for equations. Compensation is the set of principles/rules that mean that an expression remains the same if one number is changed. Balance is the set of principles where the equation remains the same after something is done to one side of the equation. For the four operations, the principles are as follows.

(1) The compensation principle for addition is "to do the opposite". In the example, 3 + 4, adding a 2 to the 3 is compensated by subtracting a 2 from the 4 (i.e. 5 + 2 is the same as 3 + 4). Similarly, subtracting 2 from the three is compensated by adding 2 to the 4 (i.e. 1 + 6). These sets of operations can also be done to the 4, which will always result in doing the opposite operation to keep the expression equivalent using the compensation principle.

(2) The compensation principle for subtraction is "to do the same" (the opposite to the principle for addition). Adding/subtracting 2 to or from the 7 in example 7-3 is compensated by adding/subtracting (same operation) 2 to or from the 3 making 9-5 and 5-1 both of which give the same answer as 7-3. Similarly, adding/subtracting 2 to or from the 3 in example 7-3 is compensated by adding/subtracting 2 to or from the 7.

(3) The compensation principles for multiplication is "to do the opposite" (the same as addition and the opposite to subtraction). Multiplying/dividing the 6 by 2 in example 6x4 is compensated by dividing/multiplying (opposite operation) the 4 by 2 making 12x1 and 3x4 both of which give the same answer as 6x4. Similarly, multiplying/dividing the 4 by 2 in 6x4 is compensated by dividing/multiplying the 6 by 2.

(4) The compensation principle for division is "to do the same" (the opposite to addition and multiplication and the same as subtraction). Multiplying/dividing the 18 by 3 in example18÷6, is compensated by multiplying/dividing the 6 by 3 to make 54÷18 and 6÷2 both of which give the same answer as 18÷6. Similarly, multiplying/dividing the 6 by 2 in example 18÷6 is compensated by multiplying/dividing the 18 by 2.

(5) The balance principle for addition is "to do the same to both sides of the equals sign". Adding/subtracting 2 to or from the 3 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side making 5+4=9 or 1+6=5, both of which are equivalent to 3+4=7. Similarly, addiing/subtracting 2 to or from the 4 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side.

(6) The balance principle for subtraction is "to do the same to both sides of the equals sign" (the same principle as for addition) but is complex when the second number is involved. Adding/subtracting 2 to the 7 in example 7-3=4 is balanced by adding/subtracting (same operation) 2 to the 4 on the other side making 9-3=6 and 5-3=2 both of which are equivalent to 7-3=4. However, adding/subtracting 2 to the 3 in 7-3=4 is adding/subtracting to a take-away so is really subtracting/adding 2 to 7-3. This means that it is balanced by subtracting/adding (opposite to what was done to the 3 but the same as effect on the 7-3) 2 to the 4 on the other side making 7-5=2 and 7-1=6 both of which are equivalent to 7-3=4.

(7) The balance principle for multiplication is to "do the same thing to both sides of the equals sign" (the same principle as addition and subtraction – but without complexity for the second number – unlike subtraction). Multiplying/dividing the 6 by 2 in example 6x4=24 is balanced by

multiplying/dividing (same operation) the 24 on the other side making 12x4=48 and 3x4=12 both of which are equivalent to 6x4=24. Similarly multiplying/dividing the 4 by 2 in 6x4=24 is balanced by multiplying the 24 by 2 on the other side.

(8) The balance principle for division is "to do the same thing for both sides of the equals sign" (but with complexity for the second number – the same as subtraction and different in this aspect from addition and subtraction). Multiplying/dividing the 18 by 3 in example 18÷6=3 is balanced by multiplying/dividing (same operation) the 3 on the other side giving 54÷6=9 and 6÷6=1 both of which are equivalent to 18÷6=3. However, multiplying/dividing the 6 by 3 in example 18÷6=3 is multiplying/dividing a division (the number sharing or the size of the group) so is really dividing/multiplying 18÷6 by 3. This means that it is balanced by dividing/multiplying (the opposite to what was done to the 6 but the same as what was done to the 18÷6) by 3 the 3 on the other side making 18÷2=9 and 18÷18=1 both of which are equivalent to 18÷6=3.

With respect to both compensation and balance, addition and multiplication act similarly, and subtraction and division also act similarly but have differences from addition and subtraction. With respect to compensation, addition and multiplication "do the opposite", while subtraction and division "do the same". With respect to balance, all operations "do the same to both sides", but subtraction and division are complex with respect to the second number where "do the opposite" appears to hold. However, the actions to both sides are "the same".

