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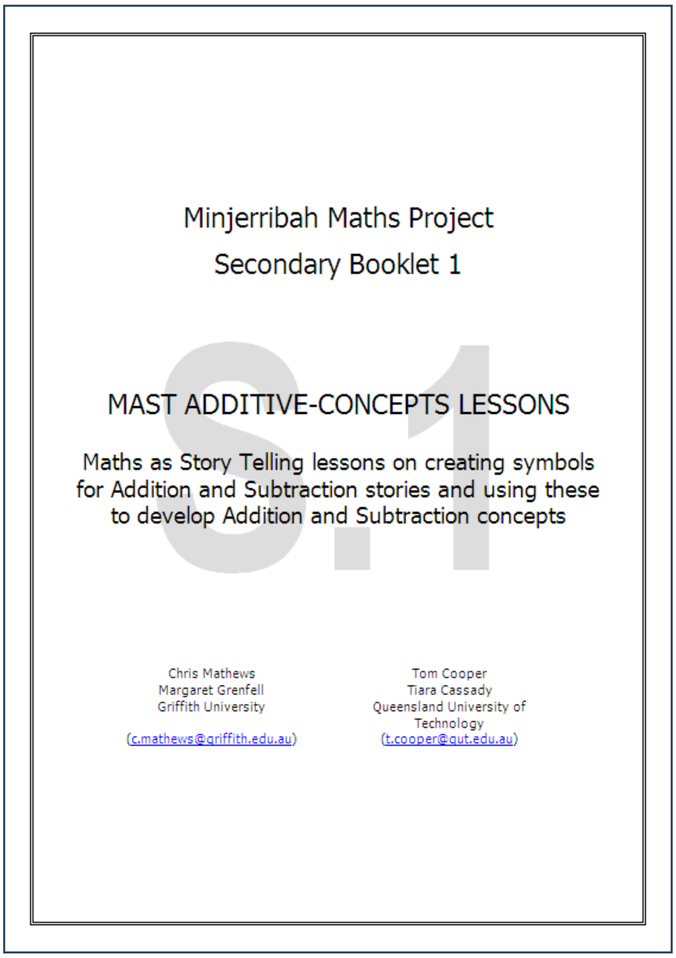
Minjerribah Maths Project

MAST Additive-Concepts Lessons

Booklet S.1: creating symbols for Addition and Subtraction stories and using these to develop Addition and Subtraction concepts


Minjerribah Maths Project

Secondary Booklet 1

S.1

MAST ADDITIVE-CONCEPTS LESSONS

Maths as Story Telling lessons on creating symbols for Addition and Subtraction stories and using these to develop Addition and Subtraction concepts

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YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

MINJERRIBAH MATHS PROJECT

MAST

MAST ADDITIVE-CONCEPTS LESSONS

BOOKLET S.1

CREATING SYMBOLS FOR ADDITION AND SUBTRACTION STORIES AND USING THESE TO DEVELOP ADDITION AND SUBTRACTION CONCEPTS

VERSION 1: 2007

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BACKGROUND

Minjerribah Maths Project

The Minjerribah Maths Project was a research study to find effective ways to teach Indigenous students mathematics, and for Indigenous students to learn mathematics. It realised that effective mathematics teaching is crucial for Indigenous students' futures as mathematics performance can determine employment and life chances. It endeavoured to find ways that will encourage and enable more Indigenous students to undertake mathematics subjects past Year 10 that lead to mathematics-based jobs.

Some educators argue that Indigenous students learn mathematics best through concrete "hands-on" tasks, others by visual and spatial methods rather than verbal, and still others by observation and non-verbal communication. However, these findings may be an artefact of Indigenous students being taught in Standard Australian English with which they may not have the words to describe many mathematical ideas and the words they have may be ambiguous. It is important to recognise that Indigenous students come from a diverse social and cultural background and investigations into Indigenous education should take this into consideration. Indigenous people also have common experiences, which can be reflected upon to suggest ways forward.

There is evidence that school programs can dramatically improve Indigenous learning outcomes if they reinforce pride in Indigenous identity and culture, encourage attendance, highlight the capacity of Indigenous students to succeed in mathematics, challenge and expect students to perform, provide a relevant educational context in which there is Indigenous leadership, and contextualising instruction into Indigenous culture. However, the majority of teachers of Indigenous students are non-Indigenous with little understanding of Indigenous culture and these non-Indigenous teachers can have difficulties with contextualisation and reject it in favour of familiar Eurocentric approaches. Thus, there is a need to build productive partnerships between non-Indigenous teachers and the Indigenous teacher assistants employed from the community to assist them and the Indigenous community itself.

There is also some evidence that Indigenous students tend to be holistic, learners, a learning style that appreciates overviews of subjects and conscious linking of ideas and should appreciate algebraic structure. Thus, algebra could be the basis for Indigenous mathematics learning. This position is interesting because algebra is the basis of many high status professions. It is also based on generalising pattern and structure, skills with which Indigenous students may have an affinity because their culture contains components (e.g., kinship systems) that are pattern-based and which may lead to strong abilities to see pattern and structure. Finally, algebra was the vehicle which enabled the first Indigenous PhD in mathematics to understand mathematics. As he reminisced:

> When reflecting back on my education, my interest in mathematics started when I began to learn about algebra in my first year of high school. … For me, algebra made mathematics simple because I could see the pattern and structure or the generalisation of algebra much clearer than the detail of arithmetic.

Therefore, the Minjerribah Maths Project was set up to answer the following questions. *Can we improve achievement and retention in Indigenous mathematics by refocusing mathematics teaching onto the pattern and structure that underlies algebra? In doing this, are there Indigenous perspectives and knowledges we can use? Can we at the same time provide a positive self-image of Indigenous students?*

The project's focus was to put Indigenous contexts into mathematics teaching and learning (making Indigenous peoples and their culture visible in mathematics instruction) and to integrate the teaching of arithmetic and algebra (developing the reasoning behind the rules of arithmetic, while teaching arithmetic, so that these can be extended to the rules of algebra). The overall aim is to improve Indigenous students' mathematics education so they can achieve in formal abstract algebra and move into high status mathematics subjects.

