YUMI DEADLY CENTRE
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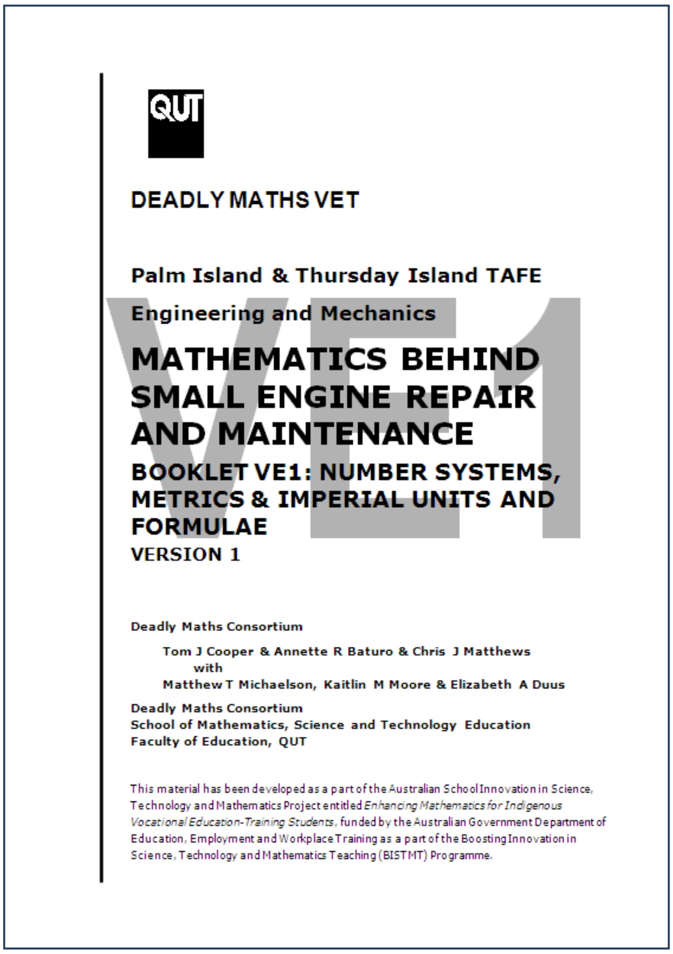
Engineering and Mechanics

Mathematics behind Small Engine Repair and Maintenance

Booklet VE1: Number Systems, Metrics & Imperial Units, and Formulae



DEADLY MATHS VET

Palm Island & Thursday Island TAFE

Engineering and Mechanics

MATHEMATICS BEHIND SMALL ENGINE REPAIR AND MAINTENANCE

BOOKLET VE1: NUMBER SYSTEMS, METRICS & IMPERIAL UNITS AND FORMULAE

VERSION 1

Deadly Maths Consortium

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This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

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NUMBER SYSTEMS, METRICS & IMPERIAL UNITS

AND FORMULAE

08/05/09

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THIS BOOKLET

This booklet VE1 was the first booklet produced as material to support Indigenous students completing certificates associated with engineering and mechanics at the TAFE campuses on Thursday Island and Palm Island. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

VB1: Mathematics behind whole-number place value and operations
Booklet 1: Using bundling sticks, MAB and money

VB2: Mathematics behind whole-number numeration and operations
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

VC1: Mathematics behind dome constructions using Earthbags
Booklet 1: Circles, area, volume and domes

VC2: Mathematics behind dome constructions using Earthbags
Booklet 2: Rate, ratio, speed and mixes

VC3: Mathematics behind construction in Horticulture
Booklet 3: Angle, area, shape and optimisation

VE1: Mathematics behind small engine repair and maintenance
Booklet 1: Number systems, metric and Imperial units, and formulae

VE2: Mathematics behind small engine repair and maintenance
Booklet 2: Rate, ratio, time, fuel, gearing and compression