Unknowns (Booklets 3 and 6). When not all numbers are known in a situation, this not-known number has to be represented by a new symbol called a variable or unknown. In formal mathematics, this is represented by a letter. Informally, it can be represented by a box or question mark.

The balance principle can be used to find the unknown. The steps are as follows: (1) the operation (or operations are worked out that will leave the unknown alone on one side; and (2) these operations are balanced on the other side to find the value of the unknown. For example:

$y + 3 = 11$	$y \div 4 = 3$	$5 \times y + 6 = 21$
change the 3 to 0	change the 4 to 1	subtract 6
by subtracting 3	by multiplying by 4	$5 \times y = 21 - 6 = 15$
$y = 11 - 3$	$y = 3 \times 4$	divide by 5
$y = 8$	$y = 12$	$y = 15 \div 5 = 3$

Approach to teaching MAST

A crucial component in any successful program to improve Indigenous students' learning outcomes in mathematics is **high teacher expectations** for these students' learning.

The MAST program has been designed to take account of the strengths of Indigenous students in terms of cultural and social background and learning style. This should be made evident to the Indigenous students at the beginning of each booklet.

Be direct. State that the mathematics they are to do now is different to the mathematics that they have done before – state that it is designed for Goori or Murri students but all students can excel in the new approach. The approach has

- never been trialled anywhere else in the world and you will be the first to try it out;
- it focuses on being creative, to think about mathematics in a different way; and
- to relate mathematics to the world around us.

Be firm in your conviction that all students will be able to do it and, in fact, will thrive and prosper.

ABOUT THIS BOOKLET

Focus of lessons

The four lessons in this booklet are the third level lessons for MAST. They cover Step 6 for addition and subtraction. They focus on introducing the idea of unknowns and using balance principle to solve for unknowns. Similar to booklets 1 and 2, the lessons relate to different representations:

(1) Unknowns in addition and subtraction

(2) Real life situations

(3) Acting out these situations

(4) Symbol constructed by students and,

(5) The formal symbols of mathematics

These Lessons are suitable from Year 5 onwards and can be used into junior secondary school. They could also be used earlier. The lessons attempt to work with unknowns as follows.

(1) **Lesson 1** introduces the concept of "unknown" or "any number" and utilises a created symbol for unknown in addition and subtraction expressions and equations. Students create their own symbol for the unknown.

(2) **Lesson2** introduces balance principle to addition with unknown and uses this balance principle to solve for unknown in addition stories. Students use the balance principle to ensure equations remain balanced when they change the equation so that the unknown is alone on one side.

(3) **Lesson 3** introduces balance principle to subtraction with unknown and uses balances principle to solve for unknown in uses balance principle to solve for unknown in subtraction stories. Students use the balance principle to ensure equations remain balanced when they change the equation so they the unknown is alone on one side.

(4) **Lesson 4** relates invented symbols to formal mathematics symbols and solve for unknown in formal addition and subtraction equations. Includes equations with more than one operation.

The first and second MAST booklets (titled *MAST Additive-Concepts and Additive-Principles Lessons*), focused on Steps 1 to 5 of the MAST sequence. Later booklets (4 to 6) repeat activities for booklets 1 to 3 but with multiplication and division.

Hints for teaching

The Lessons give detailed directions with respect to the Lessons. The first rule is that THESE DIRECTIONS DO NOT HAVE TO BE FOLLOWED. Develop your own approach to the Lessons – mix and match from different Lessons (and booklets) – spend more time on certain ideas, and give a lot more reinforcement and practice examples than in these lessons.

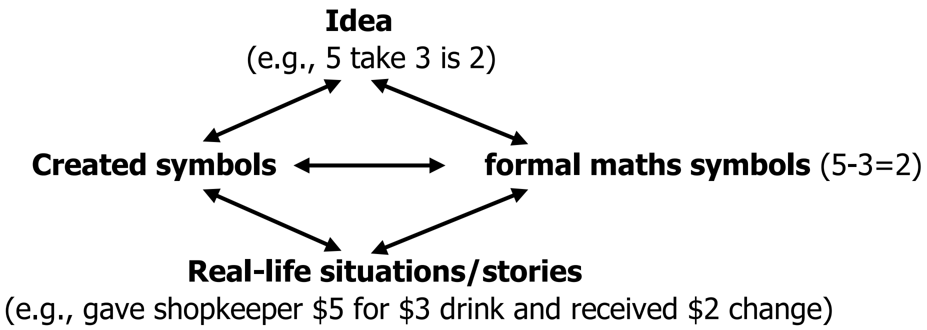
The best way to operate is to keep in mind where you want to go and let the students' responses to your earlier teaching direct your later teaching. There can be great merit even in integrating ideas from different booklets.

