YUMI DEADLY CENTRE
School of Curriculum

Enquiries: +61 7 3138 0035
Email: ydc@qut.edu.au

http://ydc.qut.edu.au

Engineering and Mechanics

Mathematics behind Small Engine Repair and Maintenance

Booklet VE2: Rate, Ratio, Time, Fuel, Gearing and Compression

DEADLY MATHS VET

Engineering and Mechanics

Palm Island and Thursday Island TAFE

MATHEMATICS BEHIND SMALL ENGINE REPAIR MAINTENANCE

BOOKLET VE2: RATE, RATIO, TIME, FUEL, GEARING AND COMPRESSION

VERSION 1

VE2

Deadly Maths Consortium

Tom J Cooper & Annette R Baturo & Chris J Matthews
with
Elizabeth A Duus, Kaitlin M Moore & Matthew T Michaelson

Deadly Maths Consortium
School of Mathematics, Science and Technology Education
Faculty of Education, QUT

This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

Restricted waiver of copyright

Electronic edition 2011

School of Curriculum
QUT Faculty of Education
S Block, Room S404, Victoria Park Road
Kelvin Grove Qld 4059
Phone: +61 7 3138 0035
Fax: + 61 7 3138 3985
Email: ydc@qut.edu.au
Website: http://ydc.qut.edu.au

CRICOS No. 00213J

This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

Queensland University of Technology

DEADLY MATHS VET

Engineering and Mechanics

MATHEMATICS BEHIND SMALL ENGINE REPAIR AND MAINTENANCE

BOOKLET VE2

RATE, RATIO, TIME, FUEL, GEARING,

AND COMPRESSION

08/05/09

Research Team:

Tom J Cooper

Annette R Baturo

Chris J Matthews

with

Elizabeth A Duus

Kaitlin M Moore

Matthew T Michaleson

Deadly Maths Group

School of Mathematics, Science and Technology Education,
Faculty of Education, QUT

THIS BOOKLET

This booklet VE2 was the second booklet produced as material to support Indigenous students completing certificates associated with engineering and mechanics at the TAFE campuses on Palm Island and Thursday Island. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

VB1: Mathematics behind whole-number place value and operations
Booklet 1: Using bundling sticks, MAB and money

VB2: Mathematics behind whole-number numeration and operations
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

VC1: Mathematics behind dome constructions using Earthbags
Booklet 1: Circles, area, volume and domes

VC2: Mathematics behind dome constructions using Earthbags
Booklet 2: Rate, ratio, speed and mixes

VC3: Mathematics behind construction in Horticulture
Booklet 3: Angle, area, shape and optimisation

VE1: Mathematics behind small engine repair and maintenance
Booklet 1: Number systems, metric and Imperial units, and formulae

VE2: Mathematics behind small engine repair and maintenance
Booklet 2: Rate, ratio, time, fuel, gearing and compression

VE3: Mathematics behind metal fabrication
Booklet 3: Division, angle, shape, formulae and optimisation

VM1: Mathematics behind handling small boats/ships
Booklet 1: Angle, distance, direction and navigation

VM2: Mathematics behind handling small boats/ships
Booklet 2: Rate, ratio, speed, fuel and tides

VM3: Mathematics behind modelling marine environments
Booklet 3: Percentage, coverage and box models

VR1: Mathematics behind handling money
Booklet 1: Whole-number and decimal numeration, operations and computation

CONTENTS

OVERVIEW

The ASISTM VET project funded in 2008 by the Australian Schools Innovations in Science, Technology and Mathematics scheme had 6 sites: Wadja Wadja HS at Woorabinda, Shalom Christian College in Townsville, Palm Island Post Year 10 Campus (run by Kirwan State High School and Barrier Reef Institute of TAFE), Tagai College Secondary Campus at Thursday Island, Thursday Island campus of Tropical North Queensland Institute of TAFE Campus, and Northern Peninsula Area College at Bamaga. All these sites have Indigenous students and the project focused on developing instruments and materials to assist the teaching of mathematics needed for certification for Indigenous VET students with little previous success in school.

The meeting held with VET staff from Palm Island and Thursday Island (TI) requested support for a program in which Indigenous VET students were taught how to repair and maintain small engines (e.g. outboard motors, motor mowers, etc). Indigenous people at Palm Island and TI run boats similar to the way city people run cars and have the same needs with respect to outside work as city people. There are many small engines used in these two islands – outboard motors, motor mowers, whipper snippers, chain saws, and so on. This means that the training will be of use both for certification and to enable these motors to be repaired by people on the Island.

The VET staff requested support with units and unit conversions associated with the engines (both metric and imperial) plus help with use of formulae for describing engines (e.g., volume). They also asked for support in teaching gearing and compression ratios, and ratios for mixing fuel. To this was added some work on rates for speed and fuel use.

To meet this, Deadly Maths VET completed two booklets as follows:

Booklet 1:

(1) Numeration basis of units - structure of number systems (around ones), building large numbers through bundling ones, powers of 10, and small numbers by partitioning ones into fractions (common, any base and decimal – base 10).

(2) Metric units and conversions on structure of the base 10 number system, multiplication relationships, connection to metrics, conversions of metric units, and use of micrometer for hundredths of a mm.

(3) Fractions and Imperial units - common fractions (part of a whole, number line), equivalent fractions, fraction basis of Imperial units, and use of micrometer for thousandths of an inch.

(4) Area and volume formulae – formulae for area of rectangle and circle, and volume of cylinder and semi sphere.

Booklet 2:

(1) Nature of rate and ratio – the mathematics of ratio and rate, models for interpreting rate and ratio, multiplication and multipliers, and use of computers to show change.

(2) The double number line method – the understanding behind the method, and its use in solving rate and ratio problems.

(3) Application of rate to small engines – time, distance and speed, and fuel consumption.

(4) Application of ratio to small engines - fuel mixes, gearing ratios, and compression ratios (including investigation of power, torque and speed).

This booklet (Booklet 2) covers the mathematics underpinning ratio and rate.

1. THE NATURE OF RATE AND RATIO

1.1 Rate vs. ratio

Rate and ratio compare two amounts using multiplication and division, called multiplicative comparisons. Two numbers, for example, 4 and 20, can be compared in each of three ways:

(a) by number: 20 is bigger than 4

(b) by addition: 20 is 16 more than 4

(c) by multiplication: 20 is 5 times as much as 4

Comparison (c) is the meaning behind rate and ratio. For example, if the 20 is money ($) and the 4 is kg of meat, then we can say that the cost of the meat is $5/kg (rate), that is, the money is 5 times the meat. Similarly, if 20 is the number of litres of water and 4 is the number of litres of cordial, then we can say that the water to cordial mix is 20:4 or 5:1 (ratio), that is, the water is 5 times the cordial.

Rate and ratio come from different aspects of our lives but have different histories. Rate is usually used when there are different measures, that is, $7 is the cost of 2 L of cordial gives us a rate of $7/2 L = $3.50 per 1 L. Ratio is usually used when attributes are the same, that is, 7 L of water to 2 L cordial gives a ratio of 7:2.

Thus, in the diagram below, rate is the unshaded areas and ratio is the shaded areas:

	$	Length	Area	Volume	Mass	Time
$						
Length						
Area						
Volume						
Mass						
Time						

However, in reality, this classification is sometimes not followed. For example, with paint, it is common to say that paint is applied to a wall 2L of paint for every 7 m^2 of wall. This is using ratio language for what would normally be a rate situation.

Activity 1.1

(1) Compare the sets of two numbers below in each of the three ways mentioned above. Write each as a rate and a ratio:

(a) 6 and 4 (b) 8 and 24 (c) 10 and 15

(2) Determine whether each of the following situations is normally a rate or a ratio:

(a) Sand and cement is mixed mass to mass.

(b) Sand costs dollars per mass.

(c) Paint covers are per volume.

(d) Cordial is mixed with water volume to volume.

(3) State a rate when there is comparison between:

(a) area and length (b) time and money (c) mass and volume.

(4) Construct a ratio when there is comparison between:

(a) area to area (b) mass to mass (c) temperature to temperature.

(5) Determine whether each of the following situations is normally a rate or a ratio:

(a) Butter to milk is 1:2.

(b) Butter costs $5.40 per kilogram.

(c) The pressure is 2 kg per cm^2.

(d) Number of bottles of coke to bottles of lemonade is 4:3.

(6) Create a relevant rate or ratio problem for:

(a) 6:4 (b) 6:1 (c) 4:7

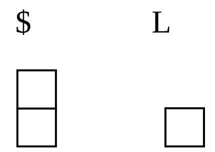
1.2 Models for rate and ratio

A pictorial model can help understanding of rate and ratio and assist with finding the correct solution for rate and ratio problems. There are three models commonly used: set/area, number line, and multiplicative change.

Set/Area. Rate and ratio can be considered in terms of sets or area. For example, if cordial is $2 per litre, then the rate can be considered in terms of Diagram A. If cordial is 1:2, then ratio can be considered in terms of Diagram B.

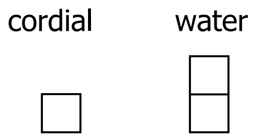
<u>Diagram A:</u>

Rate $2/L can be shown as



<u>Diagram B:</u>

Ratio 1:2 can be shown as



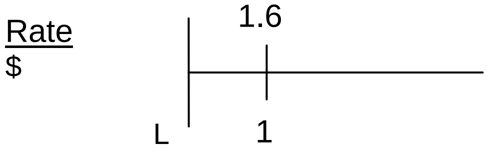
These drawings can be used to work out solutions to problem. For example, if in A there is 8 L of cordial, then each square is 8 and thus we need $16 to buy the cordial, while if in B there is 14L of water, then each square is 7L and so we need 7L of cordial to mix with the water.

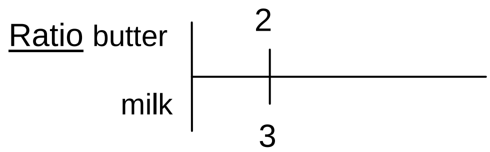
Drawing B is also useful in showing that, for ratio, there is a whole made up of the mixed cordial and water. For the example above, this whole is 21L (the 14 L of water with the 7L of cordial). This means that we can consider both the water and the cordial as fractions (i.e., 7/21 = 1/3 for the cordial and 14/21 = 2/3 for the water). This shows that rate, like fractions, is composed of two parts and a whole. However, unlike fractions, ratio is expressed in terms of the two parts not, as for fractions, in terms of part of a whole.

