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Minjerrabah Maths Project

MAST Additive-Principles Lessons

Booklet Pr.P: using created symbols to develop Addition stories, and exploring the principles of compensation and balance

Minjerrabah Maths Project

Booklet for Prep. Trial

MAST ADDITIVE-PRINCIPLES LESSONS

Maths as Story Telling lessons on: using created symbols to develop Addition Stories, and the exploring the principles of compensation and balance.

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YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called “Deadly Maths”.

“YuMi” is a Torres Strait Islander word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at <http://ydc.qut.edu.au> and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

MINJERRIBAH MATHS PROJECT

MAST

MAST ADDITIVE-PRINCIPLES LESSONS

BOOKLET Pr.P

**USING CREATED SYMBOLS TO DEVELOP ADDITION STORIES,
AND EXPLORING THE PRINCIPLES OF
COMPENSATION AND BALANCE**

VERSION 1: 2007

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BACKGROUND

Minjerribah Maths Project

The Minjerribah Maths Project was a collaboration between Griffith University and Queensland University of Technology with Dr Chris Matthews from Griffith University as coordinator. The researchers involved in the project were:

- Dr Chris Matthews, Coordinator and Principal Researcher, Centre for Environmental Systems Research, Griffith University;
- Professor Tom Cooper, Researcher, Centre for Learning Innovation, QUT;
- Ms Margaret Grenfell, Research Assistant, Centre for Environmental Systems Research, Griffith University;
- Ms Tiara Cassady, Research Assistant, Centre for Learning Innovation, QUT.
- Mr Todd Phillips, Research Assistant, Centre for Environmental Systems Research, Griffith University.
- Ms Ashlee Surha, Research Assistant, Centre for Environmental Systems Research, Griffith University.

MAST (Maths as Story Telling) pedagogy

MAST is the first product developed for the Minjerribah Maths Project. It is an attempt to work from the storytelling world of the Indigenous student through to the formal world of algebra by experiences with the creation of symbols that have personal meaning. The storytelling starts with simple arithmetic but moves quickly to algebraic thinking. It enables Indigenous students to bring their everyday world of symbols into mathematics.

It is an answer to the dilemma of contextualising the teaching and learning of algebra. It focuses on representing mathematical equations as stories which leads to contextualising of mathematical symbols. It is an approach to symbolisation based on students creating and using their own symbols, drawn from their socio-cultural background, to describe these stories as a precursor to working with the accepted mathematics symbols. It utilises Indigenous knowledge of symbols within domains such as sport, driving, art and dance as a starting point for building understanding of arithmetic symbolism in a way that can be easily extended to algebraic symbolism. The approach has five steps. These steps are explained for addition. Obviously, the other three operations could be similarly undertaken.

Step 1. Students explore the *symbols and their meaning* and how symbols can be assembled to tell and create a story. This is initially done by looking at symbols in Indigenous situations (e.g., exploring and understanding symbols in paintings) and then creating and interpreting symbols for simple actions (e.g. walking to and sitting at a desk).

Step 2. Students explore a *simple addition story* by acting it out as a story (e.g. two groups of people joining each other). A discussion is then generated to identify the story elements such as the different groups of people and the action (the joining of the two groups) and the consequences of the action (the result of the joining).

Step 3. Students *create their own symbols* to represent the story. This step could be done in a freestyle manner; however, we have opted to take a more structured approach by using concrete materials (which are familiar to the students) to represent the objects (or people) in the story. The story is then created by allowing the students to construct the two groups of people with the concrete materials and construct their own symbol for "joining two groups" and lay this out to represent the action (or history) of the story. In a similar fashion, the students then construct their own symbol for "resulting in" or "same as" to tell the story of what happens after this action has taken place. Figure 1 gives an example of an addition story that was constructed by a student in Year 2.