VE3: Mathematics behind metal fabrication
Booklet 3: Division, angle, shape, formulae and optimisation

VM1: Mathematics behind handling small boats/ships
Booklet 1: Angle, distance, direction and navigation

VM2: Mathematics behind handling small boats/ships
Booklet 2: Rate, ratio, speed, fuel and tides

VM3: Mathematics behind modelling marine environments
Booklet 3: Percentage, coverage and box models

VR1: Mathematics behind handling money
Booklet 1: Whole-number and decimal numeration, operations and computation

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OVERVIEW

The ASISTM VET project funded in 2008 by the Australian Schools Innovations in Science, Technology and Mathematics scheme had 6 sites: Wadja Wadja HS at Woorabinda, Shalom Christian College in Townsville, Palm Island Post Year 10 Campus (run by Kirwan State High School and Barrier Reef Institute of TAFE), Tagai College Secondary Campus at Thursday Island, Thursday Island campus of Tropical North Queensland Institute of TAFE Campus, and Northern Peninsula Area College at Bamaga. All these sites have Indigenous students and the project focused on developing instruments and materials to assist the teaching of mathematics needed for certification for Indigenous VET students with little previous success in school.

The meeting held with VET staff from Palm Island and Thursday Island (TI) requested support for a program in which Indigenous VET students were taught how to repair and maintain small engines (e.g. outboard motors, motor mowers, etc). Indigenous people at Palm Island and TI run boats similar to the way city people run cars and have the same needs with respect to outside work as city people. There are many small engines used in these two islands – outboard motors, motor mowers, whipper snippers, chain saws, and so on. This means that the training will be of use both for certification and to enable these motors to be repaired by people on the Island.

The VET staff requested support with units and unit conversions associated with the engines (both metric and imperial) plus help with use of formulae for describing engines (e.g., volume). They also asked for support in teaching gearing and compression ratios, and ratios for mixing fuel. To this was added some work on rates for speed and fuel use.

To meet this, Deadly Maths VET completed two booklets as follows:

Booklet 1:

(1) Numeration basis of units - structure of number systems (around ones), building large numbers through bundling ones, powers of 10, and small numbers by partitioning ones into fractions (common, any base and decimal – base 10).

(2) Metric units and conversions on structure of the base 10 number system, multiplication relationships, connection to metrics, conversions of metric units, and use of micrometer for hundredths of a mm.

(3) Fractions and Imperial units - common fractions (part of a whole, number line), equivalent fractions, fraction basis of Imperial units, and use of micrometer for thousandths of an inch.

(4) Area and volume formulae – formulae for area of rectangle and circle, and volume of cylinder and semi sphere.

Booklet 2:

(1) Nature of rate and ratio – the mathematics of ratio and rate, models for interpreting rate and ratio, multiplication and multipliers, and use of computers to show change.

(2) The double number line method – the understanding behind the method, and its use in solving rate and ratio problems.

(3) Application of rate to small engines – time, distance and speed, and fuel consumption.

(4) Application of ratio to small engines - fuel mixes, gearing ratios, and compression ratios (including investigation of power, torque and speed).

1. NUMERATION BASIS OF UNITS

There are four meanings associated with number. A strong understanding of number is based on connections between these meanings and between the real world situations, language and symbols. The four meanings are:

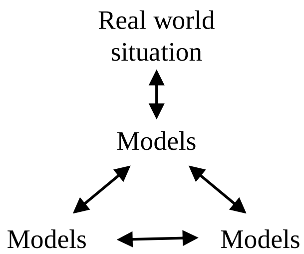
(1) Place value (separation) – digits in numbers show value through position - e.g., 27 is 2 tens and 7 ones became the 2 is on the left of the 7;

(2) Counting (odometer) – any place value counts as the ones do– e.g., counting forward in 10s is 278, 288, 298, 308, 318, while counting back in 100s is 2364, 2264, 2164, 2064, 1965;

(3) Rank (order) – regardless of its place value, a number is a simple point on a number line and its distance from zero gives order; and

(4) Multiplicative structure – adjacent place values relate by x10 to left and ÷ 10 to right.