Number line.

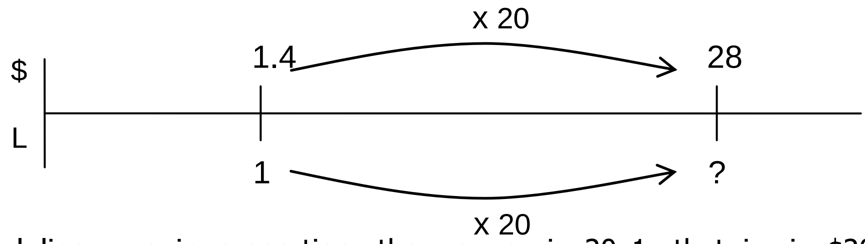
The second model for rate and ratio is what is usually called the double number line. This is a particularly good model for problem solving (as shown Section 2).

Rates and ratios are represented by giving each side of the line to the two components of the rate or ratio. Thus a rate of $1.60/L and a ratio of butter to milk of 2:3 would be as follows (Note – the rate always has one side as 1):



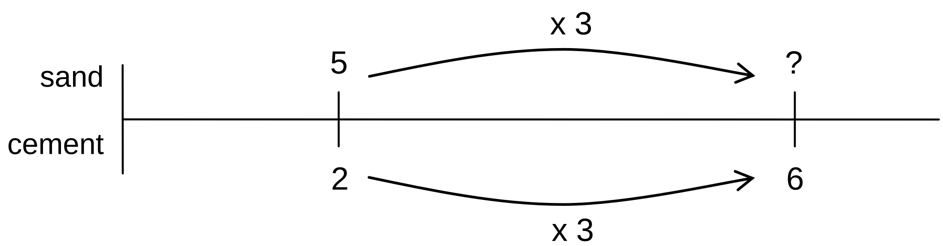


To solve problems, more information is added. For a rate example, *If petrol costs $1.40/L, then how many litres could you buy for $28?*, money ($) and volume (L) are the two sides of the line. The other numbers are then placed with equivalences on the same lines as follows.



Since all vertical lines are in proportion, the answer is 20x1, that is, is $20, because 28 = 20x1.4.

For a ratio example, *Sand and cement get mixed in ratio 5:2. how much sand is required for 6 tonnes of cement?*, the line is used in a similar manner. Since units and attributes are mostly the same, the two components, sand and cement, are placed on opposite sides and related information placed on the same vertical lines.



Once again, since 6 = 3x2 for the sand, the same relationship holds for 5 and ? and thus the sand required is 5x3 = 15 tonnes.

Multiplicative change.

Rate and ratio can be considered as multiplicative change and, as such, have a start, a multiplier (M), and an end, as follows.

$$\text{START} \xrightarrow{\text{xM}} \text{END}$$

For the rate example, *Paint is $24/L, how many litres for $192?*, the START is L, the END is $ and the M is 24. Thus, the diagram is:

$$?\ \text{L} \xrightarrow{\text{x24}} \$192$$

The litres are, therefore, 192÷24 = 8 as reversing the direction requires inverting the operation.

For the ratio example, *Milk is mixed with flour in the ratio 2:7, how much flour for 8Kg of milk?*, the START is milk (8Kg), the END is flour (?Kg), and the M is x7/2 (as follows):

$$8\text{Kg} \xrightarrow{\text{X7/2}} ?\text{Kg}$$

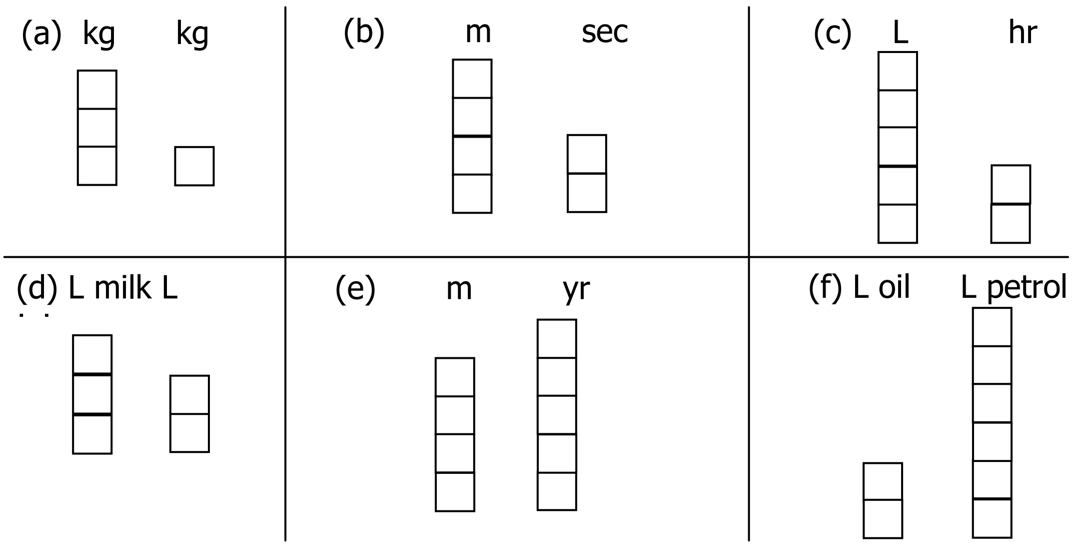
The mass of flour required is, therefore, 8x7/2 = 4x7 = 28 Kg as the direction required is multiplication.

Activity 1.2:

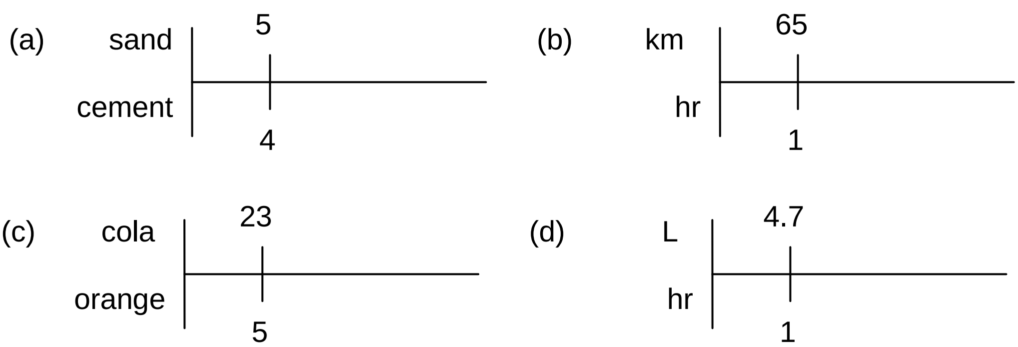
(1) Draw diagrams for:

(a) Sand to cement is 2:3 (b) Baby to man is 1:6 (c) Milk to butter is 4:3

(2) State what each of the following set/area diagrams represents in terms of rate or ratio:



(3) State what each of the following number lines diagrams represents in terms of rate or ratio:



(4) State what each of the following change diagrams represents in terms of rate or ratio:

(a) $3L \xrightarrow{X5/3} 5L$ (b) $1 \text{ hr} \xrightarrow{X7.25} 7.25 \text{ L}$

(c) $8 \text{ kg} \xrightarrow{X5/3} 3 \text{ kg}$ (d) $\$1 \xrightarrow{X3.59} 3.59 \text{ kg}$

(5) Draw set/area, number line, and change diagrams to represent the following:

a) ratio of kg tin to kg copper is 3:10
b) Rate of change of temperature is 0.07 degrees/year
c) The exchange rate is 0.957 US$ for 1 A$.
d) The chemical is mixed 25mL to 3L of water.

1.3 Multiplication and multipliers

Both rate and ratio are based on division as they are multiplicative comparison.

Rate. To find rate, say $/L for petrol, we divide the cost by the number of L. For example, 50L at $78 is 78÷50 = $1.56/L. This has two implications.

The first is that rates can easily be multiplied (and, in fact, some would argue that multiplication is a number by a rate). For example, if we by 35 L of petrol then the cost is

$$35L \times \$1.56/1L = \$35 \times 1.56 = \$54.60.$$

In fact, basic multiplication (e.g., 3 bags with 4 lollies in each bag) can be considered as a number x a ratio – 3 bags x 4 lollies/bag = 12 lollies.

The second is that divisions stay the same (i.e., are equivalent) when both numbers are multiplied by the same number, e.g.,

X2 x2.25 x100/9

$$6 \div 2 = 12 \div 4 = 27 \div 9 = 300 \div 100$$

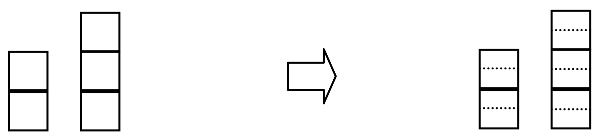
X2 x2.25 x100/9

x5 x4

$$3.2/1 = 16/5 = 64/20$$

x5 x4

Ratio. Rate is a relationship between two numbers. These numbers can be equivalent, i.e. 2:3 can become 4:6, as in the figure below.



Proportion sticks can be used to see ratios equivalent to a starting ratio (see below):

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

The sticks above are all the multiples of 1, 2, 3 and so on up to 10. This gives 10 sticks which can be used to show when ratios are equivalent (i.e., proportion).

5	2
10	4
15	6
20	8
25	10
30	12
35	14
40	16
45	18
50	20

To show equivalence, e.g., for 5:2, put the 5 stick beside the 2 stick (as on left). As look down both of the sticks, see ratios that are equivalent, i.e., 5:2 = 10:4 = 15:6 = 20:8 and so on. This can be used to get across the pattern of equivalence – that both numbers have been multiplied by the same amount, and that the multiplier between any two of the numbers is the same.

This pattern can be seen in the two ratios 2:3 and 4:6, and 2:3 and 14:21. These are in proportion and the multiples can be seen as below (both across and within ratios).

x2 x7 x1.5 x1.5

2 : 3 = 4 : 6 2 : 3 = 14 : 21 2 : 3 = 4 : 6 2 : 3 = 14 : 21

x2 x7 x1.5 x1.5

We can now work out proportions by keeping multiples the same.

x3 x8

3 : 7 = 9 : ? ?=7x3=21 ? : 7 = 24 : 56 ?x8=24, ?=3

x3 x8

Activity 1.3

(1) Calculate the following rates:

a) The cost of mobile phone calls was 75c per 30 seconds. What was the rate in terms of cents/s?

b) A dome builder worked for 11.6 hours and was paid \$141.52. What was the dome builder's rate of pay in \$/h?

c) The tap delivered 47 L of water in 5 h. What was the rate in terms of L/h?