This project was undertaken through action-research collaboration with Dunwich State School teachers by putting into practice processes to improve and sustain these enhanced Indigenous mathematics outcomes. The research is qualitative and interpretive and aims to address Indigenous mathematics-education questions in ways that give sustained beneficial outcomes for Indigenous people. It is based on the following analysis of the present situation with regard to Indigenous mathematics teaching and learning.

The Minjerribah Maths Project was a collaboration between Griffith University and Queensland University of Technology with Dr Chris Matthews from Griffith University as coordinator. The researchers involved in the project were:

- Dr Chris Matthews, Coordinator and Principal Researcher, Centre for Environmental Systems Research, Griffith University;
- Professor Tom Cooper, Researcher, Centre for Learning Innovation, QUT;
- Ms Margaret Grenfell, Research Assistant, Centre for Environmental Systems Research, Griffith University;
- Ms Tiara Cassady, Research Assistant, Centre for Learning Innovation, QUT.
- Mr Todd Phillips, Research Assistant, Centre for Environmental Systems Research, Griffith University.
- Ms Ashlee Surha, Research Assistant, Centre for Environmental Systems Research, Griffith University.

MAST (Maths as Story Telling) pedagogy

MAST is the first product developed for the Minjerribah Maths Project. It is an attempt to work from the storytelling world of the Indigenous student through to the formal world of algebra by experiences with the creation of symbols that have personal meaning. The storytelling starts with simple arithmetic but moves quickly to algebraic thinking. It enables Indigenous students to bring their everyday world of symbols into mathematics.

It is an answer to the dilemma of contextualising the teaching and learning of algebra. It focuses on representing mathematical equations as stories which leads to contextualising of mathematical symbols. It is an approach to symbolisation based on students creating and using their own symbols, drawn from their socio-cultural background, to describe these stories as a precursor to working with the accepted mathematics symbols. It utilises the current knowledge of the Indigenous student, which is drawn from their world, such as art, dancing, sport or driving, as a starting point for building understanding of arithmetic symbolism in a way that can be easily extended to algebraic symbolism. The approach has five steps. These steps are explained for addition. Obviously, the other three operations could be similarly undertaken.

***Step 1*.** Students explore the meaning of *symbols* and how symbols can be assembled to tell and create a story. This is initially done by looking at symbols in Indigenous situations (e.g., exploring and understanding symbols in paintings) and then creating and interpreting symbols for simple actions (e.g. walking to and sitting in a desk).

***Step 2*.** Students explore *simple addition story* by acting it out as a story (e.g. two groups of people joining each other). A discussion is then generated to identify the story elements such as the different groups of people and the action (the joining of the two groups) and the consequences of the action (the result of the joining).

***Step 3*.** Students *create their own symbols* to represent the story. This step could be done in a freestyle manner; however, we have opted to take a more structured approach by using concrete materials (which are familiar to the students) to represent the objects (or people) in the story. The story is then created by allowing the students to construct the two groups of people with the concrete materials and construct their own symbol for "joining two groups" and lay this out to represent the action (or history) of the story. In a similar fashion, the students then construct their own symbol for "resulting in" or "same as" to tell the story of what happens after this action has taken place. Figure 1 gives an example of an addition story that was constructed by a student in Year 2.

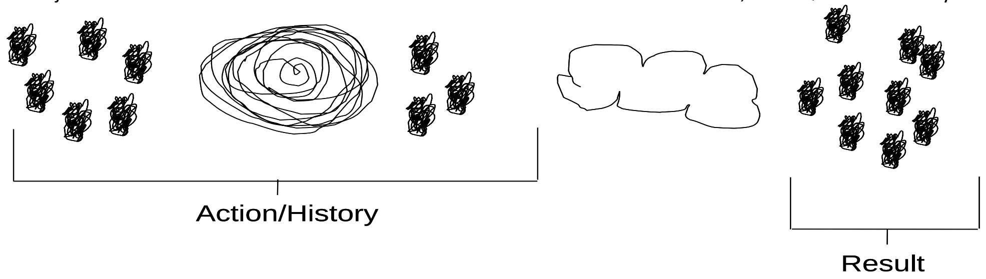


Figure 1. A Year 2 student's representation of the addition story 6 + 3 = 9.

Step 4. Students share their symbol systems with the group and any addition meanings their symbols may have. For example, in Figure 1, the student's "joining" symbol was a vortex that sucked the two groups together. The teacher then selects one of the symbol systems for all the students to use to represent a new addition story. This step is important to accustom students to writing within different symbol systems and to develop a standard classroom symbol system.

Step 5. Students *modify the story* (a key step in introducing algebraic ideas) under direction of the teacher. For example, the teacher takes an object from the action part of the story (see Figure 1), asks whether the story still makes sense (normally elicits a resounding "No"), and then challenges the students by asking them to find different strategies for the story to make sense again. There are four possibilities: (1) putting the object back in its original group, (2) putting the object in the other group on the action side, (3) adding another action (plus 1) to the action side, and (4) taking an object away from the result side. The first three strategies introduce the notion of compensation and equivalence of expression, while the fourth strategy introduces the balance rule (equivalence of equations). At this step, students should be encouraged to play with the story, guided by the teacher, to reinforce these algebraic notions.

Step 6. Students explore the meaning of *unknown* under direction of the teacher. For example, the teacher sets an example with an unknown (e.g. John bought a pie for $3 and an ice cream and he spent $7). The teacher asks the students to represent this without working out the value of the ice cream. Students invent a symbol for unknown and use it in stories with unknowns. Then the students are challenged to solve for unknowns using the balance rule. They have to first determine the operations to leave the unknown on its own. Thus, begins solutions to unknowns in linear equations.

Mathematics behind MAST

The MAST pedagogy is a way of introducing concepts, principles and unknowns for the four operations. The mathematics behind the activities in the booklets is now discussed.

Symbols and concepts (Booklets 1 and 4). The symbols for the four operations and equals are + addition, - subtraction, x multiplication, ÷ division, and = equals. Numbers and these symbols make up expressions (a number sentence without equals, such as 3+4 or 6x7-3) and equations (a number sentence that has an equals sign, such as 3+4=7 or 40-1=6x7-3). The concepts of the operations are complex and cover many situations. The best meanings for the four operations are as follows.