However, in our few trials, some things have been found useful.

(1) Drawing symbols for a simple task such as a child walking to a desk really works as an introduction to the maths drawings. It is useful to focus on objects and actions and get the students to think of the drawing being like a cartoon showing a series of symbols to tell a story.

(2) If teachers have blu tack and magnetic counters, they can set up a white board so that students can stick their A5 drawings on the board between counters to discuss their inventions.

(3) Always move the Lessons in <u>both directions</u> and then in <u>all directions</u> – from idea to created symbols and from created symbols to idea, from created symbols to story and story to created symbols, and from idea to created symbols and created symbols to idea. Act out things as well as talk about things. Overall, lessons will be trying to build all these relationships:



(4) Relate to the symbols in all ways too. A good sequence appears to be:

- Get students to draw their own symbols for your stories and then to make up their own stories for their own symbols.
- Get students to draw stories in other students' symbols and then to make up stories for other students' symbols
- Get students to draw stories in formal symbols and then to make up stories for formal symbols

(5) Take every chance to look at similarities and differences between addition and subtraction (e.g., different symbols for add and subtract but the same symbol for makes, gives or equals).

(6) Introduce formal mathematics symbols in a similar way that would get students to use other students' created symbols. Watch that the introduction of formal symbols does not change students' focus to "getting answers" rather than discussing/inventing rules.

MAST ADDITIVE UNKNOWN: LESSON 1

Objectives:

- To introduce concept of "unknown" or "any number"
- To utilise created symbol for unknown in addition and subtraction expressions and equations

Materials:

- Counters (preferably natural objects) for students.
- Magnetic counters and blu tack for the teacher
- A4 and A5 sheets
- Board set up with:

Counters	blu tack	counters	blu tack	counters

- Calculators, Worksheet 1 & 2

Language: Expression, equation, addition, subtraction, unknown, representation

What teacher does:

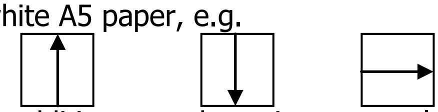
1. Recap lessons on creating symbols for addition, subtraction and equals.

   Handout white and coloured A5 paper and counters and ask students to draw symbols for addition, subtraction and equals or white A5 paper.

What students do:

Discuss previous lessons and suggest what was covered.

Draw symbols for addition, subtraction and equals on white A5 paper, e.g.



(Note: These are examples only. The students will make up their own symbols.

What teacher does:

2. Ask students to represent story: I had $5. My mother gave me $3. Now I have $8.

   State that sometimes, we do not know or cannot remember what the numbers are, for example:

   "I had some money in my pocket. My mother gave me $3. How much do I have?"

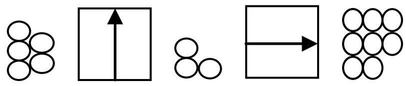
   State that in this situation, the amount I have in my pocket could be anything, so I need a new symbol that stands for "any number" or "unknown".

   Ask to make up such a symbol on coloured A5 paper. Organise for students to share and discuss symbols.

   Ask students to use created symbol to construct the example. Organise students to share what they produce.

What students do:

Draw story "5 add3 is 8":



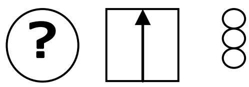
Discuss idea a "unknown" or something that could be "any number"

Create symbol for unknown eg, 

(Note: Again, this is an example only)

Share symbol with other students and discuss what makes a good symbol.

Construct story, "unknown add 3", with own symbols.



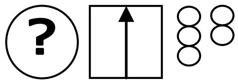
3. Ask students to use created symbols for other stories with "unknowns".

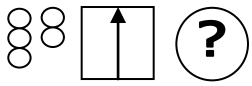
   (a) "I drove to my friends place then I drove 5km to Jan's place. How far did I drive?"

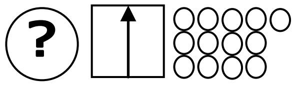
   (b) I bought a chocolate and a pie. The chocolate cost $5. How much did I spend?"

   (c) "In the second game I scored 13 points. How much did I score in both games?

Construct the stories with own symbols.