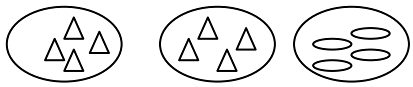
d) The painter covered 20 m² of wall with 8 L of paint. What was the painting rate in terms of m²/L?

e) A team of engine repairers can fix 20 engines in 4 days. What was the rate in terms of engines/day?

f) It costs \$108.15 to add 72.1 L to a car. What was the rate in \$/L?

(2)

a) (a) Draw a representation for three bags of lollies with 4 lollies in each bag:



b) Combine the groups, 3 × 4 = 12, and look at this in terms of rate, 4 lollies/bag × 3 bag = 12 lollies.

c) Repeat (a) and (b) for other examples: (i) 3 trees per row × 5 rows = 15 trees, and (ii) 5 blocks per runner × 6 runners = 30 blocks.

d) Find the patterns in these multiplications? [The numbers are multiplied and the attributes are "cancelled" (in this example, lollies ÷ bag × bag = lollies)].

(3) Identify which of these are rate multiplication:

a) 4 apples/box × 7 boxes

b) 6 pencils/box × 3 rulers

c) 5 carriages/train × 6 carriages

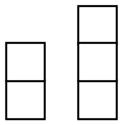
d) 8 trucks × 6 cars/truck

(4) Calculate each of the following by rate multiplication, using a calculator:

a) Rain fell at 7.5 mm per hour. How much did the swimming pool fill in 4.5 hours?

b) The painter used 0.7 L of paint for 1 m². How many litres of paint are needed to cover 7.48 m²?

(5) Cut these squares into 3 parts to show 2:3 = 6:9:



(6) Construct and cut squares to show:

(a) 1:2 = 2:4 = 3:6 (b) 4:1 = 8:2 = 12:3 (c) 3:5 = 6:10 = 9:15

(7) Take proportion sticks as required and complete proportions.

a) 3:7 = 9:_____ = _____:28 = 27:_____

b) 6:2 = 36:_____ = _____:14

c) 8:5 = 40:_____ = _____:30

(8) Complete the following

(a) $4 : 5 \xrightarrow{\times 2} 8 : \_\_\_\_$ (x2, x2)

(b) $8 : 5 = \_\_\_\_ : 20$ (x4, x4)

(c) $1 : 4 = \_\_\_\_ : 28$ (x7, x7)

(d) $\_\_\_\_ : 4 = 9 : 12$ (x3, x3)

1.4 Computer applications (stretching and squishing)

Computers can be used for ratio and proportion because, in Powerpoint, figures can be easily changed in length or width or can be changed in size with the shape left unchanged. This can be done using numbers and by pulling on the edges or boundary indicators of the figures. In this way, proportion can be shown.

Activity 1.4

(1) Use *PowerPoint* to draw two identical people, as shown below:

a) Change A so that height A:B is 2:1.

b) Change B so that width A:B is 2:3.

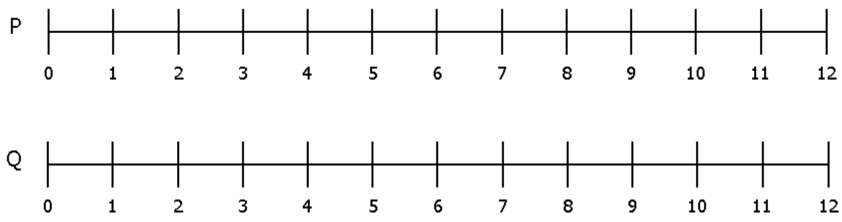
c) Change A so that height A:B is 1:3 and width A:B is 1:2.

d) Play with the two people, stretching and squishing them.

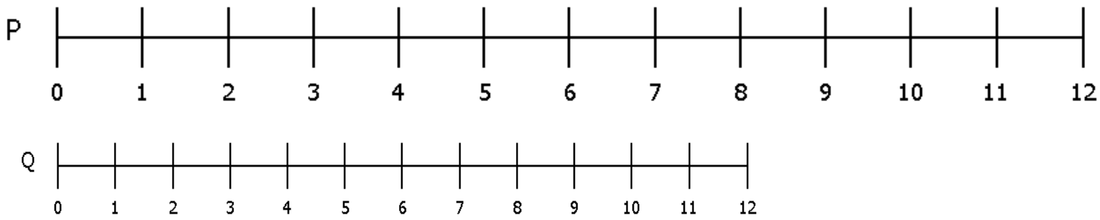
e) Can you make size A:B as 2:1?

(2)

a) Construct two identical rulers P and Q on *PowerPoint*, as shown below:



b) Stretch ruler P so P:Q is 2:3. That is, the 2 of ruler P aligns with the 3 of ruler Q



c) Now, proportions can be seen. For example, if P = 6, then what is Q?

d) Other ratios can also be seen in the diagram. What other ratios are evident?

2:3 = _______ = _______ = _______ = _______

(3)

a) Construct two identical candles X and Y on *PowerPoint*:

X Y

b) Change X and Y so that the:

(i) height X:Y is 1:4.

(ii) height X:Y is 3:4.

(iii) width X:Y is 9:1.

(iv) height X:Y is 3:2 and width X:Y is 2:1.

(4) Use your two rulers from (2) to show three ratios that are the same as (equivalent to) each of the following:

a) 1:2 = _____ = _____ = _____

b) 3:4 = _____ = _____ = _____

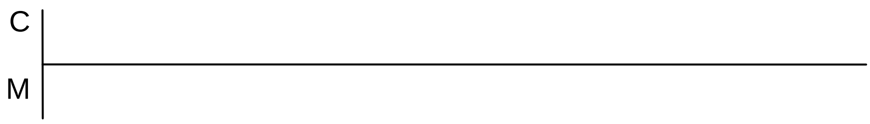
c) 2:5 = _____ = _____ = _____

2. THE DOUBLE NUMBER LINE METHOD FOR RATE AND RATIO

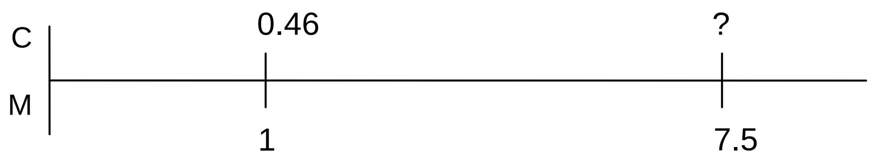
2.1 Calculating using rates

If we use a double number line, then rate problems can be easily solved in four steps. Let us look at this example: *A mobile phone call costs 0.46 cents/min, how much does a 7.5-minute call cost?*

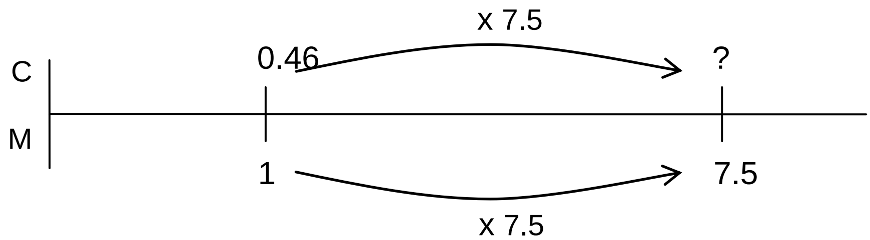
Step 1: Set up the number line. The first step is to identify the two things that will name the top and bottom of the line. Here we have call costs and minutes, thus the top and bottom should be named in terms of cost (C - cents) and time (M - minutes).



Step 2: Put in what is known. Put ? for the unknown. The second step is to put in vertical lines which relate costs and time. 0.46/min means that 1 minute is 0.46 cents so 0.46 and 1 go on one vertical line. The call is 7.5 minutes and the unknown is the cost of this call so ? and 7.5 go on another vertical line. (Note: In rate, one side of the line always has a 1).



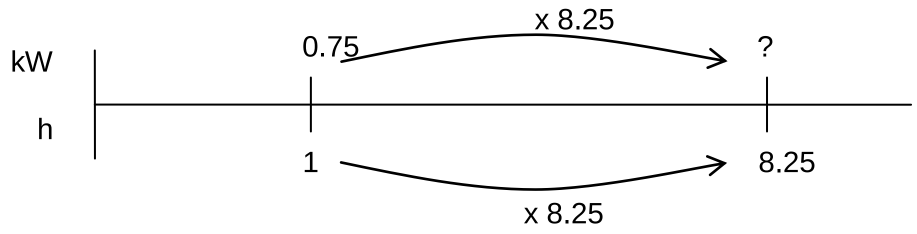
Step 3: Work out the multiplier. The third step is to calculate the multiplier that relates the numbers on the two vertical lines. To do this, look at the side of the line which has two numbers. In this example, this is the 7.5 and the 1, which means that the numbers on the second line are 7.5 x the numbers on the first line. This is shown on the diagram by an arrow.



Step 4: Calculate the unknown. The fourth step involves using the multiplier to calculate the unknown. Here, the multiplier is x 7.5, so ? = 0.46 × 7.5 = $3.45. Thus, a 7.5-minute call costs $3.45.

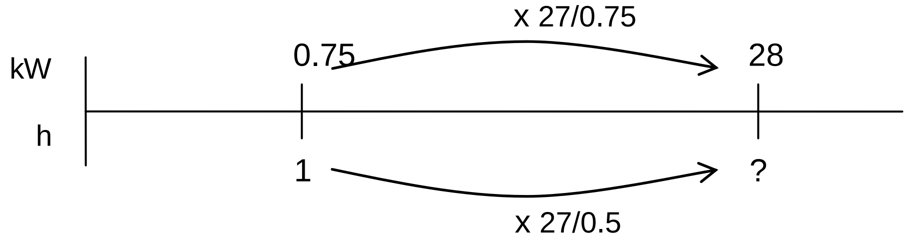
Once the double number line is identified, the multiplication of rates is straight forward. Let us look at another example: *The heater uses 0.75 kilowatts per hour (kW/h). If the heater is left on for 8.25 hours, then how many kilowatts were used?*

Steps 1 to 3 require setting up the diagram and calculating the multiplier. This is shown on the diagram below.