With these meanings, there are conventions which provide information on where the ones are - all other positions come from knowing where the ones are. The convention for the ones position is the rightmost digit for whole numbers and the digit just before the dot for decimal number. The relationship to be built is based on the Payne and Rathmell triangle.



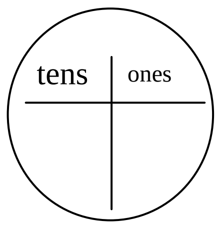
1.1 Place Value

Place value has two components: position and value. For 27.4, the 7 is the ones, so by position the 2 is tens and the 4 is tenths, with the 2 being 10 times larger than the 7 and the 4 being 1/10 the size of the 7. To teach this, we use materials that represent value (e.g., bundling sticks, MAB, and money) and put them on a place value chart (PVC) that represents position. Activities 1.1a and 1.1b cover two-digit and three-digit place value respectively.

Activity 1.1a

Materials: Wooden bundling sticks (coffee stirrers) or tongue depressors, or paper strips, place value chart (PVC), calculator, 2 dice, rubber bands.

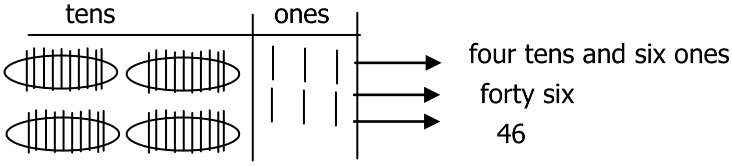
Directions:



(1) Set up PVC and place single sticks in ones' position. Count as put them in 1, 2, 3, … 8, 9. When get to 10, bundle the sticks with a rubber band and make one 10. Add a new "tens"column on the left of the ones.

(2) Relate bundles of ten and left over ones to language and symbols and vice versa – as you say the numbers, move left hand from tens to one - Note: be careful with the tens – can be useful to call 17 "onety seven" until pattern is seen.

e.g.



(3) Reverse everything – so go from materials to language and then language to materials, and materials to symbols and symbols to materials.

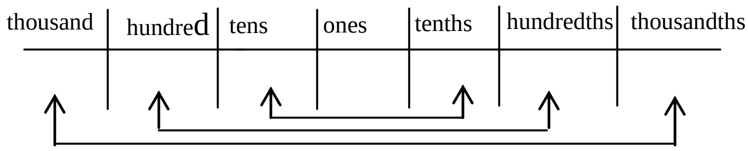
(4) Play the game *Blockbuster* – in turn, starting at zero, throw two dice and put out this number of ones on PVC, bundling to make tens and recording final numbers on calculators. As turns come around, keep throwing 2 dice and adding ones to own PVC, bundling and recording as go. First to 100 wins. Reverse the game – put 10 tens on the PVC (begin at 100) and go to 0 by removing sticks after throwing 2 dice.

(5) Take a stick and cut it into 10 equal parts, discuss what these parts are in relation to ones [say that they are tenths]. Add in new "tenths" column on right of the ones.

tens	ones	tenths

(6) Count in tenths until combined into ones. Repeat (2) and (3) for tens, ones and tenths. Discuss role of decimal point in determining where the ones are.

(7) Discuss how we make numbers ever larger and smaller – add in extra columns – look at the symmetry.



(8) For larger and more complicated numbers, we base our system on a pattern of threes – ones, tens and hundreds of ones, thousands, and so on. So we have the following structure (more of this in Activity 1.1b):

Millions			Thousands			Ones			Thousandths		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Activity 1.1b

Materials: MAB, PVC, card deck, calculators.

Directions:

(1) Set up chart and place MAB into columns on PVC chart. Repeat (2) and (3) from 1.1a.