(a) 

(b) 

(c) 

4. Repeat steps 2 and 3 above for subtraction.

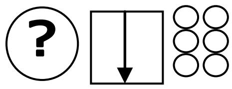
   Ask students to use invented symbols for the following stories:

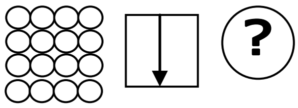
   (a) "I spent $6. How much money did have left?"

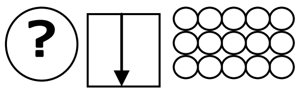
   (b) I had $17. I bought a game. How much money did I have left?"

   (c) "Jack drove to his Aunties'. He drove back 15km. How much further has he to travel to get home?

Construct the stories with own symbols.

(a) 

(b) 

(c) 

Share stories and symbols with other students.

5. State that sometimes we know the answer with an unknown and can use equals.

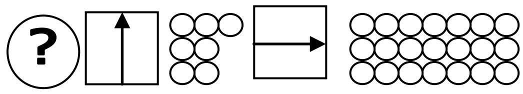
   Ask to use created symbols for these stories:

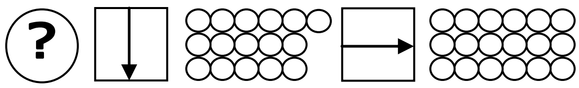
   (a) "John bought a ticket and a $7 meal. He spent $21.

   (b) "Frank paid $16 for the petrol and still had $18 in his pocket."

   Get students to share their stories and symbols.

Construct the stories with own symbols.

(a) 

(b) 

Share stories and symbols with other students.

6. Direct students to complete Worksheet 1. Lead discussion of answers

Complete Worksheet 1. Lead discussion of answers.

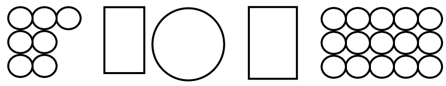
Evaluation:

- Students are engaged seem able to construct stories with unknowns.
- Students create an "unknown" symbol and understand its meaning.
- Students complete Worksheet 1.

MAST ADDITIVE UNKNOWN: WORKSHEET 1

Student name:	**School/class**:

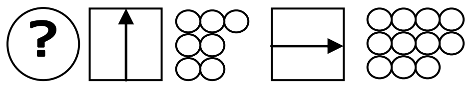
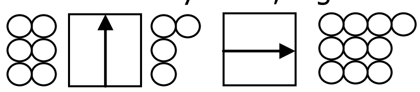
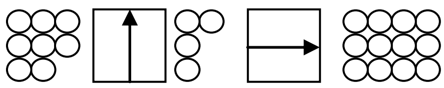
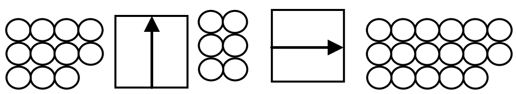
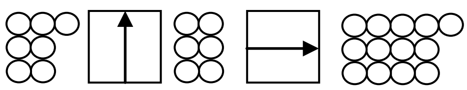
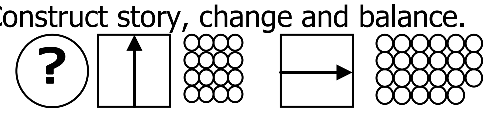
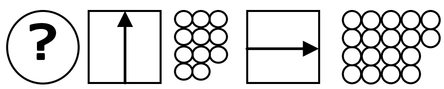
Fill in the missing sections. Use your own created symbols. We have done the first one for you.

Story	Action	Symbols
Example: Mary baked 7 cakes and then a second batch. This would make 15 cakes.	7 add unknown makes 15	
1. Sam spent $8. He has $9 left.	Unknown subtract 8 is 9.	
2. I received money form Dad and $6 from Mum, making $13.		
3. Jack took $20 bought lunch, and still had $12.		
4.	11 add unknown is 14	
5.	31 subtract unknown is 15	
6.	13 tonne on the first truck. The first and second truck carried 24 tonne.	
7. Dad and Mum gave me the same amount of money. With the $7 I already had, this made $33.		