Step 4 gives ? = 0.75 × 8.25 = 6.1875 kW/h, so the heater used 6.1875 kW/h.

The multipliers work on both sides of the line. Let us look at yet another example: *The heater uses 0.75 kW/h. If the heater uses 28 kW, then for how many hours was the heater on?* Once again, we set up the diagram and calculate the multiplier (this time using the top line).



So, ? = 28 ÷ 0.75 = 21 h, meaning the heater was on for 21 h.

Activity 2.1

(1) Draw the double number lines for the following:

a) Cost of liquorice is 2.5c/cm

b) Computer downloads are 17c/min

(2) Use a double number line to solve each of the following:

a) Paint is $12 per litre. How much would 25 litres cost?

b) Smarties were $17.06/kg. How much for 1.35 kg of smarties?

c) Paint is $12 per litre. How many litres can $72 purchase?

d) Smarties were $17.60/kg. How many kilograms of smarties can $50 buy?

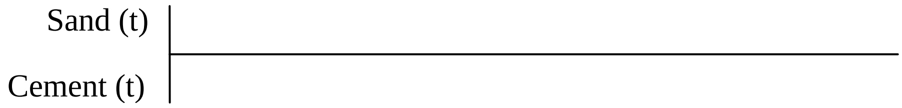
e) Cleaning is $16/h. How much does it cost for 7.25 h of cleaning?

f) Engine repairers are paid $11.50/h. How much will an engine repairer be paid for working 6.7 hours?

2.2 Calculating using ratios

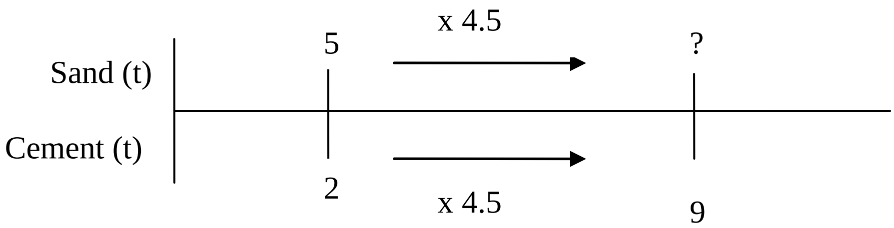
Similar to rate, ratio (and proportion) problems can be easily solved using the double number line. Again, the solution takes 4 steps. Let us look at these steps using an example: *Sand and cement are in ratio 5:2. How much sand for 9 tonnes of cement?* As we move through the steps, notice the similarity to rate.

Step 1: The first step is to set up the diagram by naming each side of the line. As this is a sand to cement ratio, we name one side sand and the other cement (as below).



Step 2: The second step is to place the numbers on the diagram. Numbers that are related are on opposite sides of vertical lines. For this example, the ratio is 5:2 so these two numbers go on one vertical line. The unknown is the sand for 9 tonnes of cement, so ? and 9 are on the other vertical line (as below).

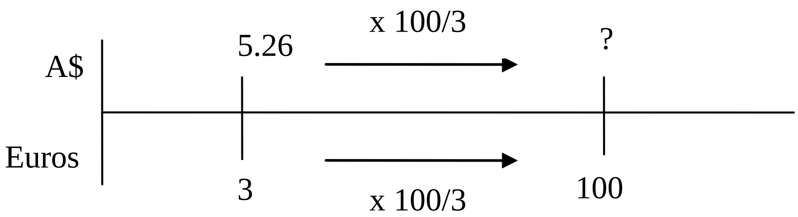
Step 3: The third step is to determine the multiplier from the side with two numbers and to mark that this is also the multiplier for the other side (as proportion means a common multiplier). For this example, the change is from 2 to 9, so the multiplier is x 4.5.



Step 4: The fourth step is to use the multiplier to work out the unknown. The multiplier is x4.5 so ? (Sand) = 5 x 4.5 = 22.5 tonne.

It is useful to look at another example: I received 3 Euro's for A$5.26. How many dollars for 100 Euros?

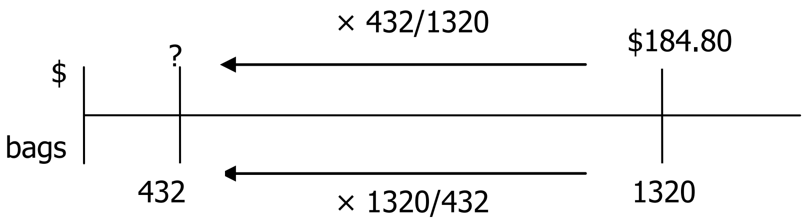
Steps 1, 2 & 3: A$ and Euros name opposite sides, the 3 and the 5.26 are opposite on one vertical line, and so are ? and 100. The 3 and 100 on one side mean that the multiplier is 100/3 (as below). Note: The easiest way to determine a multiplier is to divide the number which is opposite the ? with the other number on that side.



Step 4: Using the multiplier on the other side, ? = 5.26 x 100 ÷ 3 = $175.33. Thus 100 Euros costs A$175.33.

The multiplier can be in a reverse direction as for the following example: *It cost $184.80 to purchase 1320 bags. How much do 432 bags cost?* Note: this is an example where different attributes are stated in a ratio form, even though they would normally be stated as a rate, that is, either bags per dollar or dollars per bag. We will look at it as a ratio.

Steps 1, 2 and 3: $ and bags are opposite sides, 184.80 and 1320 are on the same vertical line, as are ? and 432, and the multiplier is reversed and is x 432/1320.



Step 4: The multiplier applied to the $184.80 gives ? = 184.80 x 432 ÷1320 = $60.48, the cost of 432 bags.

Activity 2.2

(1) Draw the double number lines for the following:

a) Apple to bananas sell in ratio 5:9

b) Chemical and water mix in ratio 0.05 L to 3 L

(2) Use a double number line to solve each of the following:

a) Cordial and water is in ratio 2:7. How much cordial for 35 litres of water? How much mixture does this make?

b) Boy to man is 3:5 in height. The boy is 102cm high. How high is the man?

c) Sand to cement is 5:2. How much tonne of cement for 35 tonne of sand?

d) Flour to butter is 4:7. How much flour for 50kg of butter?

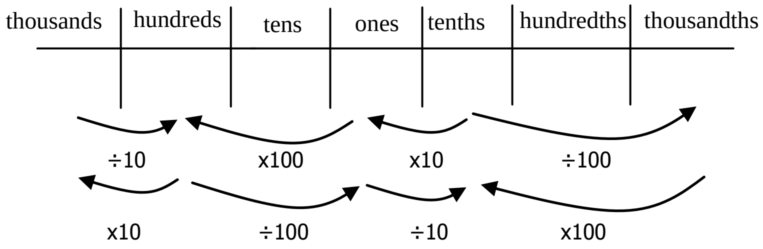
e) Paint covers walls in ratio 4L to 9m^2. How much paint is needed for 23m^2.

f) Chemical is mixed with water in ratio 0.025L to 2L. How much chemical for 23L.

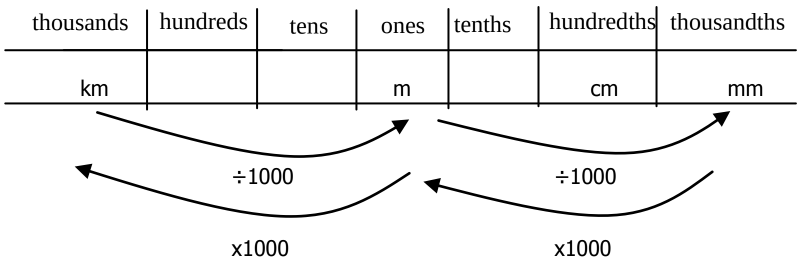
3. RATE, TIME, DISTANCE AND FUEL CONSUMPTION

3.1 Time calculations

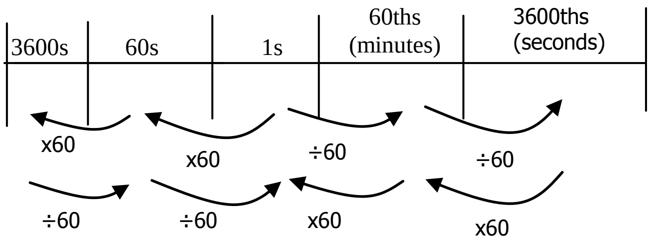
The western number system is based on multiples of 10 (see below):



Metric units are likewise based on 10 (see below)



However, not all measurement units are based on ten. From 2000 BC, the Babylonians (part of old Persia) developed a number system based on 60. That is, 23 47 is 23 x 60 + 47 = 1380 + 47 = 1427. The number 60 was chosen because the Babylonians say 6 and 12 as numbers with special religious significance (there is some discussion of this in booklet VC1) The Babylonians also used 60ths for fractions. They had two levels – 60ths which were called minutes, and 3600ths (60ths of 60ths) which were called seconds (see below)



The two measurements that are based on 60 are time (hours, minutes, seconds) and angle (degrees). As this booklet is focussing on engineering and not marine, we will focus on time.

Telling Time

The working day is defined in terms of hours and minutes, where minutes are 60ths of hours (e.g. 60 minutes in an hour) The day is divided into 2 parts (morning or am, and afternoon or pm) Time is given in digital form (e.g. 8.37 pm), 24 hour digital (e.g. 8.37 pm is 20.37) and in analogue form from a clock face (e.g. 37 past 8 or 23 to 9)

Reading in analogue form (from a clock face) has difficulties. The difference in analogue form (from a clock face) has difficulties. The difference in action of the shorter and longer hands have to be known. In particular the number 1 to 12 and the 5 spaces between them have to be understood in two ways. For the shorter hand, the numbers are hours and the 5 spaces are 12 minutes each. For the longer hand the numbers 1, 2, 3, to 12 are 5, 10, 15, to 60 and the 5 spaces are each a minute. If students have difficulty reading the clock face, the flock face needs to be acted out. Place numbers 1 to 12 in a circle and get the student to stand in the middle and act out a clock with their 2 hands.

