YUMI DEADLY CENTRE
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Minjerribah Maths Project

MAST Additive-Principles Lessons

Booklet S.2: using created symbols to develop the Addition and Subtraction principles of compensation and balance

Minjerribah Maths Project

Secondary Booklet 2

S.2

MAST ADDITIVE-PRINCIPLES LESSONS

Maths as Story Telling lessons on using created symbols to develop the Addition and Subtraction principles of compensation and balance

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YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

MINJERRIBAH MATHS PROJECT

MAST

MAST ADDITIVE-PRINCIPLES LESSONS

BOOKLET S.2

USING CREATED SYMBOLS TO DEVELOP ADDITION AND SUBTRACTION PRINCIPLES OF COMPENSATION AND BALANCE

VERSION 1: 2007

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BACKGROUND

Minjerribah Maths Project

The Minjerribah Maths Project was a research study to find effective ways to teach Indigenous students mathematics, and for Indigenous students to learn mathematics. It realised that effective mathematics teaching is crucial for Indigenous students' futures as mathematics performance can determine employment and life chances. It endeavoured to find ways that will encourage and enable more Indigenous students to undertake mathematics subjects past Year 10 that lead to mathematics-based jobs.

Some educators argue that Indigenous students learn mathematics best through concrete "hands-on" tasks, others by visual and spatial methods rather than verbal, and still others by observation and non-verbal communication. However, these findings may be an artefact of Indigenous students being taught in Standard Australian English with which they may not have the words to describe many mathematical ideas and the words they have may be ambiguous. It is important to recognise that Indigenous students come from a diverse social and cultural background and investigations into Indigenous education should take this into consideration. Indigenous people also have common experiences, which can be reflected upon to suggest ways forward.

There is evidence that school programs can dramatically improve Indigenous learning outcomes if they reinforce pride in Indigenous identity and culture, encourage attendance, highlight the capacity of Indigenous students to succeed in mathematics, challenge and expect students to perform, provide a relevant educational context in which there is Indigenous leadership, and contextualising instruction into Indigenous culture. However, the majority of teachers of Indigenous students are non-Indigenous with little understanding of Indigenous culture and these non-Indigenous teachers can have difficulties with contextualisation and reject it in favour of familiar Eurocentric approaches. Thus, there is a need to build productive partnerships between non-Indigenous teachers and the Indigenous teacher assistants employed from the community to assist them and the Indigenous community itself.

There is also some evidence that Indigenous students tend to be holistic, learners, a learning style that appreciates overviews of subjects and conscious linking of ideas and should appreciate algebraic structure. Thus, algebra could be the basis for Indigenous mathematics learning. This position is interesting because algebra is the basis of many high status professions. It is also based on generalising pattern and structure, skills with which Indigenous students may have an affinity because their culture contains components (e.g., kinship systems) that are pattern-based and which may lead to strong abilities to see pattern and structure. Finally, algebra was the vehicle which enabled the first Indigenous PhD in mathematics to understand mathematics. As he reminisced:

> When reflecting back on my education, my interest in mathematics started when I began to learn about algebra in my first year of high school. … For me, algebra made mathematics simple because I could see the pattern and structure or the generalisation of algebra much clearer than the detail of arithmetic.

Therefore, the Minjerribah Maths Project was set up to answer the following questions. *Can we improve achievement and retention in Indigenous mathematics by refocusing mathematics teaching onto the pattern and structure that underlies algebra? In doing this, are there Indigenous perspectives and knowledges we can use? Can we at the same time provide a positive self-image of Indigenous students?*

The project's focus was to put Indigenous contexts into mathematics teaching and learning (making Indigenous peoples and their culture visible in mathematics instruction) and to integrate the teaching of arithmetic and algebra (developing the reasoning behind the rules of arithmetic, while teaching arithmetic, so that these can be extended to the rules of algebra). The overall aim is to improve Indigenous students' mathematics education so they can achieve in formal abstract algebra and move into high status mathematics subjects.

This project was undertaken through action-research collaboration with Dunwich State School teachers by putting into practice processes to improve and sustain these enhanced Indigenous mathematics outcomes. The research is qualitative and interpretive and aims to address Indigenous mathematics-education questions in ways that give sustained beneficial outcomes for Indigenous people. It is based on the following analysis of the present situation with regard to Indigenous mathematics teaching and learning.

The Minjerribah Maths Project was a collaboration between Griffith University and Queensland University of Technology with Dr Chris Matthews from Griffith University as coordinator. The researchers involved in the project were:

- Dr Chris Matthews, Coordinator and Principal Researcher, Centre for Environmental Systems Research, Griffith University;
- Professor Tom Cooper, Researcher, Centre for Learning Innovation, QUT;
- Ms Margaret Grenfell, Research Assistant, Centre for Environmental Systems Research, Griffith University;
- Ms Tiara Cassady, Research Assistant, Centre for Learning Innovation, QUT.
- Mr Todd Phillips, Research Assistant, Centre for Environmental Systems Research, Griffith University.
- Ms Ashlee Surha, Research Assistant, Centre for Environmental Systems Research, Griffith University.

MAST (Maths as Story Telling) pedagogy

MAST is the first product developed for the Minjerribah Maths Project. It is an attempt to work from the storytelling world of the Indigenous student through to the formal world of algebra by experiences with the creation of symbols that have personal meaning. The storytelling starts with simple arithmetic but moves quickly to algebraic thinking. It enables Indigenous students to bring their everyday world of symbols into mathematics.

It is an answer to the dilemma of contextualising the teaching and learning of algebra. It focuses on representing mathematical equations as stories which leads to contextualising of mathematical symbols. It is an approach to symbolisation based on students creating and using their own symbols, drawn from their socio-cultural background, to describe these stories as a precursor to working with the accepted mathematics symbols. It utilises the current knowledge of the Indigenous student, which is drawn from their world, such as art, dancing, sport or driving, as a starting point for building understanding of arithmetic symbolism in a way that can be easily extended to algebraic symbolism. The approach has five steps. These steps are explained for addition. Obviously, the other three operations could be similarly undertaken.

Step 1. Students explore the meaning of *symbols* and how symbols can be assembled to tell and create a story. This is initially done by looking at symbols in Indigenous situations (e.g., exploring and understanding symbols in paintings) and then creating and interpreting symbols for simple actions (e.g. walking to and sitting in a desk).