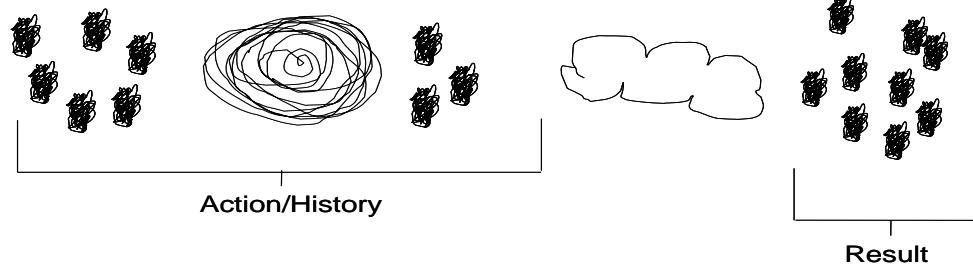


Figure 1. A Year 2 student's representation of the addition story $6 + 3 = 9$.

Step 4. Students *share their symbol* systems with the group and any additional meanings their symbols may have. For example, in Figure 1, the student's "joining" symbol was a vortex that sucked the two groups together. The teacher then selects one of the symbol systems for all the students to use to represent a new addition story. This step is important to accustom students to writing within different symbol systems and to develop a standard classroom symbol system.

Step 5. Students *modify the story* (a key step in introducing algebraic ideas) under direction of the teacher. For example, the teacher takes an object from the action part of the story (see Figure 1), asks whether the story still makes sense (normally elicits a resounding "No"), and then challenges the students by asking them to find different strategies for the story to make sense again. There are four possibilities: (1) putting the object back in its original group, (2) putting the object in the other group on the action side, (3) adding another action (plus 1) to the action side, and (4) taking an object away from the result side. The first three strategies introduce the notion of compensation and equivalence of expression, while the fourth strategy introduces the balance rule (equivalence of equations). At this step, students should be encouraged to play with the story, guided by the teacher, to reinforce these algebraic notions.

Step 6. Students explore the *meaning of unknown* under direction of the teacher. For example, the teacher sets an example with an unknown (e.g. John bought a pie for \$3 and an ice cream and he spent \$7). The teacher asks the students to represent this without working out the value of the ice cream. Students invent a symbol for unknown and use it in stories with unknowns. Then the students are challenged to solve for unknowns using the balance rule. They have to first determine the operations to leave the unknown on its own. Thus, begins solutions to unknowns in linear equations.

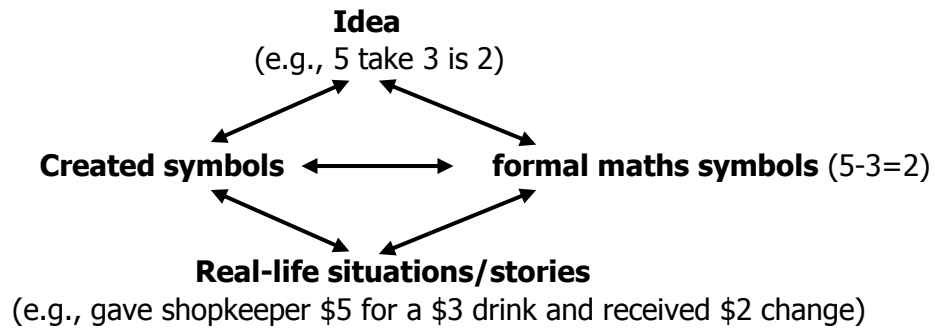
Hints for teaching

The booklets give detailed directions with respect to the Lessons. The first rule is that THESE DIRECTIONS DO NOT HAVE TO BE FOLLOWED. Develop your own approach to the Lessons – mix and match from different Lessons (and booklets) – spend more time on certain ideas, and give a lot more reinforcement and practice examples than in these lessons.

The best way to operate is to keep in mind where you want to go and let the students' responses to your earlier teaching direct your later teaching. There can be great merit even in integrating ideas from different booklets.

However, in our few trials, some things have been found useful.