(2) Play a game. Remove J, Q, K, Joker and 10s from card deck. Divide red and black cards. Shuffle the two decks. Call red 10s and black 1s. In turn, select a red and a black card and add the tens and ones the cards show to the PVC, exchanging (trading) and recording results on calculators as you go. Keep drawing cards, adding MAB, trading and recording on calculators. First to 500 wins. Reverse the game – go from 100 to 0 by removing MAB.

(3) Add in thousands and millions.

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Put in digits – read in threes as below, record on calculators, for example:

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
6	8	4	3	2	4	7	9	1

six hundred and eighty-four millions

three hundred and twenty-four thousands

Seven hundred and ninety-one (ones)

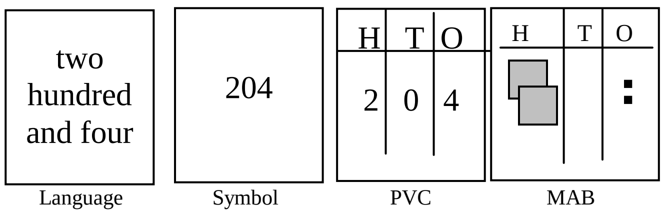
(4) Go from PVC to language to number and then reverse this process, recording the final numbers on calculators.

(5) Play the calculator game Wipeout – enter a number (say 68 743), state a digit (say 7), and students have to wipe this digit out with a single subtraction { for this example, the subtraction would be -700]

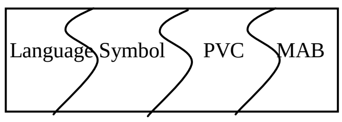
(6) Repeat (4) and (5) but adding in decimal place values to millionths.

(7) Reinforce number understanding by playing games that relate models, language and symbols for various representations of numbers, for example:

a) make the four cards on the right and play Concentration, Snap and Rummy (and "Cover the Board");



b) make up mix & match cards like on right with the four different representations (cut them up and students have to reform); and



c) make up Bingo boards with 3 representations (language, PVC and MAB) randomly on the board and flash cards with numbers on them.

1.2 Counting & Odometer

The pattern for counting forward is that any place value counts by going 2, 3, 4, … 7, 8, 9 and then the 9 becomes 0 and the place on the left moves up by 1, e.g., 378, 388, 398, 408. Similarly, the pattern for counting backward is that any place value counts by going 9, 8 7, … 2, 1, 0 and then the 0 becomes 9 and the place on the left moves down by 1, e.g., 372, 371, 370, 469, 368. It is the reverse of counting forward. Activities 1.2a and 1.2b cover counting forward and counting backward respectively (1.2b also introduces a game).

Activity 1.2a

Materials: calculators

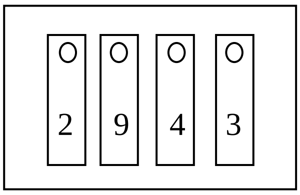
Directions:

(1) Enter 25 on calculator, press + 1, then repeatedly press =, stating the ones answer as you go. Look for the pattern - see how the ones digit goes up to 9 and then back to 0 while 10 goes up to 3, then to 4, and so on. Repeat this starting at 94. What is the difference?

(2) Enter 235 on calculator, press + 10, then repeatedly press =, stating the position of the tens as you go (e.g., if you start with 275, press +10, getting 285 etc., you will say 7, 8, 9, 0, 1, 2, …). Encourage students to see how the10s repeat the counting pattern seen in the 1s position.

(3) Repeat (2) above for hundreds, thousands, tenths, hundredths (pressing + 100 = = = …, + 1000 = = = …, + .1 = = = …, + .01 = = = ,… respectively). Keep going until students see that the pattern is the same across all place values.

(4) If problems, bring in flip cards as well as the calculator. Flip over cards in one place-value position (e.g., hundreds). When get to 9, flip that place back to 0 and increase place on the left by 1 (in example on right, changes 2 to 3).