Step 2. Students explore *simple addition story* by acting it out as a story (e.g. two groups of people joining each other). A discussion is then generated to identify the story elements such as the different groups of people and the action (the joining of the two groups) and the consequences of the action (the result of the joining).

Step 3. Students *create their own symbols* to represent the story. This step could be done in a freestyle manner; however, we have opted to take a more structured approach by using concrete materials (which are familiar to the students) to represent the objects (or people) in the story. The story is then created by allowing the students to construct the two groups of people with the concrete materials and construct their own symbol for "joining two groups" and lay this out to represent the action (or history) of the story. In a similar fashion, the students then construct their own symbol for "resulting in" or "same as" to tell the story of what happens after this action has taken place. Figure 1 gives an example of an addition story that was constructed by a student in Year 2.

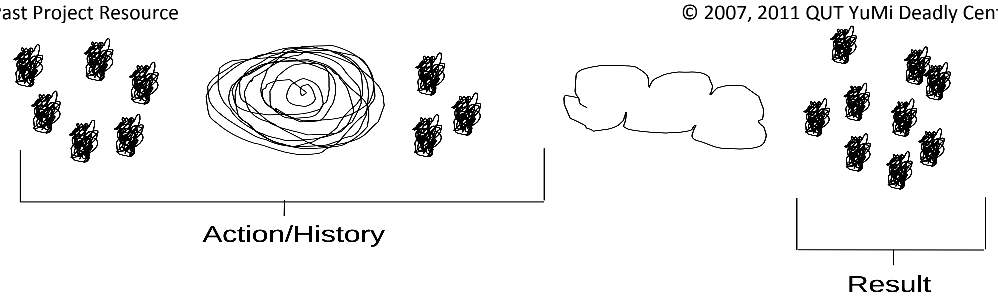


Figure 1. A Year 2 student's representation of the addition story 6 + 3 = 9.

***Step 4*.** Students share their symbol systems with the group and any addition meanings their symbols may have. For example, in Figure 1, the student's "joining" symbol was a vortex that sucked the two groups together. The teacher then selects one of the symbol systems for all the students to use to represent a new addition story. This step is important to accustom students to writing within different symbol systems and to develop a standard classroom symbol system.

***Step 5*.** Students *modify the story* (a key step in introducing algebraic ideas) under direction of the teacher. For example, the teacher takes an object from the action part of the story (see Figure 1), asks whether the story still makes sense (normally elicits a resounding "No"), and then challenges the students by asking them to find different strategies for the story to make sense again. There are four possibilities: (1) putting the object back in its original group, (2) putting the object in the other group on the action side, (3) adding another action (plus 1) to the action side, and (4) taking an object away from the result side. The first three strategies introduce the notion of compensation and equivalence of expression, while the fourth strategy introduces the balance rule (equivalence of equations). At this step, students should be encouraged to play with the story, guided by the teacher, to reinforce these algebraic notions.

Step 6. Students explore the meaning of *unknown* under direction of the teacher. For example, the teacher sets an example with an unknown (e.g. John bought a pie for $3 and an ice cream and he spent $7). The teacher asks the students to represent this without working out the value of the ice cream. Students invent a symbol for unknown and use it in stories with unknowns. Then the students are challenged to solve for unknowns using the balance rule. They have to first determine the operations to leave the unknown on its own. Thus, begins solutions to unknowns in linear equations.

Mathematics behind MAST

The MAST pedagogy is a way of introducing concepts, principles and unknowns for the four operations. The mathematics behind the activities in the booklets is now discussed.

Symbols and concepts (Booklets 1 and 4). The symbols for the four operations and equals are + addition, - subtraction, x multiplication, ÷ division, and = equals. Numbers and these symbols make up expressions (a number sentence without equals, such as 3+4 or 6x7-3) and equations (a number sentence that has an equals sign, such as 3+4=7 or 40-1=6x7-3). The concepts of the operations are complex and cover many situations. The best meanings for the four operations are as follows.

(1) Addition and subtraction are when situations involve joining to make a total or separating a total into parts – addition is when the parts are known and the total is unknown, and subtraction is when the total and one unknown is known and the other part is unknown. For example, in the story *I went to the bank and took out $7.983; this left $5,205 in the bank; how much did I have to start from?*, the operation is addition because $7,983 and $5,205 are parts and the amount at the start is the total and is the unknown (even though the action and the language is "take-away").

(2) Multiplication and division are when situations involve combining equal groups to make a total and separating the total into equal groups – multiplication is when the number of groups and the number in the group is known and the total is unknown, and division is when the total is known and one of the number of groups or number in each group is unknown. For example, in the story *There are 8 times as many oranges as apples; there are 56 oranges; how many apples?*, the operation is division because 8 is the number of groups, 56 is the total and the number of apples is an unknown group (even though the action and language is "times").

However, for these booklets, the following initial simpler (and incomplete) meanings are used:

Addition - joining Subtraction - take-away,

Multiplication - combining equal groups Division - sharing or breaking into groups equally.

The idea in Booklets 1 and 4 (addition and subtraction in 1, and multiplication and division in 4) is that mathematics symbols are a way of telling stories of everyday life with these meanings.

Principles (Booklets 2 and 5). An important component of algebra is understanding when these two things do not change (i.e., are equivalent or invariant) because in this case we can construct arguments where we replace the expression or equation with an equivalent one and find answers and solution to unknowns, such as 6x7-3 = 42-3 = 39 and 6*x*-3 = 39 is the same as 6*x*-3+3 = 39+3 is the same as 6*x* = 42 is the same as 6*x*/6 = 42/7 which is the same as *x*=7. Thus it is important in both arithmetic and algebra to teach when expressions and equations are equivalent.

Two important principles or rules for equivalence are compensation for expressions and balance for equations. Compensation is the set of principles/rules that mean that an expression remains the same if one number is changed. Balance is the set of principles where the equation remains the same after something is done to one side of the equation. For the four operations, the principles are as follows.