MAST ADDITIVE UNKNOWN: LESSON 2

Objectives:	**Materials**:
• To introduce balance principle to addition with unknown. • To use balance principle to solve for unknown in addition stories.	• Counters (preferably natural objects) for students. • Magnetic counters and blu tack for the teacher • A4 and A5 sheets • Board set up with: Counters [blu tack] counters [blu tack] counters • Calculators, Worksheet 1 & 2

Language: Expression, equation, addition, subtraction, unknown, replacement, balance, solving for unknown

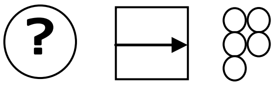
What teacher does:	**What children do:**
1. Recap the previous lesson. Go through an example. Ask students to construct story: "I bought a book for $7 and a pen. I paid $11. What did the pen cost?"	Discuss previous lesson. Construct story with own symbols,  (Note: These are examples only. The students will make up their own symbols.)
2. Ask if anyone remembers the balance principle. Do an example. Ask students to construct "I spent $6 on games and $4 on a burger. I spent $10". Ask students to balance the story if I increased $6 by $2. Repeat the process for: "I gave Jenny $11 and Frank $6. I gave away $17." Ask students to balance story if I decrease what I gave Jenny by $4. Discuss answers given by students. Discuss what balance means (i.e., What you do to one side of equals, you do to the other side of equals).	Discuss the balance principle. Construct addition example with own symbols, e.g.  Add 2 to 6 and balance story, e.g.  (Adding 2 and 6 means adding 2 to the 10?) Construct new example and balance change.   (decrease 11 by 4 means decrease 17 by 4)
3. Show balance acts with unknowns. Use the example: "I drove to Hampton. Then I drove a further 16km. In total, I drove 23km." Reduce the 16km by 5km and balance the story.	Construct story, change and balance.   (Decrease 16 by 5 means decrease 23 by 5.)

4. State that there is a way to work out unknowns by using the balance principle. Ask students to construct this addition example with an unknown: "I spent $7 on a game and bought a burger meal. I spent $12. Ask: *What change to the story will leave the unknown as the only money being spent?*

Lead discussion to see that if the 7 is reduced to zero, then the unknown would be all that is left. (If students are having trouble seeing this, ask to decrease games by $3 and argue that further decreases will leave unknown as the only spending.)

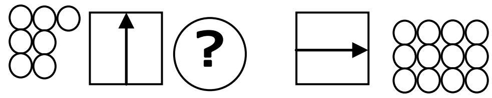
Lead discussion that if balance for $7 is being changed to nothing, this will mean that unknown is what remains.

Lead students to understand this means:



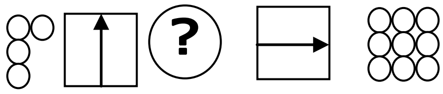
(No spending on games means $7 less than $12 is the value of burger meal.)

Construct the addition unknown example:



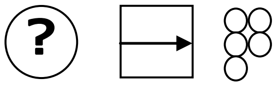
Share symbols and stories. Discuss how could only have the unknown as the only money being spent.

If necessary, balance story when spending $3 less on games:



($3 less on game means $3 less spent overall.)

Balance when spending nothing on games



($7 on games means $7 less spent overall)

5. Look at another example: "Tom scored some points. Ben scored 6 points. Together they scored 15 points.

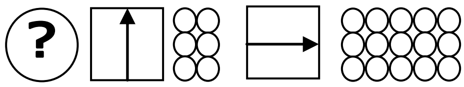
Ask: *What do we change to only have Tom's points?* Lead discussion to get students to see that we have to remove Ben's 6 points.

Ask students to make up own stories and find unknown. Discuss student's answers to the balanced story.

Ask: *"Who knows a rule for finding unknown in an addition story?*

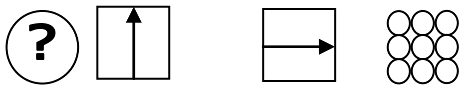
Discuss rules. Ensure students realise you have to subtract the other number.

Students construct:

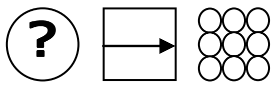


Discuss how to get  on own in addition.

When see that it requires change of the 6 to zero, do this and balance.



Which means



Make up own stories and draw them with own or modified symbols. Share symbols with class.

Discuss rules for finding unknown in addition stories.

6. Direct students to complete Worksheet 1. Lead discussion of answers. Direct students to put in numbers when they are large and use calculators. Ensure students know Lyn's symbols.

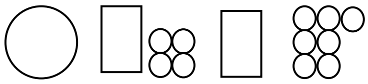
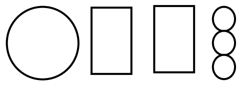
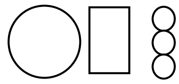
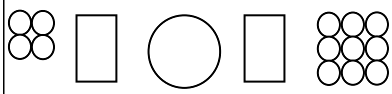
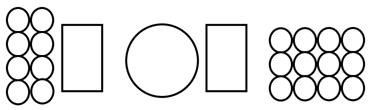
Complete Worksheet 1. Discuss the answers.