- (1) Involve students and their culture in discussion – allow them to share, explain and give point of view without labelling this a right or wrong – allow them to bring in local contexts and Indigenous context (i.e., where possible, change the examples to better represent the students' situation).
- (2) If teachers have blu tak and magnetic counters, they can set up a white board so that students can stick their A5 drawings on the board between counters to discuss their inventions.
- (3) Always move the Lessons in both directions and then in all directions – from idea to created symbols and from created symbols to idea, from created symbols to story and story to created symbols, and from idea to created symbols and created symbols to idea. Act out things as well as talk about things. Overall, lessons will be trying to build all these relationships:



(4) Relate to the symbols in all ways too. A good sequence appears to be:

- Get students to draw their own symbols for your stories and then to make up their own stories for their own symbols.
- Get students to draw stories using other students' symbols and then to make up stories for other students' symbols
- Get students to draw stories using formal symbols and then to make up stories for formal symbols

(5) Take every chance to look at similarities and differences between addition and subtraction (e.g., different symbols for add and subtract but the same symbol for makes, gives or equals).


(6) Introduce formal mathematics symbols in a similar way that would get students to use other students' created symbols. Watch that the introduction of formal symbols does not change students' focus to "getting answers" rather than discussing/inventing rules.

Summary of the resource

The main outline is as follows:

- Develop students understanding of symbols;
- Develop students confidence in creating symbols and creating stories using symbols;
- Create a simple addition story using the students own symbols;
- Discover algebra concepts (balance and compensation) through modifying the story;
- Use the balance concept to solve simple linear equations.

MAST LESSON 1: SYMBOLS AND MEANING

<p>Objectives:</p> <ul style="list-style-type: none"> • To explore symbols in the student's everyday life; • To explore the meaning of symbols; and • Gain an understanding that symbols can be interpreted in many ways. 	<p>Materials:</p> <ul style="list-style-type: none"> • A set of cards with a picture of symbols on each card; • Symbols should be drawn from the local community and from different cultural backgrounds.
<p>Language: Symbol, meaning, represents, simple drawings</p>	
<p>What teacher does:</p> <ol style="list-style-type: none"> 1. Show students a picture of a symbol from the local community (e.g. stop sign, life guard signs). <i>Ask: Have you seen this before? Where have you seen it? What does it mean?</i> <p>State that this is a "symbol" that has a particular and important meaning.</p> <p>Define what a symbol is i.e.</p> <p style="padding-left: 40px;">" a simple drawing that has a meaning."</p> <ol style="list-style-type: none"> 2. State that different cultures have different symbols. Show the students a picture of the following symbol: <div style="text-align: center; margin: 10px 0;">  </div> <p>Note: try and pick a symbol that the students may not be familiar with.</p> <p><i>Ask the students: Have you seen this before? Where have you seen it?</i></p> <p>Ask the students to guess the meaning of the symbol and write the meanings the students suggest on a white board.</p> <p>State that for many Aboriginal people the symbol means "someone sitting down".</p> <p>Ask the students how the symbol could represent "someone sitting down". Pick some other meanings from the board and ask how the symbol could represent that meaning.</p> <p>State that symbols can have different meanings for different people. Also, reinforce that symbols are "simple" draws that represents a particular "meaning". 3. Ask the students: Would you like to play a symbols guessing game. <p>Teacher will have a set of cards with a symbol on each card. The teacher will show the students a card at a time. The students will try to guess the intended meaning of the symbol. Once the</p> </p>	<p>What children do:</p> <p>Look at the picture, discuss where they have seen the symbol and discuss the meaning of the symbol.</p> <p>Look at the picture and discuss if they have seen the symbol before and where they may have seen the symbol.</p> <p>Students guess the meaning of the symbol</p> <p>Discuss the cultural meaning of the symbol and how this meaning can be represented by the symbol.</p> <p>Discuss alternative meanings of the symbol and how these meanings can be represented by symbol.</p> <p>Students look at each symbol and try to guess the meaning of the symbol.</p> <p>Students participate in a discussion for each symbol and how the simple drawing represents the meaning of the symbol.</p>


meaning of the symbol has been revealed, the teacher leads a discussion about how the symbol represents that meaning.