(5) Get an odometer from a car (or simulated on a computer) and show how the above is the same as an odometer change.

Note: The pattern for counting backwards, at any place value position, is the reverse of the forward pattern: it goes 8, 7, 6, …, 3, 2, 1, 0 and then up to 9 with the left hand position reducing by 1 (e.g., 335, 325, 315, 305, 295 …). Calculators can be used to teach it the same as for forward counting (just subtract the 100 or .01 before pressing = = = …)

Activity 1.2b

Materials: Calculator.

Directions:

(1) Repeat all directions from Activity 1.2a but subtract the number that is repeatedly taken off or subtracted. Use flip cards to act out the odometer pattern.

(2) Look at what it means to be close to a number, say, within 300 of 245.87. Use counting forward and backward three 100s from 5 100s to find limits, for example:

forward	24 587	backward	24 587
	24 687		24 487
	24 787		24 387
	24 887		24 287

Show how this is within 3 of 5 in the 100s position.

(3) Play a game: "Target" – take a target and start number, enter the start number and press x on calculator. Then the idea is to trial what to multiply the start number by to get as close to the target number as required. Press guess = (where guess is the best

estimate of what to multiply) and keep trying new guesses until target is reached. When reach target, try another target and start number. Students' score is the number of trials. Player with lowest number wins.

Use a worksheet like this:

Target	Start	Nearness	Trials too high	too low	Answer	No of Trials
595	17	exact	40	20	35	3
1750	26	within 20	90, 80	65, 67	68	5

1.3 Rank and order

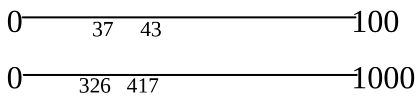
Placing numbers on a number line is an efficient way to build an understanding of order. Once initial understanding is built, the pattern that order is comparing numbers digit by digit, place-value position to place-value position, starting from the highest position. Again we start with 2-digit numbers then expand to three-digit and larger numbers. Activities 1.3a and 1.3b cover number line methods and digit methods respectively (1.3a introduces a game).

Activity 1.3a

Materials: Pegs, rope, card, number lines.

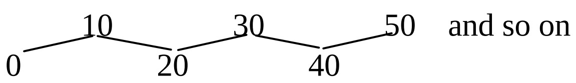
Directions:

(1) Use pegs and numbers on card. (e.g. 2 digits such as 37 and 43, or 3 digits such as 326 and 417) to put numbers on a number line. Discuss where numbers should go. Get students to share their methods for placing the numbers.



(2) If problems then do the following:

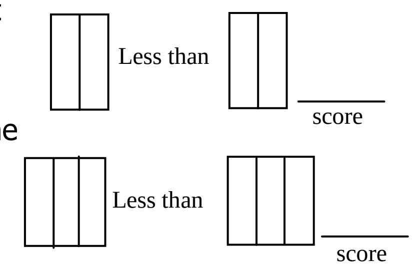
- cut 10 straws each the same length (alternate colours), thread onto thick string, stick numbers 0, 10, 20 to 100 where straws meet (as follows);



- measure things with it (guessing how many ones), and discuss the largest length and why it is the largest (it has more lengths of 10); and

- cut 100mm lengths of straws and thread onto string to make a 'metre', stick numbers 0, 100, 200, to 1000 where straws join, measure things and discuss largest/smallest and why?

(3) Play a game: "Dice Order" – Set up a playing worksheet with many rows as on right (each row is a game). Teacher selects a card at a time from a deck with only cards 1 to 9 (A is 1). Students mark this number into one of the 4 spaces as cards are chosen (have to mark before next card is chosen). If LHS number less than



RHS, students scores the value of the digit in LH tens position. Student with largest score after 5 or more trials wins.

Game can be extended to 3-digits by using playing worksheet with rows as above. The score digit in 100s place in LH number.