(1) The compensation principle for addition is "to do the opposite". In the example, 3 + 4, adding a 2 to the 3 is compensated by subtracting a 2 from the 4 (i.e. 5 + 2 is the same as 3 + 4). Similarly, subtracting 2 from the three is compensated by adding 2 to the 4 (i.e. 1 + 6). These sets of operations can also be done to the 4, which will always result in doing the opposite operation to keep the expression equivalent using the compensation principle.

(2) The compensation principle for subtraction is "to do the same" (the opposite to the principle for addition). Adding/subtracting 2 to or from the 7 in example 7-3 is compensated by adding/subtracting (same operation) 2 to or from the 3 making 9-5 and 5-1 both of which give the same answer as 7-3. Similarly, adding/subtracting 2 to or from the 3 in example 7-3 is compensated by adding/subtracting 2 to or from the 7.

(3) The compensation principles for multiplication is "to do the opposite" (the same as addition and the opposite to subtraction). Multiplying/dividing the 6 by 2 in example 6x4 is compensated by dividing/multiplying (opposite operation) the 4 by 2 making 12x1 and 3x4 both of which give the same answer as 6x4. Similarly, multiplying/dividing the 4 by 2 in 6x4 is compensated by dividing/multiplying the 6 by 2.

(4) The compensation principle for division is "to do the same" (the opposite to addition and multiplication and the same as subtraction). Multiplying/dividing the 18 by 3 in example18÷6, is compensated by multiplying/dividing the 6 by 3 to make 54÷18 and 6÷2 both of which give the same answer as 18÷6. Similarly, multiplying/dividing the 6 by 2 in example 18÷6 is compensated by multiplying/dividing the 18 by 2.

(5) The balance principle for addition is "to do the same to both sides of the equals sign". Adding/subtracting 2 to or from the 3 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side making 5+4=9 or 1+6=5, both of which are equivalent to 3+4=7. Similarly, addiing/subtracting 2 to or from the 4 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side.

(6) The balance principle for subtraction is "to do the same to both sides of the equals sign" (the same principle as for addition) but is complex when the second number is involved. Adding/subtracting 2 to the 7 in example 7-3=4 is balanced by adding/subtracting (same operation) 2 to the 4 on the other side making 9-3=6 and 5-3=2 both of which are equivalent to 7-3=4. However, adding/subtracting 2 to the 3 in 7-3=4 is adding/subtracting to a take-away so is really subtracting/adding 2 to 7-3. This means that it is balanced by subtracting/adding (opposite to what was done to the 3 but the same as effect on the 7-3) 2 to the 4 on the other side making 7-5=2 and 7-1=6 both of which are equivalent to 7-3=4.

(7) The balance principle for multiplication is to "do the same thing to both sides of the equals sign" (the same principle as addition and subtraction – but without complexity for the second number – unlike subtraction). Multiplying/dividing the 6 by 2 in example 6x4=24 is balanced by

multiplying/dividing (same operation) the 24 on the other side making 12x4=48 and 3x4=12 both of which are equivalent to 6x4=24. Similarly multiplying/dividing the 4 by 2 in 6x4=24 is balanced by multiplying the 24 by 2 on the other side.

(8) The <u>balance principle for division</u> is "to do the same thing for both sides of the equals sign" (but with complexity for the second number – the same as subtraction and different in this aspect from addition and subtraction). Multiplying/dividing the 18 by 3 in example 18÷6=3 is balanced by multiplying/dividing (same operation) the 3 on the other side giving 54÷6=9 and 6÷6=1 both of which are equivalent to 18÷6=3. However, multiplying/dividing the 6 by 3 in example 18÷6=3 is <u>multiplying/dividing a division</u> (the number sharing or the size of the group) so is really dividing/multiplying 18÷6 by 3. This means that it is balanced by dividing/multiplying (the opposite to what was done to the 6 but the same as what was done to the 18÷6) by 3 the 3 on the other side making 18÷2=9 and 18÷18=1 both of which are equivalent to 18÷6=3.

With respect to both compensation and balance, addition and multiplication act similarly, and subtraction and division also act similarly but have differences from addition and subtraction. With respect to compensation, addition and multiplication "do the opposite", while subtraction and division "do the same". With respect to balance, all operations "do the same to both sides", but subtraction and division are complex with respect to the second number where "do the opposite" appears to hold. However, the actions to both sides are "the same".

Unknowns (Booklets 3 and 6). When not all numbers are known in a situation, this not-known number has to be represented by a new symbol called a variable or unknown. In formal mathematics, this is represented by a letter. Informally, it can be represented by a box or question mark.

The balance principle can be used to find the unknown. The steps are as follows: (1) the operation (or operations are worked out that will leave the unknown alone on one side; and (2) these operations are balanced on the other side to find the value of the unknown. For example:

$y + 3 = 11$	$y \div 4 = 3$	$5 \times y + 6 = 21$
change the 3 to 0	change the 4 to 1	subtract 6
by subtracting 3	by multiplying by 4	$5 \times y = 21 - 6 = 15$
$y = 11 - 3$	$y = 3 \times 4$	divide by 5
$y = 8$	$y = 12$	$y = 15 \div 5 = 3$

Approach to teaching MAST

A crucial component in any successful program to improve Indigenous students' learning outcomes in mathematics is **high teacher expectations** for these students' learning.

The MAST program has been designed to take account of the strengths of Indigenous students in terms of cultural and social background and learning style. This should be made evident to the Indigenous students at the beginning of each booklet.

Be direct. State that the mathematics they are to do now is different to the mathematics that they have done before – state that it is designed for Goori or Murri students but all students can excel in the new approach. The approach has

- never been trialled anywhere else in the world and you will be the first to try it out;
- it focuses on being creative, to think about mathematics in a different way; and
- to relate mathematics to the world around us.

Be firm in your conviction that all students will be able to do it and, in fact, will thrive and prosper.

ABOUT THIS BOOKLET

Focus of lessons

The four lessons in this booklet are the second level lessons for MAST. They cover Step 5 for addition and subtraction. They focus on introducing the idea of using created symbols to develop the compensation and balance principles for these two operations. Similar to Booklet 1, the Lessons relate five different representations: (1) the principles for addition and subtraction, (2) real-life situations, (3) acting out these situations, (4) symbols constructed by the students, and (5) the formal symbols of mathematics.