Addition and Subtraction with Time

Adding and subtracting time is difficult because carrying does not happen at 10 but at 60. For example:

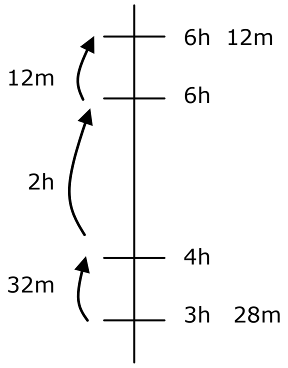
	Hours	Minutes
	3	45
+	2	37
	5	82
	6	22

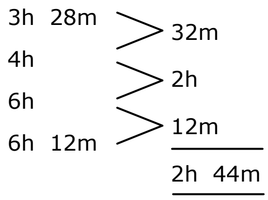
To add 3 h 45 m and 2 h 37 m, place the 4 numbers so that h and m are aligned. Add the h and m. then, if m is over 60, increase h by the number of 60s and put down the remaining m.

	Hours	Minutes
	5	72
	6	12
-	~~3~~	~~28~~
	2	44

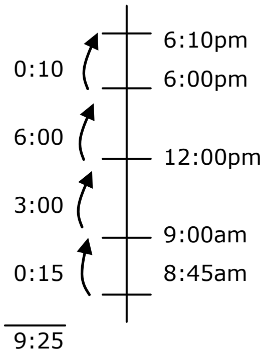
To subtract 3 h 28 m from 6 h 12 m, place the number so that h and m are aligned. Change the h and m of the larger number so that there are sufficient m (e.g. 6 h 12 m is changed to 5 h 72 m) Then both h and m are subtracted.

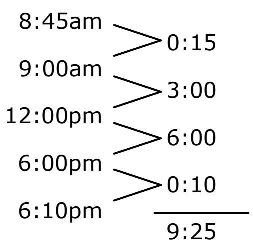
The subtraction of h and m is difficult, yet can be important in working out, for example, the hours worked. To overcome this, an additive approach can help. For example, instead of taking 3h 28m from, 6h 12m, work out how many h and m from 3h 28m to 6h 12m.





If times goes across am to pm and pm to am then have to go past 12 midnight or 12 noon. The additive method allows this to be easily done by adding in one more step, as follows We shall use the short hand 3:35 for 3h 35m





The 3hr 28m is moved towards the 6h 12m in steps. First, 4h is reached by adding 32m. Then, the move is to 6h by adding 2h, and finally the 12m. Adding all these changes means a difference of 32m + 2h +12m = 2h 44m

Activity 3.1

(1) Put out numbers 1 to 12 in a circle as on right. Stand students in the centre and give each a go at pointing to each number in turn from 12 or 1. Starting with 5, 10, 15, 20,

25, 30, 35, 40, 45, 50, 55, 60 o'clock, 5 past, 10 past, quarter past, 20 past, 25 past, half past, 25 to, 20 to, quarter to, 10 to, 5 to.

(2) Continue in centre of clock acting out times such as: 3 o'clock, half past 4, quarter past 7, quarter to 9, 20 past 1, 5 to 7, and so on.

(3) Add the following times:

a) 3h 20 m + 4h 15 m

b) 5h 22m + 4h 08m

c) 2h 47m + 3h 28m

d) 6h 18m + 5h 55m

(4) Use additive method to work out time intervals:

a) 3h 40m to 7h 50m

b) 2h 27m to 8h 48m

c) 4h 38m to 11h 06m

d) 3h 49m to 9h 38m

e) 8:20 am to 4:30 pm

f) 6:45 pm to 2:55 am

g) 8:30 am to 6:10 pm

h) 9.15 pm to 4:07 am

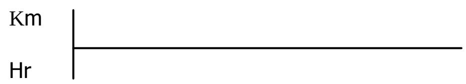
3.2 Distance and Speed

Many small engines power objects that travel, e.g. cars, motor mowers and boats. One of the most commonly used rates is speed (or velocity), i.e., distance divided by time. As a rate, speed problems can be solved with the double number line. Consider example:

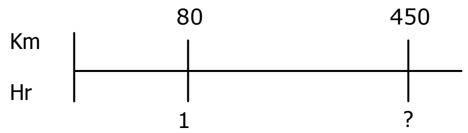
Car travels at 80 km/hr. How long to travel 450 km.

There are 4 steps

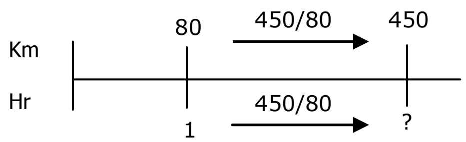
Step 1 Set up the diagram



Step 2 Put in number/unknowns



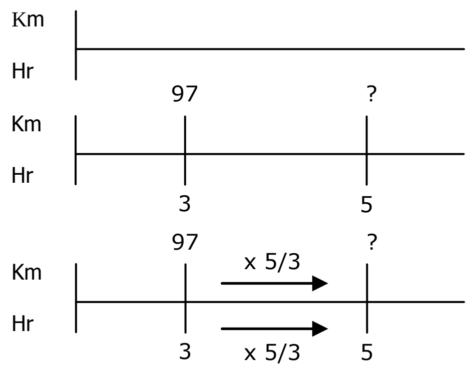
Step 3 Find multiplier



Step 4 Calculate ? ? = 1 x 450 ÷ 80 = 5 5/8 hrs, as hours are 60 mins this is 5/8 of 60 = 300 ÷ 8 = 37.5 mins

The rate, speed, is calculated by dividing distance by time. Sometimes problems do not give the rate but give distance and time from which to calculate speed. For these problems it is not necessary to calculate the rate. The distance and time information can be used in the double number line directly (similar to ratio) as for the following example:

The boat travels 97 nautical miles (nm) in 3 hours. How far can it travel in 5 hours?



<u>Step 1</u> Set up diagram

<u>Step 2</u> Put in numbers/unknowns

<u>Step 3</u> Calculate multiplier

<u>Step 4</u> Calculate? ? $= 97 \times 5 \div 3$

$= 485 \div 3 = 161 \frac{2}{3}$

The boat travels a little over 161 nautical miles in 5 hours.

(<u>Note</u>: Nautical mile (nm) is the distance given by an angle of 1 minute (i.e. 1/60th of a degree) at the equator. See booklet VC1 for information on this.)

Activity 3.2

Calculate the following:

(1) A car travels at 98 km/hr. How long to travel 500 km?

(2) A car travels at 98 km/hr. How far in 6 hrs and 15 minutes (6.25 hours)?

(3) A boat travels at 35 nm/hr. How long to travel 220 nm?

(4) A ride on motor mower travels at 9 km/hr. How far can you travel in 8 hrs?

(5) A car travels 250 km in 2.75 hours. How long to travel 800 km at the same speed?

(6) A boat travels 118 nm in 6 hours. How far will it get in 19 hours?

(7) A motor mower mows a strip 3500 m long in 8 hours. How long to mow 10 km (10 km = 10000 m)?

(8) John's boat travels the 80 mm to the coast in 2.75 hours. My boat is half the speed of John's boat. How long will it take me to go 200 nm to the coast?

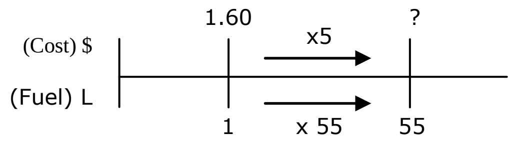
3.3 Fuel Consumption

Engines do not run without fuel and the rates of fuel price ($/L), fuel use (L/hr) e.g. economy (e.g. L/100km) are important for engines. Let us consider solving problems for these 3 types of rate. Remember that we do not have to work out the rate before we use the double number line.

Petrol price

(1) Petrol costs $1.60/L. How much for 55L?

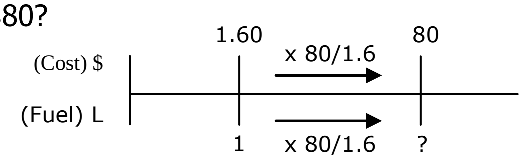
Steps 1, 2 & 3:



Step 4: ? = 1.60 x 55 = $88

(2) Petrol costs $1.60/L. How many litres for $80?

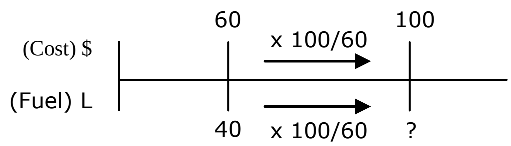
Steps 1, 2 & 3



Step 4: ? = 1 X 80 ÷ 1.6 = 50 L

(3) I bought 40L for $60. How much can I buy for $100?

Steps 1, 2 & 3

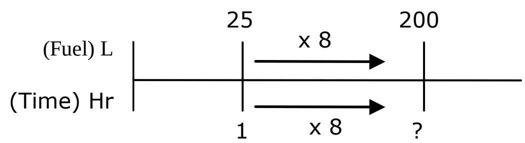


Step 4: ? = 40 x 100 ÷ 60 = 66 ⅔ or 66.67 L

Fuel Use

(1) At cruising speed a boat uses 25L/Hr. *How long can the boat travel on 200L?*

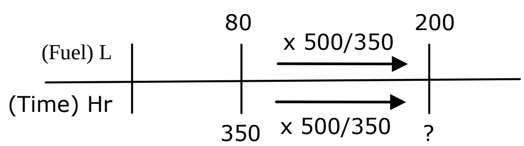
Steps 1, 2 & 3:



Step 4: Time = 1 x 8 = 8 hrs

(2) A boat uses 80L to travel 350 nm. How much fuel would it need to travel 500 nm?

Steps 1, 2 & 3



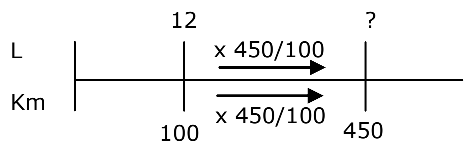
Step 4: Fuel = 80 x 500 ÷350 = 114

Fuel Economy

The third rate associated with fuel is economy which is given in terms of 100 km for cars but also could be given per distance (such as per nautical miles) or per time (such as per hour)

(1) The car uses 12 L fuel/100 km. *How much fuel for 450 km?*

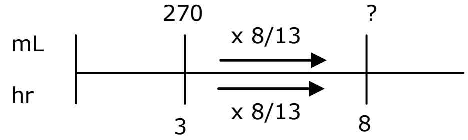
Steps 1, 2 & 3:



Step 4: Fuel = 12 x 450 ÷ 100 = 54 L

(2) The whipper snipper was 270 ml of fuel every 3 hours. *How much fuel for 8 hours?*

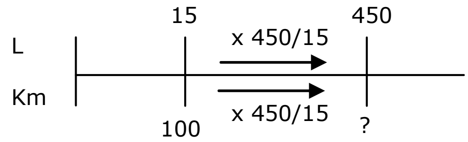
Steps 1, 2 & 3:



Step 4: *Fuel = 270 x 8 ÷ 3 = 720 mL*

(3) The tank uses 15L of diesel every 100 km. How far can the truck go on a 450 L tank?