Note: The focus of the game should be on the 10s mattering for 2 digits (e.g., 37 is less than 43 because 3 tens is less than 4 tens) and 100s mattering for 3 digits (e.g., 396 less than 417).

Activity 1.3b

Materials: Place value chart, digit cards

Directions:

(1) Consider numbers 84 762 and 128 971. How do we compare them? Put on PVC. Look at biggest place value positions. Hundred thousandths is the largest. Since 128, 971 has a 1 in this position and 84 762 has a zero, 128, 971 is the largest.

Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones
	8	4	7	6	2
1	2	8	9	7	1

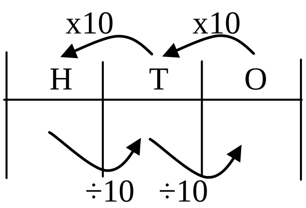
(2) Use R → L scanning to circle the largest of the two numbers, e.g., 64 175; 47 869; 71 596; 117 834; 200 808; 40 606; and 817 017; 386 086.

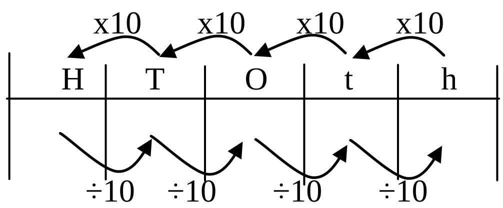
(3) Decimals are difficult. The ones have to be aligned, e.g., when 24.68 and 5.9696 have their ones places aligned, it shows that 24.68 has more 10s than 5.9696 and so is larger:

2 **4**.6 8
  5.9 6 9 6

1.4 Multiplicative structure

The basis of most metric conversion is multiplication – the multiplicative relationships between place-value positions, for example:.





x1000 x1000

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
6	8	4	3	2	4	7	9	1

÷1000 ÷1000

Activities 1.4a and 1.4b covers relationships between place value positions for whole numbers and how these relationships can be used to introduce decimal place values respectively.

Activity 1.4a

Materials: Cards as on the right, calculators, slide rules.

one ones	Ten ones	hundred ones	One thousand	Ten thousands
Hundred thousands	One millions	ten millions	hundred millions	

Directions:

(1) Give students a calculator and a slide rule that matches the cards:

2	3	5	6	8	0	0

Cut along dotted lines

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

6

5 8

3 2 9

(2) Hand out place value cards and get students to stand in front of class showing the place-value positions in order. Then get other students to stand in front of the positions holding number cards (the same as the slides). Ask students to make the numbers shown with their slide rules, and to input the numbers shown in their calculators.

(3) Repeat the activity but with the students holding the number cards moving one, two and three places left and right. Ask students to copy the movements with their slides and to change the numbers on their calculators by multiplying and dividing by 10, 100 and 1000 as appropriate for the movements. Discuss the role of zero. Get students to stand with zeros at the front.

(4) Reverse the activity and provide students with directions such as x/÷ 10, 100, etc. and ask what the movement is. Movement is acted out at the front, slides are slid, and calculators are multiplied and divided.

(5) Repeat for until students understand that 1, 2 and 3 places to the left is x 10, 100, 1000 respectively, and1, 2 and 3 places to the right is ÷ 10, 100, 1000 respectively. Ensure students understand the multiplicative structure of the numbers.

Notes: (a) If instruction focuses on moves of 3 positions (i.e., x, ÷ 10000) from one ones to one thousands to one millions, then this lays the foundation for metric conversions (e.g., km, m & mm; t, kg & g; m3, L & mL, etc.). This is done, for example, for length, by replacing the ones with mm, the thousands with m and the millions with km (see below for mm & m). (b) The relationships between x10/÷10 and one position to the left/right are important because it leads to innovative ways of introducing decimal place values (see Activity 1.4b).

(c) It is also important to see multiplicative structures in decimal places and to be flexible identifying the ones and subsequent positioning of the decimal point (see Activity 1.4b).