These Lessons are suitable from Year 5 onwards and can be used into junior secondary school although we have done the early work in Lessons 1 and 2 with Year 2 successfully. The lessons attempt to enable students to understand the principles as follows.

(1) **Lesson 1** focuses on changing an aspect of the created symbols and counters and asking students to make the story correct. Its aim are to: (1) see if students can be creative with aspect to changing a feature of the created symbols for addition, (2) to explore different ways of making addition stories correct, and (3) introduce notion of equivalence for addition.

(2) **Lesson 2** focuses on the same activity as Lesson 1 but for subtraction. Its aims are to: (1) see if students can be creative with respect to changing a feature of the created symbols for subtraction, (2) to explore different ways of making subtraction stories correct, and (3) introduce the notion of equivalence for subtraction.

(3) **Lesson 3** focuses on identifying two principles that underlie equivalence, namely, compensation and balance. Its aims are to: (1) introduce compensation and balance principles for addition and subtraction and (2) explore rules fore these 2 principles in addition and subtraction.

(4) **Lesson 4** focuses on relating addition and subtraction and created and formal symbols with respect to the principles of compensation. Its aims are to: (1) relate addition and subtraction in terms of compensation and balance, (2) explore principle differences when change happens in the second number instead of the first number, and (3) relate created symbols to formal symbols of mathematics in terms of principles.

The first booklet, (MAST booklet 1), focused on MAST Steps 1-4 for addition and subtraction. It introduced the idea of students creating their own symbols and using these symbols to tell addition and subtraction stories. This booklet (MAST booklet 2), extends this idea by introducing the principles of compensation and balance that make expressions and equations equivalent (Step 5). The next booklet (MAST booklet 3), will further extend the work with created symbols to unknowns, solving for these unknowns and solving for these unknowns using the balance principle (Step 6). The final two booklets (MAST booklets 4-6), will repeat these first three booklets but for multiplication and division instead of addition and subtraction.

Hints for teaching

The Lessons give detailed directions with respect to the Lessons. The first rule is that THESE DIRECTIONS DO NOT HAVE TO BE FOLLOWED. Develop your own approach to the Lessons – mix and match from different Lessons (and booklets) – spend more time on certain ideas, and give a lot more reinforcement and practice examples than in these lessons.

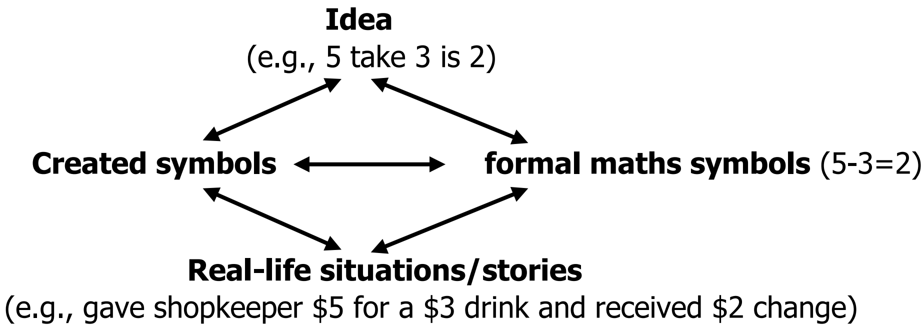
The best way to operate is to keep in mind where you want to go and let the students' responses to your earlier teaching direct your later teaching. There can be great merit even in integrating ideas from different booklets.

However, in our few trials, some things have been found useful.

(1) Involve students and their culture in discussion – allow them to share, explain and give point of view without labelling this a right or wrong – allow them to bring in local contexts and Indigenous context (i.e., where possible, change the examples to more represent the students' situation).

(2) If teachers have blu tak and magnetic counters, they can set up a white board so that students can stick their A5 drawings on the board between counters to discuss their inventions.

(3) Always move the Lessons in <u>both directions</u> and then in <u>all directions</u> – from idea to created symbols and from created symbols to idea, from created symbols to story and story to created symbols, and from idea to created symbols and created symbols to idea. Act out things as well as talk about things. Overall, lessons will be trying to build all these relationships:



(4) Relate to the symbols in all ways too. A good sequence appears to be:

- Get students to draw their own symbols for your stories and then to make up their own stories for their own symbols.
- Get students to draw stories using other students' symbols and then to make up stories for other students' symbols
- Get students to draw stories using formal symbols and then to make up stories for formal symbols

(5) Take every chance to look at similarities and differences between addition and subtraction (e.g., different symbols for add and subtract but the same symbol for makes, gives or equals).

(6) Introduce formal mathematics symbols in a similar way that would get students to use other students' created symbols. Watch that the introduction of formal symbols does not change students' focus to "getting answers" rather than discussing/inventing rules.

MAST ADDITIVE PRINCIPLES: LESSON 1

Objectives:

- To see if students can be creative with respect to changing a feature of the created symbols for addition.
- To introduce different ways of making addition stories correct.
- To introduce notion of equivalence for addition.

Materials:

- Counters (preferably natural objects) for students.
- Magnetic counters and blu tack for the teacher
- A4 and A5 sheets
- Board set up with:

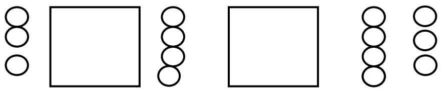
Counters | blu tack | counters | blu tack | counters

- Worksheet 1 & 2

Language: Symbol, story, addition, equals, linear, representation, replacement, equivalence

What teacher does:

1. Recap creating addition symbols for stories. Ask students to use counters and A5 sheets to represent 3 students joining 4 students to give 7 students.



Discuss some students' results. Point out the linear nature of the drawing.

What children do:

Discuss creating symbols for addition stories.

Use previous or modified symbols for 3 join 4 makes 7. Share drawings with other students.

Discuss different symbols and linear nature of drawing.

What teacher does:

2. Direct students to remove 1 counter from the 3. Ask: *Is this still correct?* Discuss why it is not.

Ask: W*hat can we do to make it correct?* Discuss options until all 4 possibilities are given. Record on board.

State that all these options are 'equivalent' to the original.

Note: Students might bring in a 5th possibility by using subtraction.