Steps 1, 2 & 3



Step 4: *Distance = 100 x 450 ÷ 15 = 3000 km*

Activity 3.3

Find the unknowns using a double number line.

(1) The forklift travels at 8 km/hr. How long will it take to travel 25 km?

(2) The truck travels 300 km in 4 hours. How long to travel 1100 km at the same speed?

(3) The truck travels 300 km in 4 hours. How far can it travel at the same speed in 11 hours?

(4) The boat uses fuel at cruising speed at 21L/Hour. How long can it cruise if its tank is 315 L?5) the whipper snipper uses 2.5 L of fuel in 2 hours. How many hours to use L?) The refuelling home delivers 50 c of petrol in 7 minutes. How long will it take to fill a 900 L tank?

(5) The car uses 8.8 L/100 Km. How far can the car to on a 55 L fuel tank?

(6) The boat uses 56 L to travel 120 nm. How much fuel will it need to travel 200 nm?

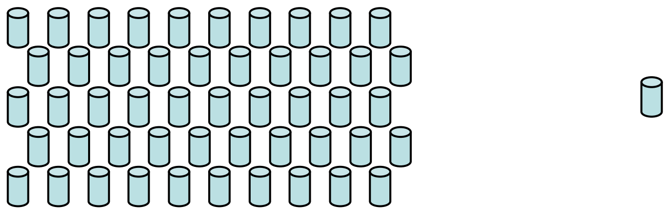
4. RATIO, FUEL MIXES, LEVERS, GEARS AND COMPRESSION

Within the engineering of small engines there are many instances where ratio is important. In this section, we study the mathematics behind 4 of these: fuel mixes, levers, gears and compression.

4.1 Fuel mixes

Two stroke engines require oil to be mixed with petrol so that the engine operates properly. The normal ratio is 1:50, that is 1 part oil to 50 parts petrol. This is the mixture that is made from 50 cans of petrol mixed with one can of oil as below.

Fuel for 2 stroke engines

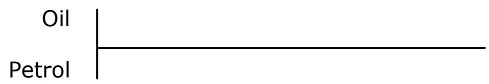


Petrol (50) Oil (1)

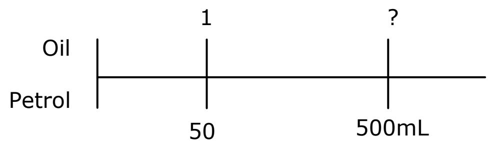
It is important to get the mixture correct for the many 2 stroke engines in use (all with different sized fuel tanks) fortunately; it is straight forward with the double number line as below.

John is mixing 2 stroke fuel from oil and petrol. How much oil is needed for a 500 mL whipper snipper fuel tank if the mix is 1 oil to 50 petrol?

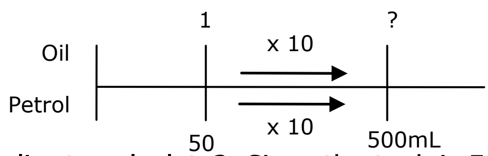
<u>Step 1</u> Name the two sides of the double number line. The problem has oil and petrol, so these are the names.



<u>Step 2</u> Place the number on the line. Numbers on opposite sides of vertical lines have to be related. We see that 1 oil is mixed with 50 petrol and this is one vertical line. The othe3r one is the ? (unknown) oil for 500 mL.

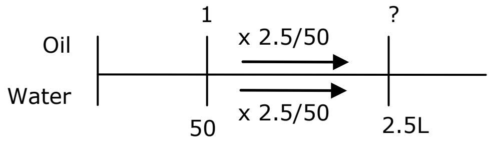


<u>Step 3</u> Use the side with 2 numbers to determine the multiplier. This is x 500/50 or x 10.



<u>Step 4</u> Use the multiplier to calculate? Since the tank is 500 mL, this means that the amount of oil is 10 mL. ? = 1 x 10 = 10.

Note: The amount of oil depends on the unit used in the volume of the tank. If the 2 stroke engine was a motor mower, the tank could be 2.5 L. Then the double number line diagram would be:



So ? = 1 x 2.5 ÷ 50 = 0.05. So amount of oil is 0.05 L or 50 mL (as there are 1000 mL in a L and a mL is 1/1000th of a L)

Activity 4.1

Complete the following using a double number line.

(1) Oil is mixed with petrol in ratio 1:50. How much oil fro a 7 L tank?

(2) Oil is mixed with petrol in ratio 1:50. How much petrol can a 750 mL can of oil be mixed with to make 2-stroke fuel?

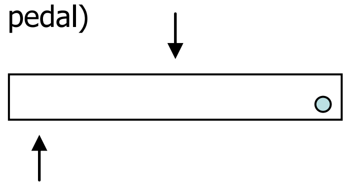
(3) Jan checked to find that 600 mL can of oil had been mixed with 2 L (2000 mL) of petrol. Was this correct? How much petrol should the can be mixed with? How much extra petrol has to be added to this mixture?

4.2 Lever Ratios

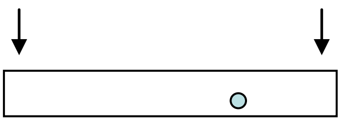
Types of Levers

Levers are common in engines. Ask your lecturer. There are two types of lever.

a) This is hinged at the end. One force pushes up and another down but at different points (e.g. the action of a brake pedal)



b) This is hinged in the "middle". Forces push the same direction at each end. (e.g. a way of lifting a car).



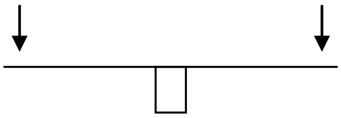
Levers are used so that a small force at one point will move a large force at the other, or so that a small movement at one point will produce a large movement at another. Or vice versa in both situations. Note: The hinge is called a 'fulcrum' in 'lever' language.

Activity 4.2 (a)

Materials: Planks, pieces of wood, bricks, fastening materials and tools.

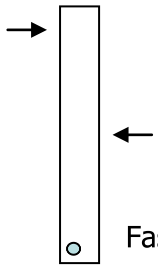
Directions:

(1) Construct a balance out of a plank and a brick like below.



(2) Put 2 students on one end and 1 student on the other. Make the balance so that the two can be balanced by the one. (Move the brick nearer to the end with the 2 students)

(3) Construct a stick as follows. Have two students push on the stick, one at the top and one half way up. What do you find? (The student pushing at the top can exert more force than the one half way up)



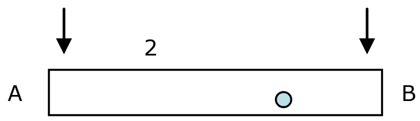
Fastened in some way

(4) In both levers, which point uses the most force? Near or away from the hinge or fulcrum? In both levers, which point moves the furthest? Near or away from the fulcrum (hinge)?

(5) Explore these questions fro different types of levers

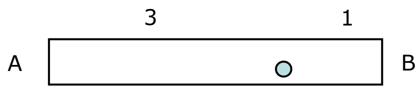
Ratios with levers

Exploration in Activity 2A will show that points which are away from the fulcrum/hinge move more but need less force to balance points near to the fulcrum/hinge. For example, in the lever on right, the force on A need only be ½ the force on B while the movement at A is twice the movement at B.

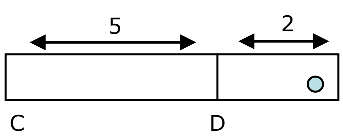


This means that the ratio or force A:force B is 1:2 (the "reverse" order) while the ratios of movements A: movement B is 2:1. The reason for this is that the momentum of a lever is force at a point multiplied by the distance that point is from the fulcrum. This means that the following holds for the 2 balance show (note: FA is force at A).

(1) $F_A \times 3 = F_B \times 1$



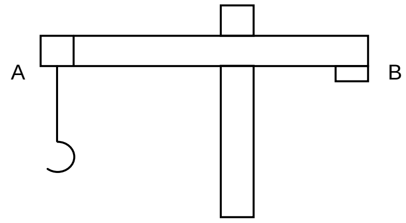
(2) $F_C \times 7 = F_B \times 2$



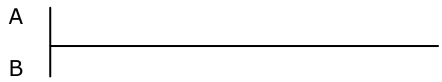
Lever Problems

The following problems are solved by use of the double number line. The only difference is to take care with force that that ratio is the opposite to what is expected. That is, the further from the fulcrum or hinge, the smaller the force that needs to be exerted.

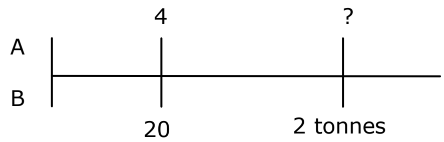
(1) The crane is set up so that a mass of 2 tonnes is at B, 4m from the fulcrum. If the other end of the crane is A is 20m from the fulcrum, what mass can be lifted from A?



Step 1: Set up a double number line (this is side A & B)

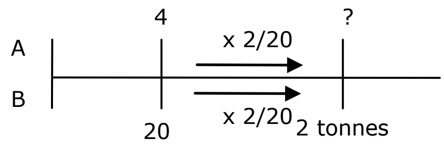


Step 2: Place in number/unknown opposite sides of lines which show things that are related. As mass is opposite to distance from fulcrum, ratio of mass at A to mass at B is 4:20. The ? mass at A for mass at B of 2 tonnes gives the opposite sides of the 2nd vertical line



Step 3: The side with the numbers is used to find the multiplier which is the same on both sides

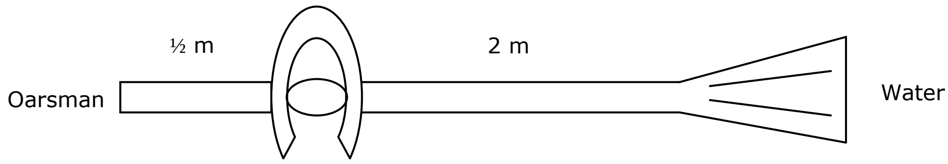
Step 4: The multiplier is used to work out the ? in tonnes.