(1) Addition and subtraction are when situations involve joining to make a total or separating a total into parts – addition is when the parts are known and the total is unknown, and subtraction is when the total and one unknown is known and the other part is unknown. For example, in the story *I went to the bank and took out $7.983; this left $5,205 in the bank; how much did I have to start from?*, the operation is addition because $7,983 and $5,205 are parts and the amount at the start is the total and is the unknown (even though the action and the language is "take-away").

(2) Multiplication and division are when situations involve combining equal groups to make a total and separating the total into equal groups – multiplication is when the number of groups and the number in the group is known and the total is unknown, and division is when the total is known and one of the number of groups or number in each group is unknown. For example, in the story *There are 8 times as many oranges as apples; there are 56 oranges; how many apples?*, the operation is division because 8 is the number of groups, 56 is the total and the number of apples is an unknown group (even though the action and language is "times").

However, for these booklets, the following initial simpler (and incomplete) meanings are used:

Addition - joining Subtraction - take-away,

Multiplication - combining equal groups Division - sharing or breaking into groups equally.

The idea in Booklets 1 and 4 (addition and subtraction in 1, and multiplication and division in 4) is that mathematics symbols are a way of telling stories of everyday life with these meanings.

Principles (Booklets 2 and 5). An important component of algebra is understanding when these two things do not change (i.e., are equivalent or invariant) because in this case we can construct arguments where we replace the expression or equation with an equivalent one and find answers and solution to unknowns, such as 6x7-3 = 42-3 = 39 and 6*x*-3 = 39 is the same as 6*x*-3+3 = 39+3 is the same as 6*x* = 42 is the same as 6*x*/6 = 42/7 which is the same as *x*=7. Thus it is important in both arithmetic and algebra to teach when expressions and equations are equivalent.

Two important principles or rules for equivalence are compensation for expressions and balance for equations. Compensation is the set of principles/rules that mean that an expression remains the same if one number is changed. Balance is the set of principles where the equation remains the same after something is done to one side of the equation. For the four operations, the principles are as follows.

(1) The compensation principle for addition is "to do the opposite". In the example, 3 + 4, adding a 2 to the 3 is compensated by subtracting a 2 from the 4 (i.e. 5 + 2 is the same as 3 + 4). Similarly, subtracting 2 from the three is compensated by adding 2 to the 4 (i.e. 1 + 6). These sets of operations can also be done to the 4, which will always result in doing the opposite operation to keep the expression equivalent using the compensation principle.

(2) The compensation principle for subtraction is "to do the same" (the opposite to the principle for addition). Adding/subtracting 2 to or from the 7 in example 7-3 is compensated by adding/subtracting (same operation) 2 to or from the 3 making 9-5 and 5-1 both of which give the same answer as 7-3. Similarly, adding/subtracting 2 to or from the 3 in example 7-3 is compensated by adding/subtracting 2 to or from the 7.

(3) The compensation principles for multiplication is "to do the opposite" (the same as addition and the opposite to subtraction). Multiplying/dividing the 6 by 2 in example 6x4 is compensated by dividing/multiplying (opposite operation) the 4 by 2 making 12x1 and 3x4 both of which give the same answer as 6x4. Similarly, multiplying/dividing the 4 by 2 in 6x4 is compensated by dividing/multiplying the 6 by 2.

(4) The compensation principle for division is "to do the same" (the opposite to addition and multiplication and the same as subtraction). Multiplying/dividing the 18 by 3 in example18÷6, is compensated by multiplying/dividing the 6 by 3 to make 54÷18 and 6÷2 both of which give the same answer as 18÷6. Similarly, multiplying/dividing the 6 by 2 in example 18÷6 is compensated by multiplying/dividing the 18 by 2.

(5) The balance principle for addition is "to do the same to both sides of the equals sign". Adding/subtracting 2 to or from the 3 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side making 5+4=9 or 1+6=5, both of which are equivalent to 3+4=7. Similarly, addiing/subtracting 2 to or from the 4 in example 3+4=7 is balanced by adding/subtracting 2 to and from the 7 on the other side.

(6) The balance principle for subtraction is "to do the same to both sides of the equals sign" (the same principle as for addition) but is complex when the second number is involved. Adding/subtracting 2 to the 7 in example 7-3=4 is balanced by adding/subtracting (same operation) 2 to the 4 on the other side making 9-3=6 and 5-3=2 both of which are equivalent to 7-3=4. However, adding/subtracting 2 to the 3 in 7-3=4 is adding/subtracting to a take-away so is really subtracting/adding 2 to 7-3. This means that it is balanced by subtracting/adding (opposite to what was done to the 3 but the same as effect on the 7-3) 2 to the 4 on the other side making 7-5=2 and 7-1=6 both of which are equivalent to 7-3=4.

(7) The balance principle for multiplication is to "do the same thing to both sides of the equals sign" (the same principle as addition and subtraction – but without complexity for the second number –

unlike subtraction). Multiplying/dividing the 6 by 2 in example 6x4=24 is balanced by multiplying/dividing (same operation) the 24 on the other side making 12x4=48 and 3x4=12 both of which are equivalent to 6x4=24. Similarly multiplying/dividing the 4 by 2 in 6x4=24 is balanced by multiplying the 24 by 2 on the other side.

(8) The balance principle for division is "to do the same thing for both sides of the equals sign" (but with complexity for the second number – the same as subtraction and different in this aspect from addition and subtraction). Multiplying/dividing the 18 by 3 in example 18÷6=3 is balanced by multiplying/dividing (same operation) the 3 on the other side giving 54÷6=9 and 6÷6=1 both of which are equivalent to 18÷6=3. However, multiplying/dividing the 6 by 3 in example 18÷6=3 is multiplying/dividing a division (the number sharing or the size of the group) so is really dividing/multiplying 18÷6 by 3. This means that it is balanced by dividing/multiplying (the opposite to what was done to the 6 but the same as what was done to the 18÷6) by 3 the 3 on the other side making 18÷2=9 and 18÷18=1 both of which are equivalent to 18÷6=3.

With respect to both compensation and balance, addition and multiplication act similarly, and subtraction and division also act similarly but have differences from addition and subtraction. With respect to compensation, addition and multiplication "do the opposite", while subtraction and division "do the same". With respect to balance, all operations "do the same to both sides", but subtraction and division are complex with respect to the second number where "do the opposite" appears to hold. However, the actions to both sides are "the same".