What children do:

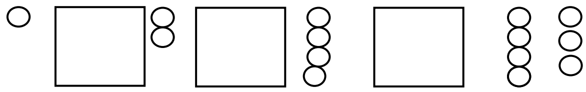
Remove one counter. Discuss why addition story is not correct and suggest possibilities to make it correct again. For e.g.

(1) Put counter back (replacement)

(2) Add extra counter to 4

(3) Remove one counter from 7

(4) Add extra joining symbol to left hand side, e.g.



Discuss term "equivalence".

What teacher does:

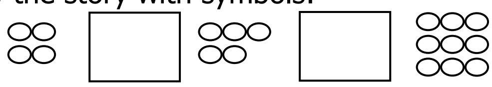
3. Repeat 1 and 2 above for this story, "Bill ate 4 pies, then he ate 5 more, making 9 pies eaten", but this time direct students to remove 2 counters from the 4.

Discuss why this makes the story wrong. Ask students to propose ways to make it true again. Try to draw out as many different ways as possible.

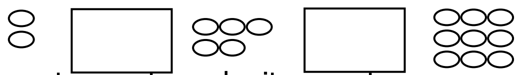
Discuss whether it matters if 2 counters are removed from the 5 instead of the 4 counters.

What children do:

Draw the story with symbols:

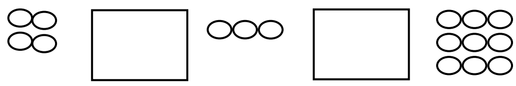


Remove 2 counters from the 4.

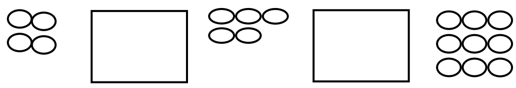


Suggest ways to make it correct.

Remove 2 counters from 5.



Discuss how to make this correct again and

	whether there are any differences to removing 2 from the 4.
4. Repeat 3 above but add 2 counters to the 4 in 4 joins 5 gives 9. Discuss how this is different to removing 2 from the 4. What is the same? What is different? Ask: *How would add 2 to the 5 change things?* Discuss difference & similarities. Ask: *Can anyone see a pattern or rule to follow?* If students have difficulty with rules, try further examples – adding or subtracting counters form the first and second number. Ensure students can see that it make no difference which number is affected, (first or second).	Repeat 3 above for:  but with 2 added to the 4. Offer options and discuss: 'what is the same?' and 'what is different?' Look at adding 2 to the 5 and discuss this. Think of a rule that encompasses all examples.
5. Direct students to complete Worksheet 1. Lead into discussion of answers.	Complete Worksheet 1. Discuss answers.

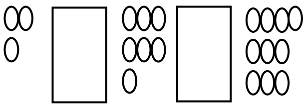
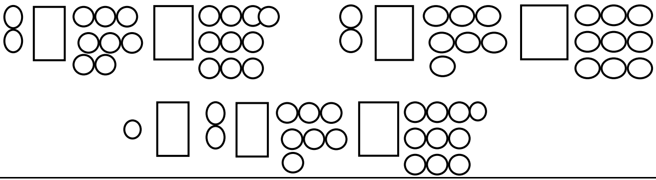
Evaluation:

- Students engage and offer opinions.
- Students suggest sensible rules.
- Students successfully complete Worksheet 1.

MAST ADDITIVE PRINCIPLES 1: WORKSHEET 1

Student name:	**School/class**:

Fill in the missing sections. Use your own symbols. We have done the first one for you.

Addition story	**Your symbols**	**Change**	**Ways to make it correct (other than replace)**
Example: 3 children join 7 children to make 10		Remove 1 from the 3	
1. 4 joins 5 to make 9		Remove 1 from the 5	
2. 8 joins 3 to make 11		Add 1 to the 8	
3. 4 joins 6 to make 10		Remove 2 from the 6	
4. 5 joins 9 to make 14		Add 2 to the 9	
5. 3 joins 4 to make 7		Add 3 to the 4	

Addition story	**Your symbols**	**Change**	**Ways to make it correct (other than replace)**
6. 1 joins 6 to make 7		Add 4 to the 1	
7. 8 joins 2 to make 10		Remove 3 from the 8	
8. 3 joins 7 to make 10		Remove 4 from the 7	
9. 4 joins 8 to make 12		Add 3 to the 8	
10. 6 joins 7 to make 13		Remove 2 from the 6	

MAST ADDITIVE PRINCIPLES: LESSON 2

Objectives:

- To see if students can be creative with respect to changing a feature of the created symbols for subtraction.
- To explore ways of making subtraction stories correct.
- To introduce notion of equivalence for subtraction.

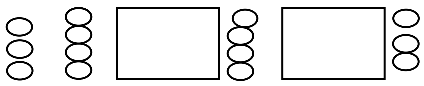
Materials:

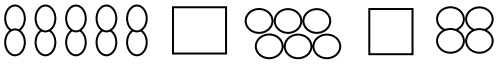
- Counters (preferably natural objects) for students
- Magnetic counters and blu tack for teacher
- A5 sheets
- Board set up with:

Counters [blu tack] counters [blu tack] counters

- Worksheet 1

Language: Symbol, story, addition, subtraction, equals, linear, representation, replacement, equivalence, principle

What teacher does:	**What children do:**
1. Recap creating subtraction symbols for stories.	Discuss creating symbols for subtraction stories.
Ask students to use counters and A5 sheets to represent 4 students walking away from 7 students leaving 3 students. 	Use previous or modified symbols for 7 students take-away 4 gives 3. Share symbols with other students. Discuss different symbols and linear nature of drawing.
Discuss some students' results. Ask: *What are the characteristics of these drawings?*	
2. Direct students to remove one counter from the 7.	Remove one counter. Discuss correctness and possibilities to make it correct again:
Ask: *Is this drawing still correct?* Discuss why it is not correct. Ask: *What can we do to make it correct?* Discuss options until all possibilities are given. Ask: *Can anyone think of a rule for subtraction that is different to addition?* Discuss what is different to addition? State that all these possibilities are equivalent to the original. Note: Someone may find a creative way to use addition.	(1) Put counter back (replacement) (2) Remove 1 counter from the 4 (3) Remove one counter from the 3, and (4) Add in extra symbols. Discuss the meaning of 'equivalence'. Discuss how subtraction equivalence different to addition.