Again, reinforce that the symbol is a “simple” drawing that represents the intended meaning.

Evaluation:

- Students gain an understanding of what a symbol is and that symbols carry a certain meaning;
- Symbols can be interpreted differently;
- Symbols can have different meanings for different people.

MAST LESSON 2: CREATING YOUR OWN SYMBOL

<p>Objectives:</p> <ul style="list-style-type: none"> • Students to think creatively; • Students create their own symbol for a particular meaning; • Students confidently share their symbol with the class. 	<p>Materials:</p> <ul style="list-style-type: none"> • A4 paper • Colour pencils • Blue tack.
<p>Language: Symbol, meaning, creative.</p>	
<p>What teacher does:</p> <ol style="list-style-type: none"> 1. Recap previous lesson. Ask: What did we do last lesson? 2. The teacher creates his/her own symbol (or gets an existing symbol that is not commonly seen), draws the symbol on the white board and challenges the class to guess the meaning of the symbol. For example, the symbol  , which I have decided means "wind". <p>Note: try to create a symbol that has the potential of having more than one meaning.</p> <p>Ask the students to try and guess the meaning of the symbol. Write all possible meanings on the white board. The teacher writes his/her meaning on the board or, if already on the board, circles the intended meaning. This could be played more like a game where the intended meaning is written on a card and placed in an envelop etc.</p> <p>Generate a discussion about why someone may think the symbol could mean "water" for example.</p> <p>The teacher should reinforce that symbols can be interpreted differently and sometimes the same symbol can have different meanings for different people.</p> <ol style="list-style-type: none"> 3. The teacher tells the students that we are going to do a challenge. The teacher places a word on the white board. Note that the word will initially be associated with an object/subject (e.g. a chair). The teacher asks the students as a challenge to create their own symbol that has the same meaning as the word (i.e. a symbol for chair). <p>Teacher needs to reinforce that the challenge is to draw a simple representation (not a whole picture) that conveys the above meaning.</p> <p>Teacher will ask students to share the symbol by sticking it on the board and asks the student to explain why the symbol represents a chair.</p>	<p>What children do:</p> <p>Discuss last lesson. Suggest things that were done in the last lesson.</p> <p>Students guess the meaning of the symbol while the teacher writes then on the board.</p> <p>Students draw on an A4 paper their symbol for "chair".</p> <p>Students share their symbol with the class, explain the meaning of the symbol and how their symbol represents the object.</p>

4. The teacher puts forward an action word, for example jumping, and challenges the students to create a symbol for the action walking.

Teacher reinforces that a symbol will be a "simple" drawing that "represents" the action of walking.

Teacher will ask students to share the symbol by sticking it on the board and asks the student to explain why the symbol represents "walking".

Teacher should generate a discussion about how the different students represented "walking".

Note: point 3 and 4 can be repeated for different object/subject and action words. Ensure that one of the words is a subject like a bird or a person. Have a range of symbols with different meanings will assist with point 5 below.

5. Teacher introduces the idea that symbols can be combined to give a new meaning or a story.

Teacher combines two symbols on the board to give a new meaning. For example, put the symbol for "bird" next to a symbol for "jumping". These symbols should be drawn from the set of symbols developed by the students.

Teacher tells the class that I have combined the symbol for "jumping" with the symbol for "bird". Teacher then asks the class where the symbols together make sense and what would be the new meaning of the combined symbols.

The teacher should then do a nonsense example like "chair" combined with "walking" and ask the student whether it makes sense and ask the students to explain why it doesn't make sense.

Teacher will then direct the class to play with their created symbols and combine the symbols to create new meanings.

Teacher encourages students to share their combined symbols with the class.

Students draw on an A4 paper their symbol for "walking".


Students share their symbol with the class, explain the meaning of the symbol and how their symbol represents the action of "walking".

Students guess the meaning of the combined symbols and discuss with the teacher this new meaning.

Students then combined their own symbols to create new meaning.