Unknowns (Booklets 3 and 6). When not all numbers are known in a situation, this not-known number has to be represented by a new symbol called a variable or unknown. In formal mathematics, this is represented by a letter. Informally, it can be represented by a box or question mark.

The balance principle can be used to find the unknown. The steps are as follows: (1) the operation (or operations are worked out that will leave the unknown alone on one side; and (2) these operations are balanced on the other side to find the value of the unknown. For example:

$y + 3 = 11$	$y \div 4 = 3$	$5 \times y + 6 = 21$
change the 3 to 0	change the 4 to 1	subtract 6
by subtracting 3	by multiplying by 4	$5 \times y = 21 - 6 = 15$
$y = 11 - 3$	$y = 3 \times 4$	divide by 5
$y = 8$	$y = 12$	$y = 15 \div 5 = 3$

Approach to teaching MAST

A crucial component in any successful program to improve Indigenous students' learning outcomes in mathematics is **high teacher expectations** for these students' learning.

The MAST program has been designed to take account of the strengths of Indigenous students in terms of cultural and social background and learning style. This should be made evident to the Indigenous students at the beginning of each booklet.

Be direct. State that the mathematics they are to do now is different to the mathematics that they have done before – state that it is designed for Goori or Murri students but all students can excel in the new approach. The approach has

- never been trialled anywhere else in the world and you will be the first to try it out;
- it focuses on being creative, to think about mathematics in a different way; and
- to relate mathematics to the world around us.

Be firm in your conviction that all students will be able to do it and, in fact, will thrive and prosper.

ABOUT THIS BOOKLET

Focus of lessons

The four lessons in this booklet are the beginning lessons for MAST. They cover Steps 1, 2, 3 and 4 for addition and subtraction. They focus on introducing the idea of creating symbols for these two operations. In particular, across the four lessons, they are designed to relate five different representations:

(1) the central concepts for addition and subtraction - addition is joining and subtraction is take-away that enable 3+4=7 to be thought of as "3 joining 4 to make or equal 7" and 8-2=6 to be thought of as "8 take-away 2 giving or equalling 6";

(2) real life situations for addition and subtraction – for example, 3+4=7 could be "there were three cakes and I baked another four, this made 7 cakes in all", while 8-2=6 could be "I had $8, I bought an ice-cream for $2, this left me with $6";

(3) acting out of addition and subtraction stories by students;

(4) symbols constructed by the students (in a drawing and integrated with objects) that tell stories of addition and subtraction defined by the concepts and illustrated in the real world situations; and

(5) the formal symbols of mathematics, that is, the "3+4=7" and the "8-2=6".

These Lessons are suitable from Year 2 onwards and can be used into junior secondary school. The lessons attempt to enable students to create their own symbols as follows.

(1) **Lesson 1** focuses on symbols in the world of the student, for example, (road symbols and symbols in Indigenous art) and then encourage the students to invent their own symbols for simple actions in the classroom and simple addition situations where students join other students. Its aims are to (1) relate mathematics symbols to other symbols in students' world, and (2) create symbols to tell an addition maths story.

(2) **Lesson 2** focuses on the addition situation from Lesson 1 and spends time exploring various students' creations that represent addition in a linear fashion with counters and symbols. It is designed to enable students to share their own creations and to read and use other students' creations. It begins the process of relation symbols to real-life situations (i.e., to stories). Its aims are to (1) create linear symbols for addition, and (2) tell addition stories from own and other students' symbols and to write addition stories using own and other students' symbols (i.e., showing both directions - symbols to stories, and stories to symbols).

(3) **Lesson 3** changes the focus from addition to subtraction and encourages students to invent symbols for subtraction situations. It does not teach using other students' creations. Its aims are to (1) create linear symbols for subtraction, and (2) tell subtraction stories from own symbols and to write subtraction stories using own symbols (i.e., showing both directions - symbols to stories and stories to symbols).

(4) **Lesson 4** continues with subtraction enabling students to read and use other students' creations (as was done for addition in Lesson 2). It attempts to relate subtraction to addition and to recognise the commonality of equals. It concludes by focusing on the formal symbols of mathematics. Its aims are to (1) tell subtraction stories from others students' symbols and to write subtraction stories using other student's symbols (i.e., symbols to stories, and stories to symbols), (2) integrate addition and subtraction symbols and stories, and (3) relate created symbols to formal mathematics symbols.

In the second MAST booklet (titled *MAST Additive-Principles Lessons*), the lessons focus on Step 5 of the MAST sequence and look at what happens if the numbers are changed. This leads to the powerful and basic algebraic notions of equivalence through compensating in expressions and balancing in equations. It looks particularly at the similarities and differences in addition and subtraction.

Later booklets (Booklets 3-6) will move onto unknowns and variables, using student-created symbols to solve for unknowns and operate with variables, and repeat the Lessons for multiplication and division.

Hints for teaching

The Lessons give detailed directions with respect to to teaching and learning the MAST ideas. The first rule is that THESE DIRECTIONS DO NOT HAVE TO BE FOLLOWED. Develop your own approach to the Lessons – mix and match from different Lessons (and booklets) – spend more time on certain ideas, and give a lot more reinforcement and practice examples than that which is detailed in these lessons.

The best way to operate is to keep in mind where you want to go and let the students' responses to your earlier teaching direct your later teaching. There can be great merit even in integrating ideas from different booklets.

However, in our few trials, some things have been found useful.

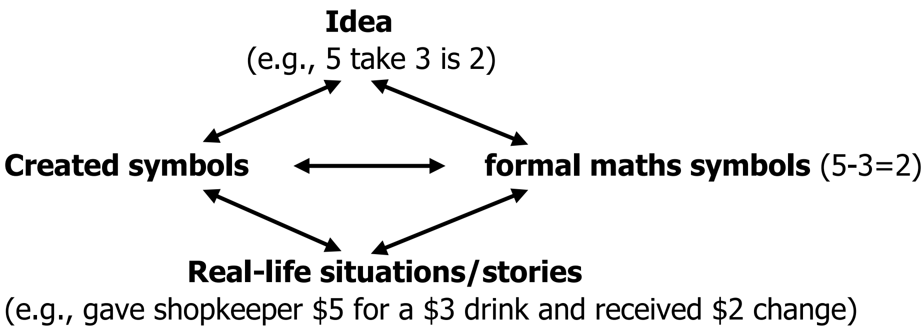
(1) Drawing symbols for a simple task such as a child walking to a desk really works as an introduction to the maths drawings. It is useful to focus on objects and actions and get the students to think of the drawing being like a cartoon showing a series of symbols to tell a story.