3. Repeat 1 and 2 above for the story, "Jack paid $10 for a $6 burger meal, and received $4 change". Ask students to remove 2 counters from the 10. Discuss why this makes the drawing wrong. Ask the students to suggest ways to make it correct again. Try to draw out as many different ways as possible. Discuss whether it matters if 2 counters are removed from 10 or 6. Check suggestions by doing with materials and symbols.	Draw the 10 subtract 6 gives 4 story with own symbols:  Remove 2 counters from 10 and discuss how this makes it incorrect. Look at different ways to make it correct again. (E.g. put back 2, decrease 6 by 2, decrease 4 by 2 and so on). Discuss whether it matters if the 2 is removed from the 10 or the 6? Check this by actually removing 2 from the 6.
4. Direct students to repeat 3 above but add 2 counters to the 10. Discuss ways to make drawing correct. Ask: *Is anything different to subtracting 2 from the 10?* Discuss whether things change if 2 counters are added to the 6. Check this by acting things out. Ask: Can anyone think of a rule for relating addition?? Can anyone think of rules for making subtraction the same again after a change? Can anyone think of a rule that explains difference if change first or second number? How are these different to addition?	Repeat 3 above but with 2 counters added to the 10 counters in the story. Suggest options to make drawing correct. Discuss how this is the same or different to subtracting 2 counters from the 10. Suggest rules for subtraction, for difference between affecting first or second numbers, and for differences to subtraction.
5. Direct students to complete Worksheet 1. Lead discussion of answers.	Complete Worksheet 1. Discuss answers.

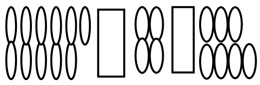
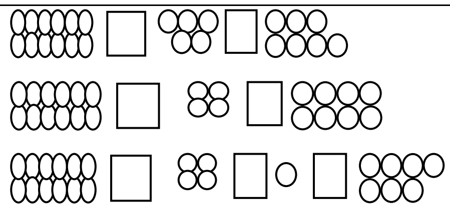
Evaluation:

- Students engage and offer opinions.
- Some students can suggest reasonable options and discuss different possibilities.
- Students successfully complete Worksheet 1.

MAST ADDITIVE PRINCIPLES 2: WORKSHEET 1

Student name:	**School/class**:

Fill in the missing sections. Use your own symbols. We have done the first one for you.

Subtraction Story	Your symbols	Change	Ways to make it correct (other than replacement)
Example: I had $11, spent $4, now I have $7.		Add 1 to the 11	
1. I had 7 cakes, ate 5, and was left with 3.		Add 2 to the 7	
2. 6 cars were parked, 2 left, 4 remained.		Add 1 to the 4	
3. Had 8 pies, ate 6, was left with 2.		Subtract 2 from the 8	
4. Joe bought 7 cards, gave 3 cards away and had 4 left.		Add 4 to the 3	
5. 11 beetles were on a leaf, 6 left, 5 remained.		Subtract 3 from the 11	

Subtraction Story	Your symbols	Change	Ways to make it correct (other than replacement)
6. Ann had $16, spent $9 and was left with $7.		Subtract 4 from the 16	
7. Jack bought 8 drinks, gave 5 away and was with 3.		Add 2 to the 5	
8. Ann had $18, spent $12, had $6 left.		Subtract 2 from the 12	
9. 3 lions were in a cage and 1 left which left 2.		Add 4 to the 1	
10. A chicken laid 9 eggs, 4 were eaten which left 5.		Add 1 to the 4	

MAST ADDITIVE PRINCIPLES: LESSON 3

Objectives:

- To introduce compensation and balance principals for addition and subtraction.
- To explore rules for these 2 principles in addition and subtraction.
- To compare addition and subtraction with respect to the rules.

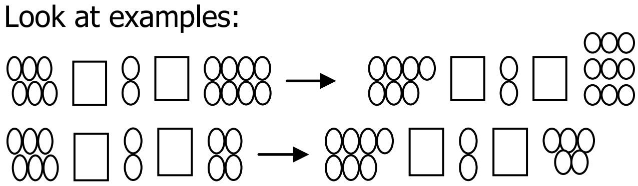
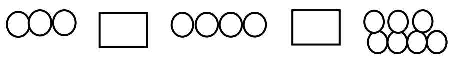
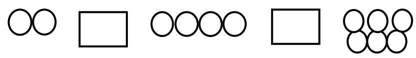
Materials:

- Counters (preferably natural objects)
- A4 and A5 sheets
- Board set up with:

Counters | blu tack | counters | blu tack | counters

- Worksheets 1 and 2

Language: Symbol, story, linear, addition, subtraction, compensation, balance, represent, principle, equivalence, principal, rule

What teacher does:	What children do:
1. Recap Lessons 1 and 2. Ask students what was covered in the lessons. Write answers on the board until have everything needed. Look at the options for examples. "6 add 2 gives 8" and "6 takes 2 gives 4" where the change is that 1 is added to the 6. State we are going to look at special options – point them out.	Think back to Lessons 1 and 2 and suggest ideas that were covered in these lessons. Look at examples: 
2. Discuss the 2 options that are left when do not allow replacement or using another joining. Use example 3 joining 4 to make 7 to look at 4 possibilities for making the example correct. Then remove replacement and using another join them so that only options A and B are left. Lead students to see that A is compensation and B is balance. See if any students can state what is happening in them as a rule. To help discussion look at 5 joining 3 to make 8 and remove 2 from the 5. Ask: *What is compensation? What is balance?* Discuss whether the rule changes if 2 was added to the 5. State: *Compensation is when you change the other number before equals. Balance is when you change the number on the other side of equals.*	Draw and look at example 3 and 4 give 7:  Remove one counter. Redraw example and discuss ways to make it correct which do not use replacement or a second operation. Look at these 2 changes: A  B  Discuss A being compensation and B being balance. Create rules for each.
3. Direct students to complete Worksheet 1. Go through first example to ensure students understand Fred's symbols. Lead into discussion of answers.	Complete Worksheet 1. Discuss Fred's symbols. Discuss answers.

4. Repeat 2 above for subtraction. Use example 9 remove 3 to make 6. Remove 1 counter from the 6. Look at the possibilities for making the example correct.

   Remove all replacement and second operation options so that only two options A and B are left.