Evaluation:
- Students are engaged.
- Students understand how balance rule finds unknown for addition.
- Students complete Worksheet 1.

MAST ADDITIVE UNKNOWN 2: WORKSHEET 1

Student name:	**School/class**:

1. Complete the missing sections. We have done the first one for you.

Addition	Symbols	Change	Balance	Unknown
Example: John bought a drink and a pie. The pie cost $4. He spent $7. What was the cost of the drink?		Subtract 4		
(a) Frank scored 4 points in the first game. In the two games he scored 9 points. How many did he score in the second game?				
(b) Mary baked 2 trays of cakes. There were 8 in the first tray and 11 overall. How many cake were there in the second tray?				
(c)The meal cost $12. The burger was $8. How much were the chips?				
(d) In the second day the car travelled 360 km. The total trip was 840km. How many kilometres were travelled on the first day?				

MAST ADDITIVE UNKNOWN 2: WORKSHEET 1

Student name:	**School/class**:

2. Complete the Missing Sections with Lyn's symbols: addition is ◇◇, equals is ⬦, unknown is (?)

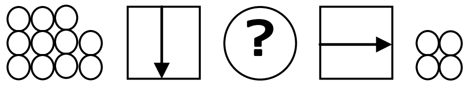
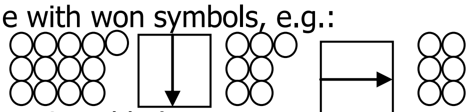
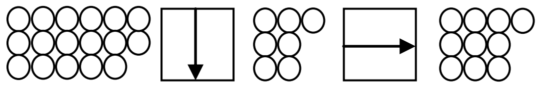
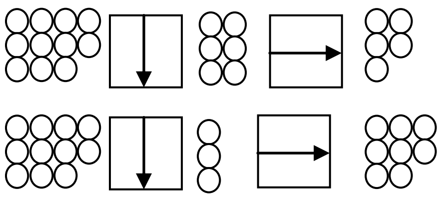
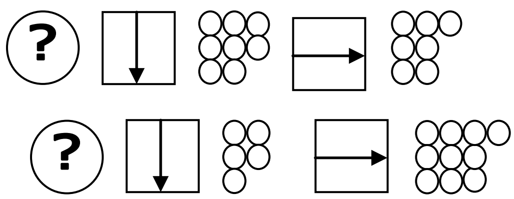
The first one is done for you. Write numbers instead of drawing the counters.

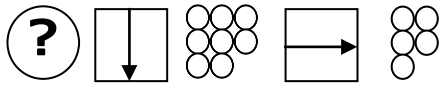
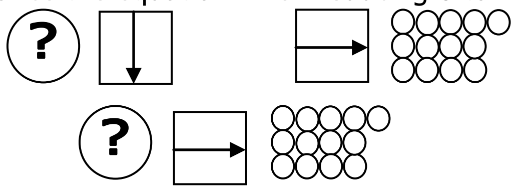
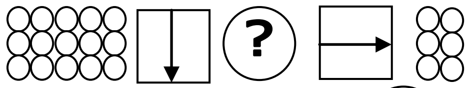
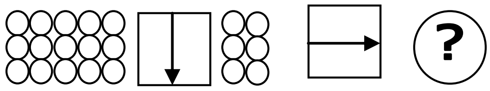
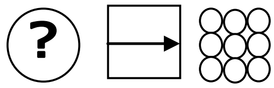
Unknown Story	Symbols	Change	Balance	Unknown
Example: John and Frank caught 30 fish. Frank caught 17. How many did John catch?	17 ◇◇ (?) ⬦ 30	Subtract 17	◇◇ (?) ⬦ 13	(?) ⬦ 13
(a) Dad handed out $37. Jim got $23. How much did Jack get?				
(b)	(?) ◇◇ 56 ⬦ 101			
(c)	29 ◇◇ (?) ⬦ 42			
(d) Susie cycled 87km. She did 38km on the first day. How many did she do on the second day?				
(e)	(?) ◇◇ 168 ⬦ 238			

MAST ADDITIVE UNKNOWN: LESSON 3

Objectives:	**Materials**:
• To introduce balance principle to subtraction with unknown. • To balance principle to solve for unknown in subtraction stories.	• Counters (natural objects) for students • Magnetic counters and blu tack for teacher • White and coloured A5 sheets of paper • Magnetic white board set up with: counters | blu tack | counters | blu tack | counters • Calculator • Worksheet 1

Language: Expression, equation, addition, subtraction, unknown, representation, balance, solving for unknown.