(2) Some students (and teachers) do not draw linear symbol-drawings for addition and subtraction – they see these operations in a more integrated fashion. So discuss this aspect – which is better – something that moves left to right or something that moves in from each side? Regardless of the answers from the students (and the teacher), the formal symbols of mathematics are linear left to right in their drawing and so this symbol structure has to take our attention in Lesson 2.

(3) Involve students and their culture in discussion – allow them to share, explain and give point of view without labelling this a right or wrong – allow them to bring in local contexts and Indigenous contexts (i.e., where possible, change the examples to be more representative of the students' situation).

(4) If teachers have Blu tak and magnetic counters, they can set up a white board so that students can stick their A5 drawings on the board between counters to discuss their inventions.

(5) Always move the Lessons in both directions and then in all directions – from idea to created symbols and from created symbols to idea, from created symbols to story and story to created symbols, and from idea to created symbols and created symbols to idea. Act out things as well as talk about things. Overall, lessons will be trying to build all these relationships:



(6) Relate to the symbols in all ways too. A good sequence appears to be:

- Get students to draw their own symbols for your stories and then to make up their own stories for their own symbols.
- Get students to draw using other students' symbols and then to make up stories for other students' symbols
- Get students to draw stories using formal symbols and then to make up stories for formal symbols

(7) Take every chance to look at similarities and differences between addition and subtraction (e.g., different symbols for add and subtract but the same symbol for makes, gives or equals).

MAST ADDITIVE CONCEPTS: LESSON 1

Objectives:	**Materials**:
• To relate mathematics symbols to other symbols in students' world. • To create symbols to tell an addition maths story	• Picture of a traffic symbol (e.g., stop sign) • Examples of Indigenous art which uses symbols • A4 paper

Language: Symbol, story, linear (in a line), not linear, join, addition, representation

What teacher does:	**What children do:**
1. Show students pictures of traffic symbol. Ask: *Have you seen this? Where? What does it mean?* State that this is a "symbol". Ask: *Do you know of other symbols? Draw them!*	Look at pictures, discuss where they have seen the items in the pictures, suggest other symbols, and go to the front and draw their symbols.
2. Show students examples of Indigenous art. Ask: *Who knows the symbols here? What story do they tell?* If appropriate, ask students to make up their own picture with the symbols. Hand out A4 paper. <u>Note</u>: This could be extended into a full art lesson.	Look at the art, discuss the symbols, and suggest what they mean. Draw their own art picture on A4 paper to tell their own story.
3. Tell students, we are going to make up own symbols to tell a story. Direct a student to stand at the door and walk inside and sit at his/her desk. State: *This story has objects and actions.* Ask students to identify them. Ask students to make up own symbols for the objects and actions of this story (student, desk, student at desk, walking, and sitting down) and to draw the story. Hand out A4 paper for the drawings. If students have difficulty, discuss possibilities for the drawing – say that it is like doing a cartoon. Say it "represents" the story. Ask students to show their drawings and explain why they made the symbols as they did. Introduce word "linear". <u>Note</u>: This could be extended into a full lesson by drawing other simple actions.	Watch the students' movement and then suggest objects and actions (objects – student, desk and student at desk; actions – walking and sitting down). Discuss with teacher what symbols could be and how the story could be drawn, for example: ○ → ┌ ↓ ┌○ where ○ is the student, → is walking, ┌ is the desk, ↓ is sitting down, and ┌○ is the student sitting at the desk. Draw own symbols for the story on A4 paper. Show story to other students and explain symbols. Discuss word linear (in a line). Discuss the joining of two symbols can create a new meaning e.g. ┌○
4. Select 5 students and put 3 on one side of the room and 2 on the other. Ask the 2 to walk towards the 3 and form 5. Once again, ask: *Can you identify the objects and actions in this story?* Hand out A4 paper and state: *Make up your own symbols and draw the story.* Pick students to share their stories and explain their symbols (observe as drawing to select good examples but allow others to share if wish to). Discuss the differences between representations or drawings and lead discussion on linear and not	Watch the students walking, and discuss objects and actions (objects – 2 people, 3 people, and 5 people; actions -2 joining 3, and the same as 5). Make up symbols and draw story on A4 paper. Share story with other students and explain what symbols mean. Identify whether stories are linear or not linear. Discuss which is better. ○○ → ○○○ — ○○○○○ OR ○○ ↔ ○○○ / ○○○○○ LINEAR NOT LINEAR

linear.

5. State that 2 walking to 3 to give 5 is an "addition maths story" which "adds" 2 and 3 to equal 5. Discuss how drawings would change if the story was 4 joined 5. Ask students to draw this.	Recognise that story is addition. Discuss changes that would have to be made to the drawings for 4 join 5 gives 9 and draw this story with on A4 paper

Evaluation:

- Do students engage in the lesson?
- Do they see the use of symbols, particularly in the Indigenous art?
- Are students able to create their own symbols and put them together for their own story?

MAST ADDITIVE CONCEPTS: LESSON 2

Objectives:

- To create linear symbols for <u>addition</u>
- To tell addition stories from own and other students' symbols and to write addition stories using own and other students' symbols (i.e., showing both directions - symbols to stories and stories to symbols).