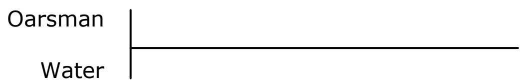
? = 4 x 2 ÷ 20 = 8 ÷ 20 = 0.4 tonnes or 400kg

(2) The oar is fixed as follows.

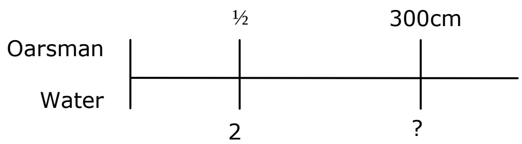
If the oarsman moves 300cm, how far does the end of the oar move in the water?



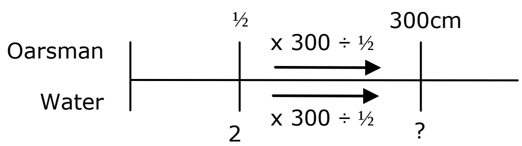
Step 1: Set up the line



Step 2: Put in numbers. Note that movement goes with distance from fulcrum – it is not opposite



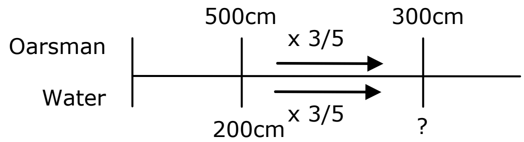
Step 3: Find multipliers



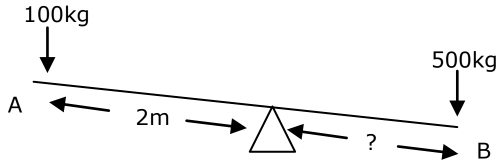
<u>Step 4:</u> Use multiplier to find ?

? = 2 x 3 ÷ ½ = 600 ÷ 0.5 = 1200cm or 1.2m

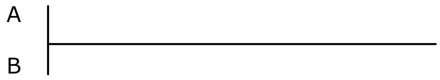
<u>Note</u>: Some people like to change everything to the one unit e.g., cm. Then the double number line is as follows. It gives the same answer.



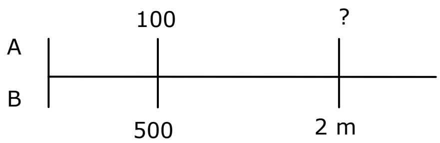
(3) John can pull down on the lever A with a force of 100kg. *What does the distance from B to the fulcrum have to be for John to life a load of ½ tonne* (500kg)?



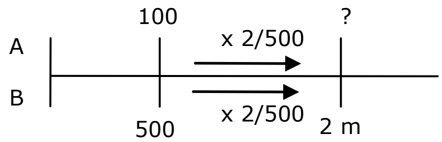
<u>Step 1</u>: Set up the line. We have the two each A and B as for (a)



<u>Step 2:</u> Put in the numbers. The ratio of A to B is 100:50. The ratio of distance is the reverse, that is, distance from B to fulcrum to distance from A to fulcrum



<u>Step 3:</u> Put in multipliers



<u>Step 4:</u> Use multipliers to find ?

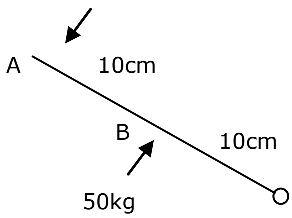
? = 100 x 2 ÷ 500 = 0.4

This is distance form B to fulcrum and has to be 0.4m or 400cm

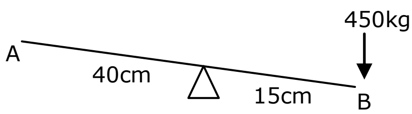
Activity 4.2b

Complete the following examples:

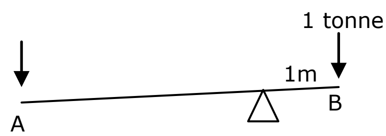
(1) A lever acts as on right: What mass at A will balance the 50kg at B? If B moves 5cm, how far will A move?



(2) A lever acts as on right. What lift at A will balance 450kg at B? If A moves 20cm, how far will B move?



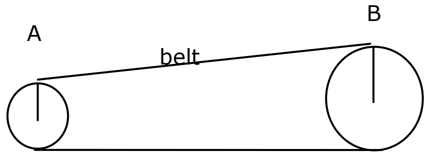
(3) A lever is lifting a 1 tonne weight at B, 1m from a fulcrum as on right. How far does A have to be from the fulcrum for 125kg to lift this 1 tonne? How far does A have to move if B has to life 15cm?



4.3 Gearing Ratios

Exploring gears

Gears are used to make machines operate faster or slower or more or less powerfully. The gearing ratios are determined by the circumference of the circles that relate to each other. For example, these two wheels A:B are in radius 1:2 (and other connected by a belt):



5cm radius; 31.4cm circumference

10 cm radius; 62.8cm circumference

This means that B turns or spins one time when A turns twice. This has two outcomes: A spins twice as fast as B; and the momentum force on A is 1/2 that on B.

Thus the gearing or spin ratio A:B is 2:1 (the reverse of the radii) and the torque ratio A:B is 1:2 (the normal way).

Note: Torque or momentum is the amount of strength in the turn. Not necessarily power but the strength of the turn

Activity 4.3a

(1) Obtain a bike with gears.

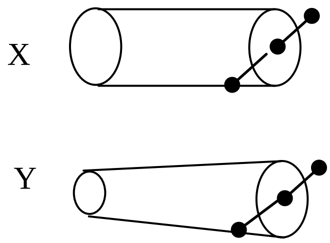
(2) Explore the action of the gears – change the back wheel cog from a large cog to a small cog (with on change to the pedal cog).

(3) Which cog gives a faster back wheel? Which cog enables the bike to go up hills easier? (Has more torque?)

(4) What can we say about ratio of the radius of the cogs

a) to make the bike go faster?

b) to make the bike have more torque (more power)?



(5) What can we say about the bike cogs on right?

a) Which is faster, X or Y?

b) Which has more torque, X or Y?

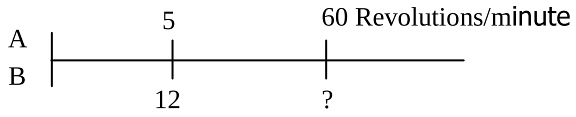
Solving Gearing Problems

Once again, we can use the double number line.

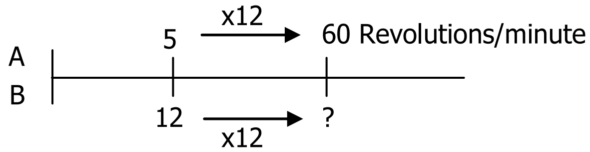
(1) *The engine is spinning at 60 revolutions a minute. A gear is chosen that relates a 12cm radius cog A to a 5cm radius cog B. What changes occur in revolutions? What changes in torque? (Assume that the original torque was 10).*

Revolutions:

Steps 1 and 2: Set up line and put in numbers. (Note that the ratio is opposite to radii – A:B is 4:10)



Step 3: Calculate multiplier

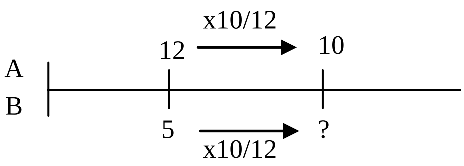


Step 4: Calculate

? = 12 x 12 = 144 revolutions/minute

Torque:

Steps 1, 2 and 3:



Step 4:

? = 5 x 10/12 = 4.17

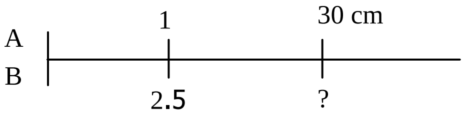
So power reduces from 10 to 4.17, .17% of original torque

(2) *A motor drives a shaft with a 30 cm radius wheel A. This needs to be connected to another wheel B so that the torque measures 2.5 tonnes. What radius wheel is needed? By how much will revolutions be decreased (assume start at 10).*

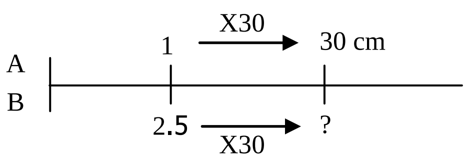
Torque:

Steps 1 and 2:

Note: for torque the radius ratio is direct



Step 3:

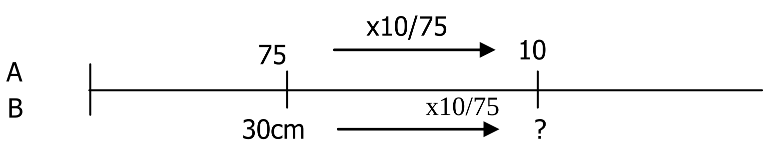


Step 4:

Radius of wheel? = 2.5 x 30 = 75cm

Revolutions

Steps 1, 2, and 3:



Step 4:

? = 30 x 10 ÷ 75 = 4.

Thus revolutions for B are 4/10 or 40% of what they were for A.

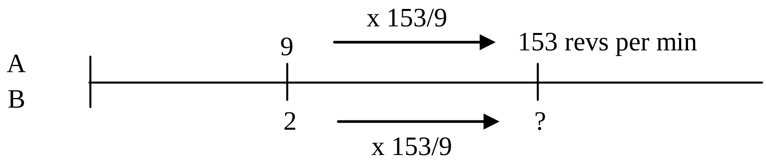
(3) *The engine in the mower is doing 153 revolutions per minute. Gears are engaged that relate a 2cm radius wheel A by belt 9cm radius wheel B. How do revolutions change? How does torque change? (Assume torque was 10)*

Revolutions:

Steps 1, 2 & 3:

Step 4:

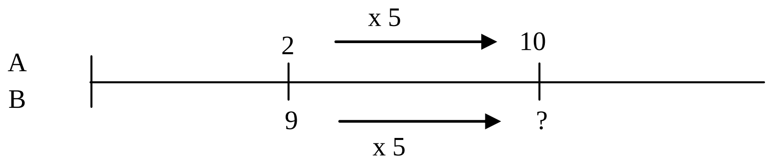
? = 2 x 153 ÷ 9 = 34 revolutions per minute.



Torque:

Steps 1, 2 & 3:

Step 4:



New torque is 45 or 450% of original torque or 4½ times original torque.

Activity 4.3b

Complete the following using double number lines.

(1) The engine of a boat is doing 100 revolutions a minute. Gears are engaged that relate 8cm radius cog on engine to a 20cm radius cog. What changes in revolutions and torque will occur? (Assume torque at start is 10.)