Students share combined symbols with class and articulate the new meaning of the combined symbols.

MAST LESSON 3: CREATING YOUR OWN STORY USING SYMBOLS

<p>Objectives:</p> <ul style="list-style-type: none"> • Understand that many symbols can be used to create a story • Symbols are placed within a 'structure' or a particular order to convey a story. • To construct a story using a set of symbols from a simple story. 	<p>Materials:</p> <ul style="list-style-type: none"> • A4 paper • Colour pencils • Blue tak
<p>Language: Symbol, meaning, story, structure</p>	
<p>What teacher does:</p> <ol style="list-style-type: none"> 1. Recap previous lesson. Ask: What did we do last lesson? 2. Tell students, we are going to create our own symbols and arrange them to tell a simple story. Direct a student to stand at the door and walk inside and sit on his/her chair. Teacher asks the student what the simple story was about. <i>What happened at the start? How did the story finish? What happen in the middle of the story?</i> Teacher leads a discussion with the students to identify the actions (walking and sitting) and the objects/subjects in the story (student, chair). 3. In a similar fashion to last lesson, ask students to create their own symbols for the objects and actions of the story (student, chair, student sitting on chair, walking, and sitting down) and to draw the story. Hand out A4 paper for the drawings. Ask students to show their drawings and explain why they made the symbols as they did. Note: This could be extended into a full lesson by drawing other simple stories. 	<p>What children do:</p> <p>Discuss last lesson. Suggest things that were done in the last lesson.</p> <p>Watch the students' movement</p> <p>Participates in discuss to identify objects and actions (objects – student, chair; actions – walking and sitting down) in the story.</p> <p>Discuss with teacher what symbols could be and how the story could be drawn, for example:</p> <div style="text-align: center;">  </div> <p>where ☺ is the student, is walking, H is the chair, ↓ is sitting down, and ☺ is the student sitting on the chair.</p> <p>Draw own symbols for the story on A4 paper.</p> <p>Show story to other students and explain symbols.</p>

MAST LESSON 4: CREATING A SIMPLE ADDITION STORY**Objectives:**

- To use symbols to create a simple addition story
- To tell addition stories from own and other students' symbols and to write addition stories using own and other students' symbols (i.e., showing both directions - symbols to stories and stories to symbols).

Materials:

- Counters for students (preferably natural objects)
- Magnetic counters for white board and blue tack for teacher
- A5 sheets (half A4)
- Magnetic white board set up with:

counters

blu
tack

counters

blu
tack

counters
- Worksheet 1 and 2

Language: Symbol, story, linear, not linear, join, addition, the same as, equals

What teacher does:

1. Recap previous lessons. Ask: *What did we do last lesson?*
2. Select 5 students and direct them to act out 2 students joining 3 students to give 5 students.
Teacher generates a discussion about what had occurred and directs the students to identify the objects/subjects in the story and the action in the story.
Hand out counters and A5 sheets and ask them to create action symbols to tell the addition story using the counters to represent the numbers.
Direct the students to put out 2, 3 and 5 counters and place the A5 sheets between them.
3. Select students to show and explain their symbols. Ensure students understand that the first symbol means 'joining' and second symbol means 'same as'.
4. Get students to make up an addition story and draw it with their own symbols. Select students to come to the front, draw and explain their symbols.
Ask: *Can you make up an addition story that this drawing describes?* Give students a context (e.g. money).
5. Choose one students' (e.g. Cam's) symbols. State: *Use Cam's symbols to represent 3 children joining 1 child to give 4 children.* Hand out A5 sheets.
Use Cam's symbols to draw 2 joining 5 to give 7 on board. Ask students to read Cam's symbols if the story was about pencils.