   Discuss ways to make it correct which do not use replacement or a second operation.

   Lead students to see that A is compensation and B is balance.

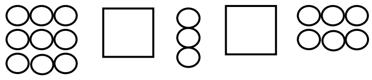
   See if any students can state what is happening as rules.

   To help discussion, look at 8 remove 5 to make 3 and remove 2 from the 8. Ask: *What is compensation? What is balance?*

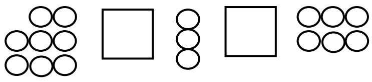
   Discuss what would happen if 2 was added to the 8 instead of being removed or if the 2 was taken from or added to the 5.

   See of students can state what happens as rules.

Draw and look at example of 9 remove 3 to make 6.



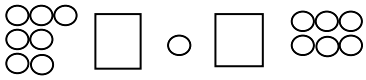
Remove one counter from the 9. Redraw example:



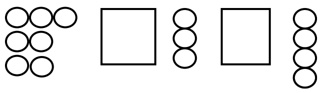
Discuss ways to make it correct which do not use replacement or a second operation.

Look at these two remaining options:

A



B



Discuss A being compensation and B being balance. Create rules for the 2 options.

5. Direct students to complete Worksheet 1. Lead discussion of answers.

Complete Worksheet 1. Discuss answers.

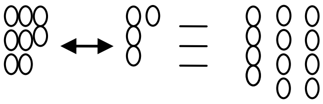
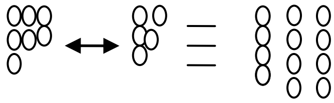
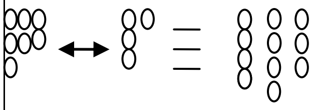
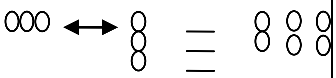
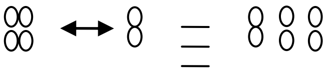
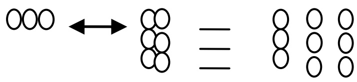
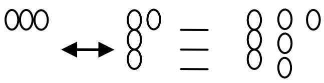
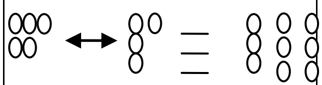
Evaluation:

- Students engage and offer opinions.
- Students can differentiate between compensation and balance for both subtraction and addition.
- Some students can suggest reasonable rules.

MAST ADDITIVE PRINCIPLES 3: WORKSHEET 1

Student name:	**School/class**:

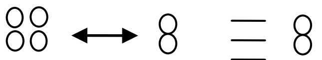
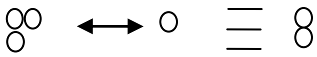
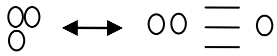
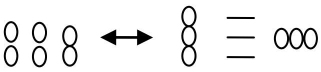
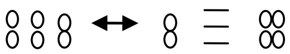
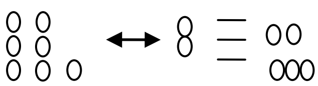
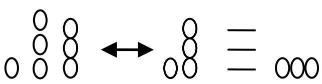
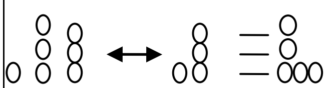
Use Fred's symbols (↔ as joining, ↓ as take-away, and ☰ as equals) to fill in the missing sections. The first is done for you. Be creative with stories.

Addition story	Symbol	Change	Compensation	Balance
Example: Bob had $8, got $4 more and made $12.		Remove 1 from 8		
1. Sue had $5 got $2 more and made $7.		Remove 1 from the 5		
2.		Remove 1 from the first 3		
3. Jenny scored 3 points then another 8.. Jenny scored 11 points altogether.		Add 1 to the 3		
4.		Add 1 to the 2		
5. Five cars were in the car park, 4 more joined them. This made 9 cars altogether.				
6.				

MAST ADDITIVE PRINCIPLES 3: WORKSHEET 2

Student name:	**School/class**:

Use Fred's symbols (↔ as joining, ↓ as take-away, and ≡ as equals) to fill in the missing sections. The first is done for you. Be creative with stories.

Subtraction story	**Symbol**	**Change**	**Compensation**	**Balance**
Example: Lisa had $4 but she lost $2. She was left with $2.		Remove 1 from the 4		
1. Sam was paid $15 and then spent $5. He was left with $10		Remove 2 from the 15		
2.		Remove 2 from the 6		
3. Jane bought 5 lollies and gave 2 to her friend. She was left with 3.		Add 1 to the 2		
4.		Add 3 to the 2		
5. Daniel made 8 paper planes but 3 were crushed. He was left with 5.				
6.				

MAST ADDITIVE PRINCIPLES: LESSON 4

Objectives:

- To relate addition and subtraction in term of compensation and balance.
- To explore principle differences when change happens to second instead of the first number.
- To relate created symbols to formal symbols of mathematics in terms of principles.

Materials:

- Counters (preferably natural objects
- A4 and A5 sheets
- Board set up with:

Counters [blu tack] counters [blu tack] counters

- Worksheets 1 and 2

Language: Symbol, story, addition, subtraction, compensation, balance, represent

What teacher does:

1. Recap previous lesson. Remind students of what compensation and balance are. Recap that they are used to keep equivalence when a number is changed.

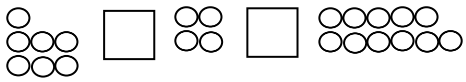
   Use example, "7 add 4 is 11" and removing 1 from the 7.

   Ask: What is compensation and what is balance?

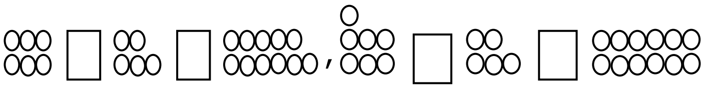
   Draw results on the board.

What children do:

Suggest ideas from lessons 3 and discuss compensation and balance. Use example to look at how original relates to compensated and balanced change.



Take 1 from 7



Compensation Balance

2. Organise students into groups to explore <u>compensation</u> for similarities and differences between addition and subtraction and between change acting on first and second number. Use examples:

   (a) 7 add 3 gives 10: remove 2 from the 7/add 2 to the 7.

   (b) 7 add 3 gives 10: remove 2 from the 3/add 2 to the 3.