What teacher does:	**What children do:**
1. Recap lesson 1 for subtraction with unknown. Ask students to construct story: "I had $11. I bought a book. I have $4 left"	Discuss Lesson 1. Construct a story with own symbols, e.g.:  Note: these symbols are examples. Students will use their own.
2. State we are going to use the balance principle for subtraction as we did for addition in Lesson 2. Do same example – Ask students to construct: "I had $13. I bought a burger meal for $7. This left me with $6." Ask students to balance the story if $4 is added to $13. Repeat process for change to second term. Again ask students to balance story. Example: "I had $11. I spent $6. This left me with $5" – remove the $3 from the $6.	Discuss balance principle. Construct subtraction example with won symbols, e.g.:  Add 4 to 13 and balance story:  (adding 4 to 13 means adding 4 to 6 to make 10) Construct and balance second example.  (removing 3 from 6 is the same as adding 3 to 5, this means adding 3 to 5 to make 8)
3. Show how balance acts with unknowns. Use the example: "I rode 8km. I still had 7km to go. How far was it?" – decrease the 8km by 3km Discuss how reducing the take-away number increases the answer (i.e., the less you take-away, the more you have). Reinforce the balance rule. (What you do to one side of equals you have to do the same with the other side of equals.)	Construct story with own symbols, change and balance eg:  (decreasing 8 by 3 means 7 increases by 3 because adding 3 to both sides)

4. Similar to Lesson 2, show how balance enables unknowns to be calculated. Ask students to construct this subtraction example with an unknown: "I made cakes. 8 were eaten and 5 were left." Ask: *What change to the story will have only the unknown left?* Lead discussion to see that if subtract 8 is removed by adding 8, you get only the unknown. Ask students to balance the story with this change. Discuss results and argue that it shows unknown is 13. Discuss rule that adding a number to a subtraction of that number removes the subtraction.	Construct the example, with own symbols:  Share symbols and stories. Discuss how could only have the unknown on left hand side of equals. See that removing the 8 will do this and that this is equivalent to adding 8. Balance the equation when reducing 8 to zero:  Discuss rule.
5. Direct students to complete Worksheet 1. Lead discussion of answers. Direct students to put in numbers instead of drawing of counters and use a calculator. Ensure students know Lyn's symbols.	Complete Worksheet 1. Discuss answers.
6. Organise students to explore how to use balance to find unknown when the unknown is the second number. Use example: "I had $15, I bought a present. I had $6 left. How much was the present? Act out with selected students symbols what is happening (allow students to use numbers instead of drawing of counters). Discuss what would happen if added to unknown. Ensure students known what to do when subtracting an unknown.	Construct example with own symbols, eg:  Discuss what could be done. Add  to both sides and balance.  Discuss what could be done now. Subtract 6 to find unknown  Discuss rule for subtracting unknowns.
7. Direct students to complete Worksheet 2. Lead discussion of answers.	Complete Worksheet 2. Discuss answers.

Evaluation:

- Students are engaged.
- Students understand how to use balance rule.
- Students can complete worksheets.

MAST ADDITIVE UNKNOWN 3: WORKSHEET 1

Student name:	**School/class**:

1. Complete the missing sections. We have done the first one for you. Use your own symbols.

Unknown Story	Symbols	Change	Balance	Unknown
Example: Sue spent $16 and was left with $11. How much money did she have?	○ □ 16 □ 11	Add 16	○ □ □ 27	○ □ 27
(a) Fred sold 26 ice-creams. He had 21 left. How many did he have at the start		Add 26		
(b) Jan cut 12m of the paths. This left 8m to cut. How many meters were there at the start?				
(c) Jack gave away 35 fish. He had 19 left. How many did he catch?				
(d) Joan rode 37km. She had 26 still to go. How many kilometres did she have to start with?				

2. Fill in the missing gaps using Jen's symbols: ⊟ for addition, □ for subtraction and ◇ for equals. Be creative! The first one is done for you.

Unknown Story	Symbols	Change	Balance	Unknown
Example: Wendy bought a $56 present. She got $28 change. How much money did she have?	? ⊟ 56 ◇ 28	Add 56	? ⊟ ◇ 84	? ◇ 84
(a) Dad gave $47 to Jenny. This left hi with $38. How much did he have to start with?				
(b)	? □ 128 ◇ 58			
(c)	? ⊟ 69 ◇ 156			
(d) John paid Sue $256 leaving 349 for May. How much did John have to pay with?				