X 1000

m			mm		
Hundreds	Tens	Ones	Hundreds	Tens	Ones **.**
					.

÷1000

Activity 1.4b

Materials: PVCs, calculators, children holding cards as in 1.4a, slide rules or digit cards (see end of this activity for digit cards), pen and paper.

Directions:

(1) Put 2 in the tens position and 3 in the ones position (use children in the front, a slide rule or digit cards). Ask students to put number on calculator and write it on paper.

thousands			ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones **.**
				2	**3**

(2) Ask students to multiply by 10 on calculator and then move digits and write number to match that on the calculator. Repeat this for three more x10 (till get to 230,000). Ask students what is the pattern for what happens as x10. [Note – adding a zero is a consequence of the pattern – say correct but looking for underlying important pattern – this is digits move one place to the left.]

(3) Ask students (starting from the 230,000) to divide by 10 on calculator and then move digits and write number to match the calculator. Again repeat this for three more ÷10 and ask for the pattern. [Note – removing a zero is a consequence – the pattern is digits move one place to right.]

(4) Ask the question what will happen if ÷10 again (and again). Discuss need to introduce new positions, tenths and hundredths, as we keep dividing. Discuss how we can write the new position – e.g. 23 is 2 tens and 3 ones, so how can we write 2 ones and 3 tenths? Focus discussion on the need to have some way of identifying the ones, introduce the dot (decimal point), and validate it by showing what happens on the calculator when divide 23 by 10. [Note – stress importance of only using dot to show the ones – it does not have a position – symmetry is around the ones (not the dot).]

(5) Then ask students if dot or decimal point can change position? Ask how many tens in 23? Show if dot moved to the right hand side of 10s position (that is, the tens becomes the one for this instance), get 2.3 tens for 23. Discuss other position for dot, for example, how many hundreds in 23 [0.23] or how many tenths in 23 [230]?

(6) Discuss special movements of the decimal point such as pecentages. For example, making hundredths the one means 0.23 = 23 hundredths or 23%. This means 100% = 1, and 0.345 is 34.5%, and so on.

PLACE VALUE CHART & DIGIT CARDS

Millions			Thousands			Ones/Units		
H	T	One	H	T	One	H	T	One

Cut out and use with the Place Value Chart

0	1	2	3	4	5	6	7	8	9

0	1	2	3	4	5	6	7	8	9

0	1	2	3	4	5	6	7	8	9

2. METRICS AND CONVERSIONS

Metric units are measurement units that relate to base 10 numeration. These require an understanding of measurement and of our numeration system. As Section 1 covered numeration for whole numbers, we begin this section with basic ideas on measurement.

Measurement is best introduced through 5 steps.

(1) Identify the attribute: The first step is to understand what the metric is. Length is straight forward – it is being further along a line, area is amount of coverage, volume is the amount of space/fill in something, while mass is heft (the weight forcing down on the hand). Students should experience this first hand with their bodies, walking lengths, covering things (e.g. painting), filling things (e.g. sand, water, cubes, etc.) and feeling weights of things (the extent to which the object presses down on hand).

(2) Comparing amounts with no units: Measurements are continuous entities. They go on forever. Need to get to know them as unnumbered constructs by comparing without numbers. E.g. length – use string to compare; area –cut out newspaper to cover one object, then cut and rejoin to cover the other; volume – pour something from one to the other or immerse in water; and mass – beam balance or how far the mass pulls down on a rubber band.

(3) Introduction of unit/informal units: Western culture (for it own reasons) decided to apply numbers to measures – it has 'discretified' the continuous. It did this by developing the notion of unit. This unit is compared to the length, area, volume or mass under consideration and the number times the unit fits into what is being measured is the number of the measure. This means that measurement has a number and a unit. There is no such thing as 2, it is 2L or 2kg and so on. Then we introduce units by looking at everyday units – hands, pencils, cups full, dusters, etc. (Note - informal units are an opportunity to develop rules/principals for metrics, e.g., (a) same units to compare, (b) smaller units are more accurate but more time consuming, (c) the smaller the unit the larger the number, and so on.