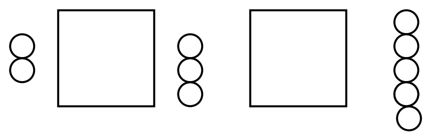
Materials:

- Counters for students (preferably natural objects)
- Magnetic counters for white board and blu tak for teacher
- A5 sheets (half A4)
- Magnetic white board set up with:

counters	blu tack	counters	blu tack	counters

- Worksheet 1

Language: Symbol, story, linear, not linear, join, addition, the same as, equals

What teacher does:	**What children do:**
1. Recap previous lessons. Ask: *What did we do last lesson?*	Discuss last lessons. Suggest things that were done in the last lesson.
2. Select 5 students and, as in lesson 1, direct them to act out 2 students joining 3 students to give 5 students. Hand out counters and A5 sheets and ask them to create action symbols to tell the addition story using the counters to represent the numbers. Direct the students to put out 2, 3 and 5 counters and place the A5 sheets between them.	Watch the acting out of 2 joining 3 to make 5. Create own action symbols (joining, the same as). Use counters and symbols on A5 sheets to represent the addition story, e.g. 
3. Select students to show and explain their symbols. Ensure students understand that the first symbol means 'joining' and second symbol mean 'same as'.	Discuss other students' symbols. Using blu tak, put own A5 symbols on board between counters and explain what the symbols mean.
4. Organise students to act out 4 joining 2 giving 6. Ask students to reuse their symbols for this new addition situation. Introduce the term 'equals' for 'same as'. Lead discussion of what makes an effective symbol. Ask students what would happen if 4 joined 0. Direct students to represent this with their symbols and counters	Students use their previous symbols and their counters to tell the 4 joining 2 giving 6 story. Discuss what makes a good symbol – modify own symbols if necessary. Discuss adding zero. Construct 4 joining zero with symbols and counters.
5. Direct students to complete worksheet 1. Lead students in discussing answers.	Complete worksheet 1. Discuss answers.
6. Get students to make up an addition story and draw it with their own symbols. Select students to come to the front, draw and explain their symbols. Ask: *Can you make up an addition story that this drawing describes*? Give students a context (e.g. money).	Make up own stories and draw them with own or modified symbols. Share symbols with class. Create stories using context given by teacher.

7. Choose one students' (e.g. Cam's) symbols. State: *Use Cam's symbols to represent 3 children joining 1 child to give 4 children.* Hand out A5 sheets. Use Cam's symbols to draw 2 joining 5 to give 7 on board. Ask students to read Cam's symbols if the story was about pencils.	Use Cam's symbols, counters and A5 paper to show 3 joining 1 to make 4. Create an addition story about 2 joining 5 to make 7 using pencils as the context (e.g., Wendy had 2 pencils. Peter gave her 5 more. Now she has 7 pencils).
8. Direct students to complete Worksheet 2. Lead students in discussing answers.	Complete Worksheet 2. Discuss answers.

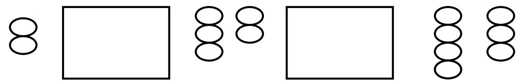
Evaluation:

- Students are engaged in activities and producing creative symbols.
- Students can successfully complete worksheets 1 & 2.

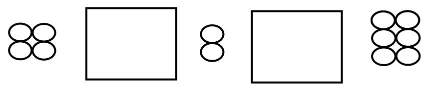
MAST ADDITIVE CONCEPTS 2: WORKSHEET 1

Student name:	**School/class**:

1. Fill in the missing sections by using your symbols. We have done the first one for you.

Addition	Symbols
Example: 2 girls join 5 others to make 7 girls	
(a) 4 girls join 6 boys to make 10 children	
(b) 3 students join 5 students to make 8 students	
(c) 1 boy joins 4 other students to make 5 students	
(d) 7 students join no students to make 7	

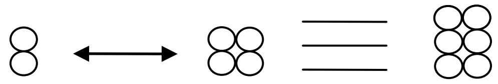
2. Fill in the missing sections using your symbols. The first is done for you.

Addition	Money Story (Be creative)	Symbols
Example: 4 joining 2 to make 6	Jack had $4. Dad gave him $2. Now Jack has $6 altogether.	
(a) 7 joining 4 to make 11		
(b) 5 joining 9 to make 14		
(c)	Henry only had $8. He received $4 for working. This gave him $12.	
(d)	Sue received $7 from her mum and $6 from her grandma. This gave her $13.	

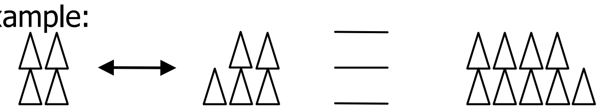
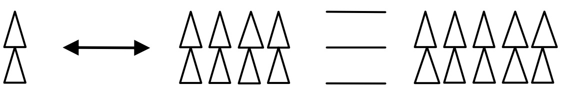
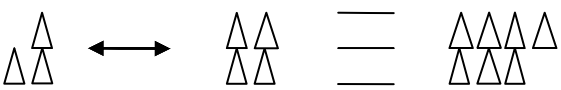
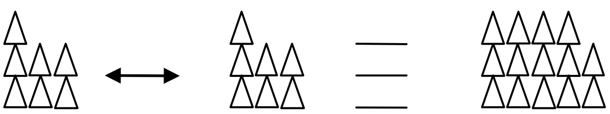
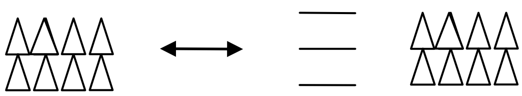
MAST ADDITIVE CONCEPTS 2: WORKSHEET 2

Student name:	**School/class**:

1. James' symbols are ↔ for joining and ≡ for same as. Fill in the missing sections using James' symbols. The first is done for you.

Story	Symbols
Example: Frank had two pencils. He collected 4 more. This made 6 pencils.	
(a) John had 2 ice-creams, he bought another 4. This made 10 ice-creams.	
(b) Sue had $6. Her uncle gave her $3. She then had $9.	
(c) There were 6 golf balls in a bag. Jenny had no more golf balls to put in. This made 6 golf balls.	
(d) Frank ran 11km then ran 2 km more. This made 13km.	

2. What are the stories told by James's symbols? All stories are about cars. Be creative. The first is done for you.

SYMBOLS	STORIES
Example: 	There were 4 cars at the park. 5 more drove up. This made 9 cars at the park.
(a) 	
(b) 	
(c) 	
(d) 	

MAST ADDITIVE CONCEPTS: LESSON 3

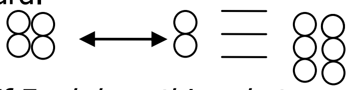
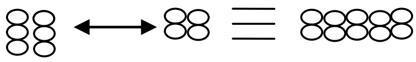
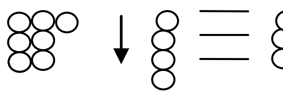
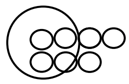
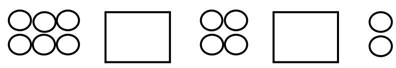
Objectives:

- To create linear symbols for <u>subtraction</u>.
- To tell subtraction stories from own symbols and to write subtraction stories using own symbols (showing both directions - symbols to stories and stories to symbols).