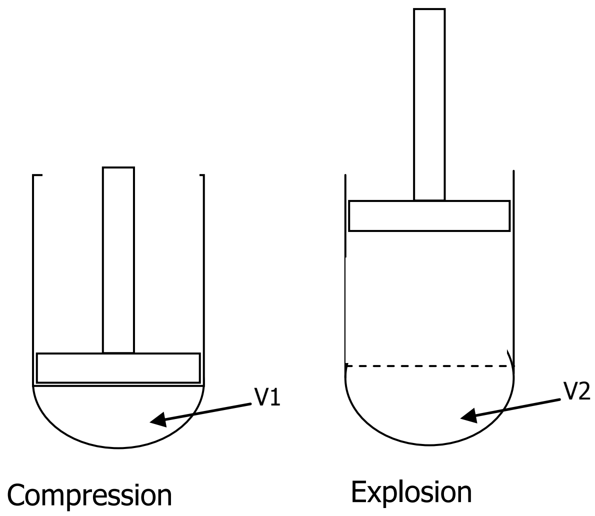
(2) A car engine is doing 240 revolutions/minute. It engages a gear that links a 5cm radius gear wheel A to a 4cm radius gear wheel B. What is the change in revolutions?

Another gear change links the 5cm radius gear wheel A to a 2cm radius gear wheel B. What changes in revolutions across each gear change? What changes in Torque?

4.4 Compression Ratios

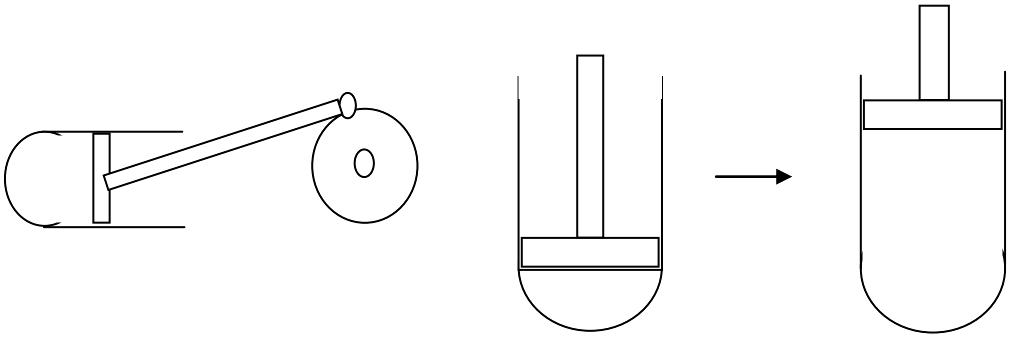
The final ratio associated with engines is compression ratios. Both 2 and 4 stroke engines involve a cylinder sucking in petrol vapour and oxygen, compressing it and then exploding it.

The difference between the compressed volume and the exploded volume is the compression ratio, as on right. The larger this is the more efficient the engine. On right, the compression ratio is V2: V1.



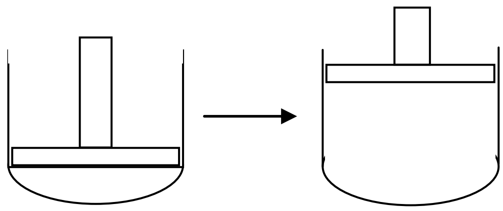
A good compression ratio can be had in two ways:

(1) *Long Stroke*: Long thin cylinder with great movement of the piston as on the above right. As the piston is connected to a crankshaft, as below, a long stroke also means a large radius on the crank and thus greater <u>torque.</u>



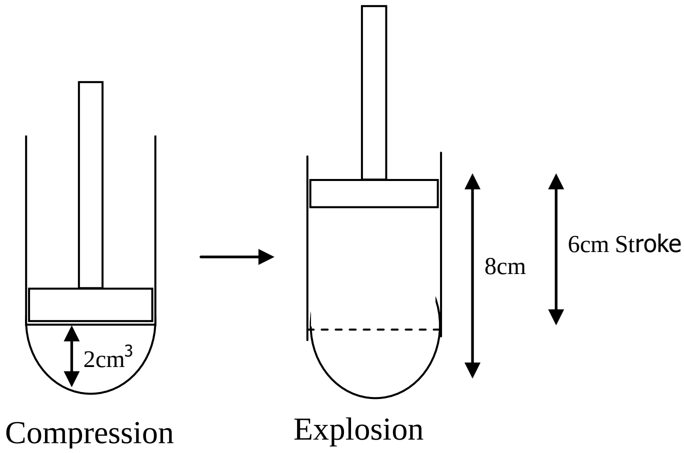
However, a long stroke means less backwards and forwards per minute and so less revolutions/min, thus less speed and less power.

(2) *Short Stroke*: A Short fat cylinder with little movement of pistons (but still large measure in volume), as on right. This means a small radius on crank and low torque but high revolutions and power.

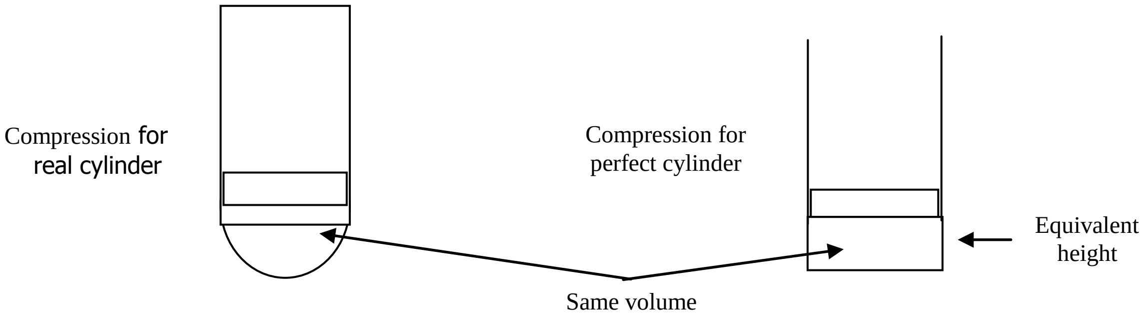


Note: Historically, old cars were long stroke – not much power and speed by high torque so they could "plug" on through bad roads - While modern cars are mainly short stroke for high revolutions and power.

If the volume of an engine was a perfect cylinder, then as the volume of a cylinder $=\prod r^2 h$, the volume change would be determined by how the length of the stroke changes the starting and finishing position. For example, if the cylinder has a stroke of 6cm and moves from 2cm from the end to 8cm from the end, this would make a compression ratio of 8:2 or 4:1



However, the end of a cylinder is so complicated that its volume is best measured by filling it with water and measuring the water. However, knowing the radius of the cylinder, a height could be worked out that was equivalent to the volume. That is:

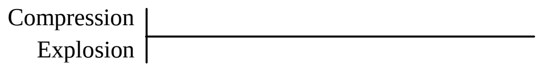


Thus problems could be worked out using height to find the ratio.

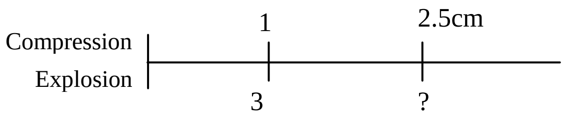
This enables problems to be worked out with the double number line as follows:

(1) *A cylinder's compressed volume is equivalent to 2.5cm of the height of a perfect cylinder. What stroke would give a compression ratio of 3:1?*

<u>Step 1:</u> Set up Line

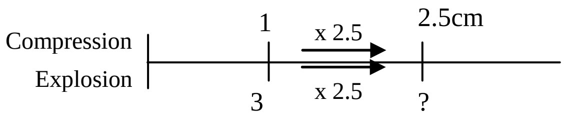


<u>Step 2:</u> Put in Numbers



<u>Step 3:</u> Determine multiplier

<u>Step 4:</u> Calculate

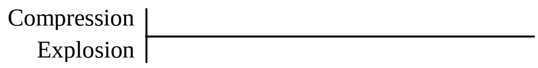


? = 3 x 2.5 = 7.5

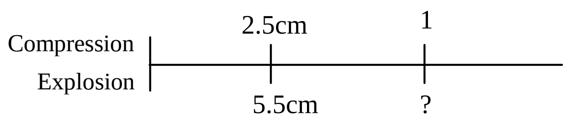
So Stroke is 7.5 – 2.5 = 5 cm

(2) *The same cylinder as above has a stroke of 3 cm. What is its compression ratio?*

<u>Step 1</u>: Set up Line

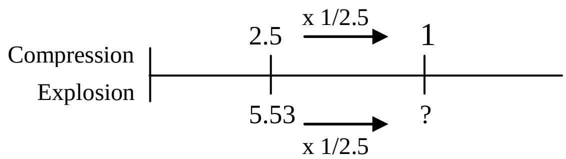


<u>Step 2</u>: Put in Numbers



<u>Step 3:</u> Determine multiplier

<u>Step 4:</u> Calculate



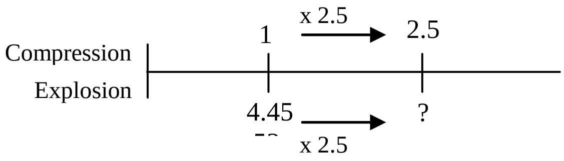
? = 5.5 x 1 ÷ 2.5 = 11/5 = 2.2

Compression ratio = 2.2:1 OR 11:5

(c) *What change has to be made to the stroke for the same cylinder as above in (b) to double its compression ratio?*

<u>Steps 1, 2 and 3:</u>

(Note: doubling give 4.4:1)



<u>Step 4:</u>

? = 4.4 x 2.5 =11

So Stroke has to be=11 – 2.5 = 8.5cm

Activity 4.4

Complete the following using a double number line.

(1) A cylinder has a compressed volume equivalent to 3.3cm height and a stroke of 4.2cm

a) What is its compression ratio?

b) What stroke will halve the compression ratio?

(2) A cylinder has a compression ratio of 8:3, a compressed volume equivalent to a height of 2.8cm.

a) What is the length of its stroke?

b) What stroke would increase the compression ratio to 15:4?

(3) The <u>torque of an engine is</u> related to the length of stroke of the piston. Double the stroke length means double the radius of the crankshaft circle and this results in double the torque.

a) What happens to the torque in activity (1) above?

b) What happens to the torque in activity (2) above?