MAST ADDITIVE UNKNOWN 3: WORKSHEET 2

Student name:	**School/class**:

Complete the missing sections using Jen's symbols: ⊟ for addition, □ for subtraction and ◇ for equals. The first one is done for you.

Unknown Story	Symbols	Change	Balance	Unknown
Example: I had $36. I bought a bag. I had $17 left. How much did the bag cost?	36 □ (?) ◇ 17	Add (?)	Add Unknown 36 ◇ 17 ⊟ (?) Subtract 17 from 36 9 ◇ (?)	9 ◇ (?)
1. It was 63km. I rode for a while then I had 27km left. How far did I ride?				
2.	47 ⊟ (?) ◇ 138			
3.	164 □ (?) ◇ 38			
4. I caught 49 fish. I gave away some and kept the last 22. How many did I give away?				

MAST ADDITIVE UNKNOWN: LESSON 4

Objectives:	**Materials**:
• To relate invented symbols to formal mathematics symbols. • To solve for unknowns in formal addition and subtraction equations.	• Counters (preferably natural) for students • Magnetic counters and blu tack for teacher • Magnetic white board set up with: counter [blu tack] counter [blu tack] counter • Worksheets 1 & 2

Language: Expression, equation, addition, subtraction, unknown, representation, balance, solving for unknown.

What teacher does:	**What children do:**
1. Recap on Lessons 2 and 3. If necessary try an example from the worksheets of these lessons.	Discuss what happened in Lessons 2 and 3. Try examples from Lesson 2 and 3 Worksheets.
2. Ask: *What are the formal operations for multiplication, division and equals?* Ask: *Is there a formal symbol for unknown? What is it?* Focus attention on x. Say: *Letters like x, y, v, a, b are used as unknowns.*	Discuss the formal symbols: add +, subtract -, multiply x, divide ÷ and equals =. Suggest formal symbols for unknown, e.g.: ☐, ?, or x, y, a, b etc
3. Use formal symbols to represent stories: e.g. (a) I bought a coat and trousers for $162. I spent $255. How much was the coat? (b)I gave away $56. I have $27 left. How much money did I have? (c) I travelled 227km, had lunch and travelled on to Grandmas. I travelled 602km. How far did I drive after lunch? (d) I had 125m of rope. I sold a length leaving 67m unsold. How many did I sell?	Use formal symbols to tell the stories: (+, -, x, ÷, = and x, y) (a) $a + 62 = 255$ (b) $x - 56 = 27$ (c) $227 + y = 602$ (d) $125 - x = 67$
4. Ask the students to think of these formal symbols as if they were someone's invented symbols. Say: *Use the methods of Lessons 2 and 3 to solve them for unknown x.* Discuss different students' options. Remind students they have to make a change or changes that leave x alone on one side. Then they have to balance this change.	Use balance principle to solve for unknown x in each example. (a) $x + 62 = 255$ [subtract 62] $x = 255 - 62$ $= 193$ (b) $x - 56 = 27$ [add 56] $x = 27 + 56$ $= 83$

	(c) $227 + x = 602$ [subtract 227] $x = 602 - 227$ $= 375$ (d) $125 - x = 67$ [add x] $125 = 67 + x$ [subtract 67] $125 - 67 = x$ $58 = x$ or $x = 58$
5. Direct students to complete Worksheet 1. Lead students in discussing answers.	Complete worksheet 1. Discuss answers.

Evaluation:

- Students are engaged and complete worksheets.
- Students see x as a symbol for unknown.
- Students complete Worksheet 1 and can solve for x in addition and subtraction situations.

MAST ADDITIVE UNKNOWN 4: WORKSHEET 1

Student name:	**School/class**:

Complete the missing sections. Use formal symbols. The first is done for you.

Unknown Story	Formal Symbols	Change(s)	Working	Unknown
Example: The crowd increased by 367. This made 587 people. How many were there in the crowd to start with?	$x + 367 = 587$	Subtract 367	$x + 367 = 587$ [subtract 367] $x = 587 - 367$ $= 220$	x - 220
1. My mum bought a $245 boom box and some CDs. She paid $300. How much did the CDs cost?	$245 + x = 300$			
2. I spent $65 and got $85. How much did I give the shopkeeper?	$x - 65 = 85$			
3.	$x + 241 = 342$			
4.	$x - 82 = 47$			
5.	$65 - x = 42$			
6. I had 48 marbles. I gave over half away. I only had 17 left. How many did I give away?				