Materials:

- Counters (preferably natural objects) for students
- Magnetic counters and blu tack for teacher
- A4 and A5 sheets
- Magnetic white board set up with:
  counters [blu tack] counters [blu tack] counters
- Worksheet 1

Language: Symbol, story, linear, not linear, take away, subtraction, equals, represent

What teacher does:	What children do:
1. Recap previous lessons. Ask: *What did we do last lesson.* Write ⟷ and ☰ on board. Say: *If joining is* ⟷ *and equals is* ☰ *, draw symbols for "Jenny had $6, she earned another $4 making $10".* Hand out A4 sheets. Draw on board.  Say: *If Fred drew this, what would be the story is we were talking about CDs.*	Discuss previous lesson and suggest things that had happened in the lesson. Draw symbols for 6 joining 4 to make 10 on A4 sheets e.g.  Discuss stories for CDs that are told in the symbols (e.g. I had 2 CDs and my uncle gave me 5. This made 7 CDs).
2. Select 7 students and bring to the front of the class. Ask 4 of the students to walk to the other side of the room. Ask: *What objects and actions are in this story.* Hand out A4 sheets. Say: *Draw the story in your own symbols.* Select students to share their symbols. Ask students: *Explain what your symbols mean?* If there are problems, get the 7 students to act out the story again while other students show their symbols. Discuss different drawings. Differentiate between linear and not linear drawings.	Watch students act out 7 remove 4 to give 3. Discuss objects and actions (objects - 7, 4 and 3 students; actions – "take-away" or "remove", and "same as" or "equals"). Create and draw symbols on A4 sheets. Share symbols with other students and explain what they mean (i.e., how they relate to the story). Differentiate between linear and not linear drawings. Discuss which is better.  OR  LINEAR NOT LINEAR
3. Hand out A5 sheets and counters. Say: We now want everyone to draw a linear story. Ask students to use counters and create symbols on the A5 sheets to tell the following story: *Jenny had $6. She gave Jane $2. This left her with $4.* Select students to share their symbols and to explain why the symbols are like they are. Ask students what would happen if Jenny gave Jane nothing ($0). Direct students to create this with symbols and counters.	Create own symbols for take-away and same as and place these between counters as below:  Share with and explain symbols to other students (use blu tak to place A5 drawings on board between the magnetic counters). Discuss what happens when subtract zero. Construct "Jenny had $6. She gave Jane nothing, leaving her with $6" with symbols and counters.

4. Ask students to use their symbols to describe: *Jack downloaded 5 songs. He deleted one. This left 4 songs.* Ask a few students to share what they did. Discuss what makes a good symbol. Ask students: *What kind of story is this?* After discussion, state (ensure they know): *This is a take-away or subtraction story.*	Students use the previous symbols to tell the "5 take-away 1" story. Discuss what makes a good symbol – modify own if necessary. See that these are take-away or subtraction stories.
5. State: *Make up your own subtraction story and write it in your own symbols.* Select students to bring out and show and explain their symbols (e.g., 10 take 7 is 3). Ask other students to make up a story for these symbols using a shopping context.	Make up own subtraction stories and represent with counters and own symbols. Show other students and explain symbols. Create stories for others symbols using the shopping context (e.g., I gave the shopkeeper $10 for a $7 gift. He gave me $3 change).
6. Direct students to complete worksheet 1 and lead into discussion of answers.	Complete worksheet 1. Discuss answers.

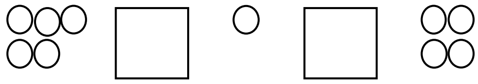
Evaluation:

- Students are engaged in activities creating interesting symbols.
- Students understand subtraction situations.
- Students successfully complete worksheet 1.

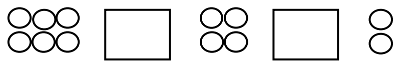
MAST ADDITIVE CONCEPTS 3: WORKSHEET 1

Student name:	**School/class**:

1. Fill in the gaps using your symbols. The first is done for you.

Subtraction	Symbols
Example: 5 children, 1 walks away , 4 left	
(a) 7 children, 5 walk away, 2 left	
(b) 12 children, 8 walk away, 4 left	
(c) John had $9, he spent nothing, he had $9 left	
(d) Fourteen cars were in a car park and 8 left, leaving 6 in the car park.	

2. Fill in the missing gaps using your symbols. Be creative! The first one is done for you.

Subtraction	Money Story (Be creative)	Symbols
Example: 6 take-away 4 make 2	Andy had $6 and spent $4, leaving him with $2	
(a)10 take-away 3 make 7		
(b) 13 take-away 7 make 6		
(c)	Sally had $9 and gave her sister $6. Sally had $3 left.	
(d)	Matt had $4 and bought a chocolate bar for $1. He was left with $3.	

MAST ADDITIVE CONCEPTS: LESSON 4

Objectives:

- To tell subtraction stories from others students' symbols and to write subtraction stories using other students symbols (symbols to stories, and stories to symbols).
- To integrate addition and subtraction symbols and stories.
- To relate created symbols to formal mathematics symbols.