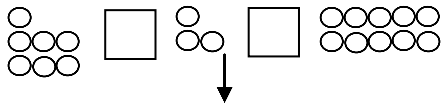
   (c) 7 subtract 3 gives 4: add 2 to the 7/remove 2 from the 7

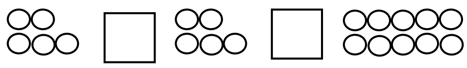
   (d) 7 subtract 3 gives 4: add 2 to the 3/remove 2 from the 7

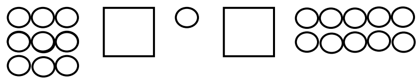
   Discuss conclusions. Allow students to write rules for compensation on board under headings:

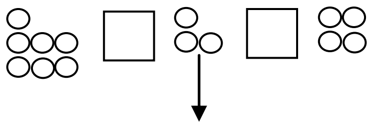
   - Addition vs Subtraction
   - First number vs Second number

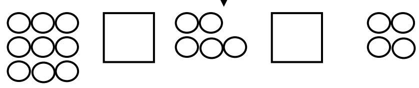
Form groups and adopt 1 member's created symbols. Use these symbols to explore compensation for the examples:

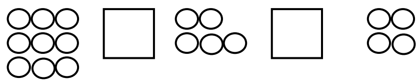


(a) 

(b) 



(c) 

(d) 

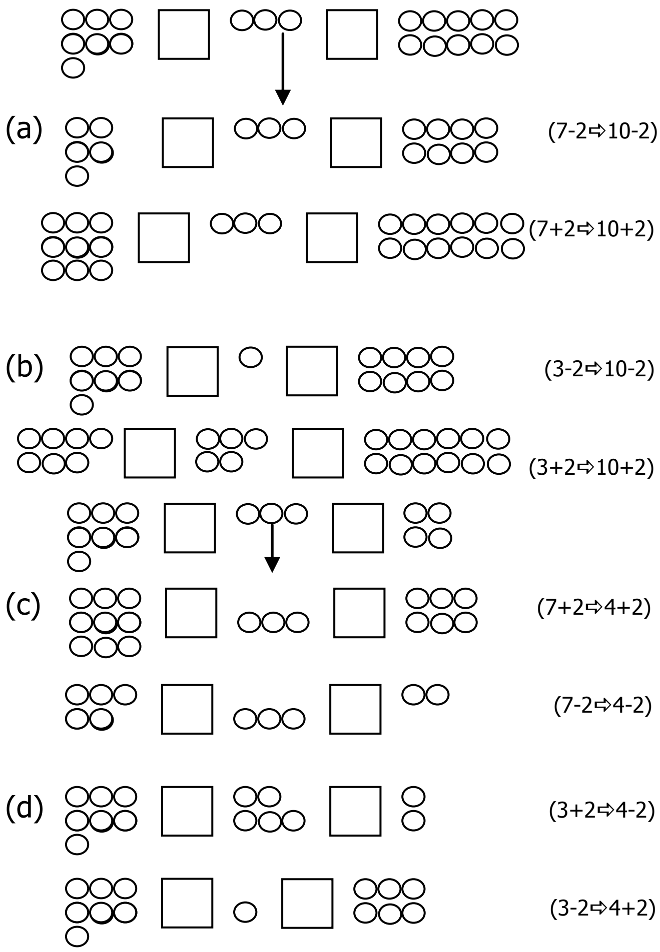
3. Similar to 2 above, organise students into groups to explore balance similarities and differences between addition and subtraction and between change acting on first number and second number. Use examples as for 2 above.

Discuss conclusions. Allow students to write rule on board under 2 headings:

- Addition vs Subtraction
- First number vs Second number

Discuss example (d) – what make this one so different. (Adding 2 to 3 means subtracting another 2 so the right-hand-side number is reduced by 2. It is still the same rule, "do the same to both sides," but have to interpret adding "2 more counters to the 3 counters" as subtraction).

Form groups and adopt one member's created symbols. Use these symbols to explore balance for 4 examples:



Suggest and discuss rules.

4. Discuss the formal symbols we use in maths for addition and subtraction.

Write these on board: +: addition, -: subtraction, =: equals. Ask: *Relate these formal symbols to your created symbols. Which do you prefer?*

Discuss how compensate and balance works with formal symbols. Use simple examples.

Restate rules for compensation and balance.

Ensure students understand it is the same rules as for their created symbols.

Discuss formal symbols. Relate there to created symbols. Look at examples:

(a) 4 + 5 = 9; add 2 to 4

(b) 6 + 8 = 14; take 3 from 8

(c) 11 – 4 = 7; add 5 to 11

(d) 12 – 7 = 5; subtract 4 from 7

Discuss how these examples compensate and balance.

5. Direct students to complete Worksheet 1. Go through examples. Lead discussion of answers.

Complete Worksheet 1. Discuss answers.

Evaluation:

- Students engage and offer opinions.
- Students can differentiate between compensation and balance for both subtraction and addition.
- All students can understand rules for application of compensation and balance for created and formal symbols.

MAST ADDITIVE PRINCIPLES 4: WORKSHEET 1

Student name:	**School/class**:

Complete missing sections. The first has been done for you.

Story	Symbol	Change	Compensation	Balance
Example: Fred had $9 and spent $4. This left $5.	9 – 5 = 4	Increase spending $4 by $2	11 – 6 = 5	9 – 6 = 3
1. John ate 11 cakes and then another 7. This meant he ate 18 cakes.	11 + 7 = 18	Decrease first eating from 11 to 8		
2. 12 cars were in the park, 14 more drove in, making 26 cars.		Only 7 cars in the park at the start		
3. 36 dresses on the rack were sold, leaving 23 dresses.		Only 31 dresses were on the rack to start with		
4. Margie had $50, she spent $27. This left $23.		Increase what she spent by $5		
5.	31 + 26 = 57	Add 11 to 26		
6.	48 – 21 = 27	Subtract 13 from 48		
7.	11 + 18 = 29	Subtract 7 from 18		
8.	66 – 28 = 38	Subtract 22 from 28		
9.	23 – 7 = 16		28 – 12 = 16	
10.	38 + 26 = 64		26 + 26 = 52	