What children do:

- Discuss last lessons. Suggest things that were done in the last lesson.
- Watch the acting out of 2 joining 3 to make 5. Create own action symbols (joining, the same as). Use counters and symbols on A5 sheets to represent the addition story, e.g.
- ○

○

○

○

○

○
- Discuss other students' symbols. Using blue tack, put own A5 symbols on board between counters and explain what the symbols mean.
- Make up own stories and draw them with own or modified symbols.
Share symbols with class.
Create stories using context given by teacher.
- Use Cam's symbols, counters and A5 paper to show 3 joining 1 to make 4.
Create an addition story about 2 joining 5 to make 7 using pencils as the context (e.g., Wendy had 2 pencils. Peter gave her 5 more. Now she has 7 pencils).

MAST LESSON 5: MODIFYING THE STORY

Objectives:

- To see if students can be creative with respect to changing a feature of the created symbols for addition.
- To introduce different ways of making addition stories correct.
- To introduce notion of equivalence for addition.

Materials:

- Counters (preferably natural objects) for students.
- Magnetic counters and blu tack for the teacher
- A4 and A5 sheets
- Board set up with:
 Counters

blu
tack

 counters

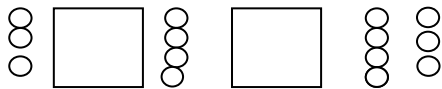
blu
tack

 counters
- Worksheet 3

Language: Symbol, story, addition, equals, linear, representation, replacement, equivalence

What teacher does:

1. Recap creating addition symbols for stories. Ask students to use counters and A5 sheets to represent 3 students joining 4 students to give 7 students.



Discuss some students' results. Point out the linear nature of the drawing.

2. Direct students to remove 1 counter from the 3. Ask: *Is this still correct?* Discuss why it is not. Ask: *What can we do to make it correct?* Discuss options until all 4 possibilities are given. Record on board. State that all these options are 'equivalent' to the original. Note: Students might bring in a 5th possibility by using subtraction.

Introduce the students to the concept "balance" and "compensation".

3. Repeat 1 and 2 above for this story, "Bill ate 4 pies, then he ate 5 more, making 9 pies eaten", but this time direct students to remove 2 counters from the 4.

Discuss why this makes the story wrong. Ask students to propose ways to make it true again. Try to draw out as many different ways as possible.

Discuss whether it matters if 2 counters are removed from the 5 instead of the 4 counters.

What children do:

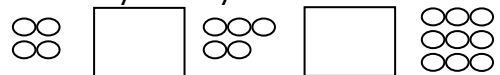
Discuss creating symbols for addition stories. Use previous or modified symbols for 3 join 4 makes 7. Share drawings with other students. Discuss different symbols and linear nature of drawing.

Remove one counter. Discuss why addition story is not correct and suggest possibilities to make it correct again. For e.g.

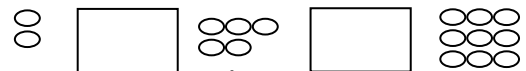
- (1) Put counter back (replacement)
- (2) Add extra counter to 4
- (3) Remove one counter from 7
- (4) Add extra joining symbol to left hand side, e.g.

Discuss term "equivalence".

Draw the story with symbols:



Remove 2 counters from the 4.



Suggest ways to make it correct.

Remove 2 counters from 5.



Discuss how to make this correct again and

whether there are any differences to removing 2 from the 4.

4. Repeat 3 above but add 2 counters to the 4 in 4 joins 5 gives 9. Discuss how this is different to removing 2 from the 4. What is the same? What is different?

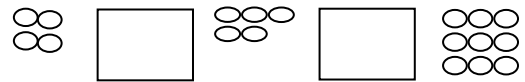
Ask: How would adding 2 to the 5 change things?

Discuss difference & similarities. *Ask: Can anyone see a pattern or rule to follow?*

If students have difficulty with rules, try further examples – adding or subtracting counters from the first and second number.

Ensure students can see that it makes no difference which number is affected, (first or second).

Repeat 3 above for:



but with 2 added to the 4.

Offer options and discuss: 'what is the same?' and 'what is different?'

Look at adding 2 to the 5 and discuss this.

Think of a rule that encompasses all examples.

Evaluation:

- Students engage and offer opinions.
- Students suggest sensible rules.