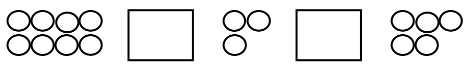
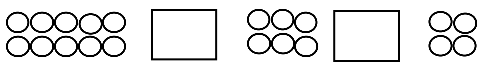
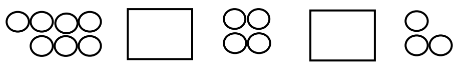
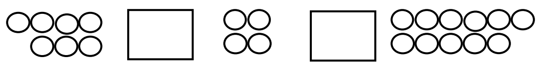
Materials:

- Counters (preferably natural) for students
- Magnetic counters and blu tak for teacher
- Magnetic white board set up with:

  counter [blu tack] counter [blu tack] counter

- Worksheets 1 & 2

Language: Symbol, story, linear, not linear, join, take-away, addition, subtraction, equals, represent

What teacher does:	What children do:
1. Recap last lessons. Ask students to get out their subtraction story symbols and show 8 take-away 3 is 5. Hand out counters and A5 sheets. Ask students to make up their own subtraction story and represent it with symbols and counters. Select student to share symbols and stories. Ask for stories in terms of food.	Recall previous lesson. Use symbols and counters to show 8 take-away 3 is 5, e.g.:  Create subtraction food stories, make them with symbols and counters and share with other students (e.g., I had 11 cakes and I ate 4, this left 7 cakes).
2. Select one students' (e.g. Joe) symbols. Draw them on the board. Ask: *Use these symbols to draw this story: We had 10 drumsticks for tea, we ate 6. This left 4.* Write on board 9 take-away 7 to give 2 on board with Joe's symbols. Ask: *Make up a clothing story which tells what I have written.*	Draw "10 subtract 6 leaves 4" with Joe's symbols, e.g.:  Discuss what Joe's symbols mean. Create a clothing story for the symbols (e.g., I had 9 t-shirts, but 7 were in the wash, so I only had to 2 to choose from).
3. Direct students to complete Worksheet 1. Lead discussion of answers.	Complete worksheet 1. Discuss answers.
4. Ask: *Do you remember your addition symbol – your symbols for joining? Draw them on A5 sheets*. Direct students to show: "Jack had 7 cakes, he ate 4. This left 3", and "Jack had 7 cakes he collected 4 more. This gave him 11 cakes". Ask students to share their symbols. Discuss whether symbols can be the same.	Remember and draw symbol. Construct two representations:  and  Share these with other students. Discuss second symbol, recognise that it could be the same.

5. Choose one students' (e.g. Ann) symbols. Ask students to make up shopping stories for addition and for subtraction and draw them with Ann's symbols. Organise the students to share stories and the corresponding symbols.	Students make up addition and subtraction shopping stories and draw them with Ann's symbols. Share and discuss stories and symbols with other students.
6. Direct students to complete Worksheet 1. Lead discussion of answers.	Complete Worksheet 1. Discuss answers.
7. Direct students to look at Ann's symbols. Ask: *Are there symbols that we normally use in maths for what our symbols do? What are these symbols?* Discuss and show the formal symbols +, - and =. Ask students to use formals symbols for "8 joins 4 to make 12" and "11 take-away 5 is 6" and to make up stories based on bicycles. Organise students to discuss symbols and share questions.	Discuss formal symbols (i.e., join/add +; take-away/subtract -; same/equals =) Write the addition and subtraction situations with formal symbols (e.g., 8 + 4 = 12, 11 – 5 = 6). Discuss stories and share symbols with other students.
8. Direct students to complete worksheet 2. Lead discussion of answers.	Complete worksheet 2. Discuss answers.

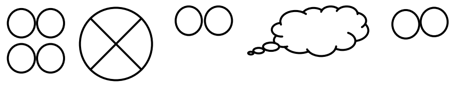
Evaluation:

- Students are engaged and complete worksheets.
- Students see the commonality of =.
- Students can relate their symbols to +, - and =.

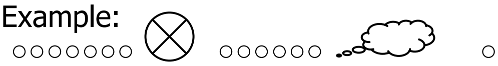
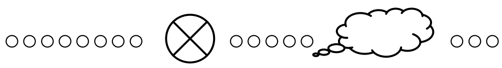
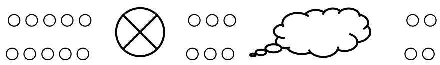
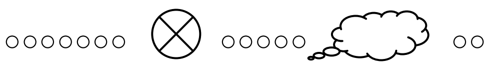
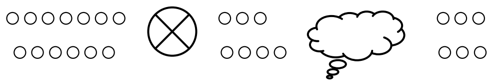
MAST ADDITIVE CONCEPTS 4: WORKSHEET 1

Student name:	**School/class**:

1. Frank's symbols were  for "take-away" and  for "same as". Draw these using Frank's symbols. We have done the first for you.

Subtraction Story	Symbols
Example: Jack had 4 Mars Bars, he ate 2, and this left 2.	
(a) Sue had $6, she spent $5. She had $1 left.	
(b) Jenny baked 13 cakes and ate 5. This left 8 cakes.	
(c) Bill had $11. He spent $3. This left $8.	
(d) John scored 12 and Bob scored 5 less. Bob scored 7.	

4. What is the story told with Frank's symbols? All stories are sport. Be creative! We have done for first for you.

Symbols	Subtraction Story
Example: 	Jane had 7 balls. She gave out 6 balls. This left one.
(a) 	
(b) 	
(c) 	
(d) 	

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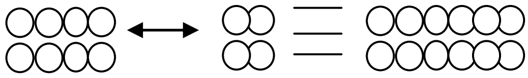
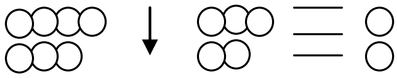
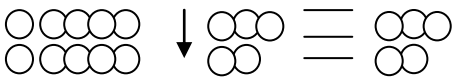
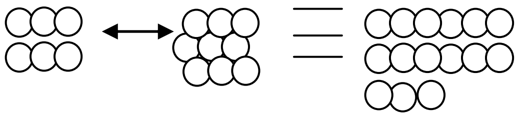
MAST ADDITIVE CONCEPTS 4: WORKSHEET 2

Student name:	**School/class**:

1. Sue's symbols are ↔ for addition, ↓ for subtraction and ≡ for equals. Using these symbols and the formal symbols for mathematics to fill in the missing gaps. Be creative with your stories. We have done the first for you.

Story	Sue's symbol	Maths symbol
Example: Bob has $8. He receives an extra $4 from his grandpa. He now has $11.		8 + 4 = 12
Example: Jane has 7 cakes. She eats 5 leaving 2.		7 – 5 = 2
(a) Mum baked 7 pies. Dad ate 2. This left 2.		
(b) Mary had $12. She spent nothing. She still have $12.		
(c)		
(d)		
(e)		11 + 7 = 18
(f)		8 – 3 = 5
(g) Jack collected 5 lollies then another 2 and finally another 7 making 12 lollies altogether.		
(h)		8 – 3 + 2 = 7
(i)