(4) Formal Units: Once we are familiar with the notion of units, we introduce metrics. This is a 4 steps process: (a) need for a standard; (b) identification (make it); (c) internalisation (find measurement in everyday objects); and (d) estimation (learn to estimate with it). Metrics are based on base 10 number system and therefore relate as per place value.

(5) Application and Formulae: There are rules/procedures for calculating measure such as perimeter, area and volume. The formulae are best learnt by discovery or connections to known formulae.

2.1 Introducing the metric system

A **metric** is any standard unit of measure that is based on 10. Therefore it covers all measures except time and angle (based on 60). A need for a standard system of measurement should precede the introduction of metric measures. Activities 2.1a, 2.1b, 2.1c and 2.1d cover need for a standard, cm and m, cm^2, mL and L, and g and Kg respectively. Activities 2.1b, 2.1c and 2.1d have three steps: identification (make it); internalisation (find measurements of everyday objects); and estimation (learn to estimate with it).

Activity 2.1a

Materials: Students themselves, sand, containers, beam balances, streamers.

(1) Get two students of different sizes. Ask both to pace out an 8-pace wall. Check the differences. Ask two students to measure out lengths of streamers with hand spans and compare lengths. Ask, what problems would we have if houses were built by pacing or fabric bought and sold by arm measurements.

(2) Give students a container of sand – ask how it feels when hefted? Give students another container (different shape) and ask them to fill it to the same weight. Compare the weight of the containers of sand on a beam balance. Ask what would this mean if apples were sold by heft?

(3) Discuss problems with informal units – look at the need for standards. Look up the history of the development of standard measures.

(4) Discuss with class what a reasonable standard unit could be. For example, a piece of dowelling (about the length of an arm) is an excellent common measure to be adopted by a class – what could be a suitable standard for area, volume and mass. Temperature is a little more difficult. Is there anything we can do?

Activity 2.1b

Materials: Cardboard, paper, pens, straws, 2m tape, metre rulers. 30m tape

Direction:

(1) *Identification*. Get students to cut 1 cm pieces from different coloured drinking straws, thread these pieces along a string in groups of 10 of one colour followed by 10 of another colour.

(2) Get students to cut ten strips from 1cm graph paper that are 10cm in length. Tape these together to form a folding 1m measuring strip. This should be placed on cardboard to make it more durable.

(3) *Internalisation*. Get students to use a measuring tape to measure and record their personal body measures in m and cm:

Height	Arm span	Waist	Chest
Hip	Head	Neck	Leg
Arm	Foot	Length of hand	Ankle to knee
Wrist to elbow	Left hand	Thumb	Index finger
Middle finger	Ring finger	Little finger	

(4) Get students to find reference lengths in your body which is approximately 1cm, 10 cm and 1m.

(5) Get students to mark out a 10m distance using a large measuring tape and determine how many of their paces equal this 10 m. Direct students to pace distances and convert their paces to metres.

DISTANCE	PACES	METRES
Length of room		
Length of side of building		
Length across carpark		

(6) Get students to mark out 25 m, using a stop watch to time how long it takes them to walk this distance. Ask them to use this time to work out how long it would take them to walk a kilometre.

(7) *Estimation*. Get students to first estimate, measure the length of the objects and to complete the table below. Estimate/measure each distance before starting the next.

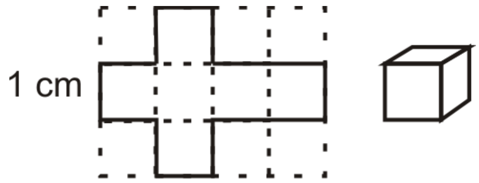
OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Lecturer's height			
Blackboard's length			
Height of top shelf			
Height of student			
....			
....			

Activity 2.1c

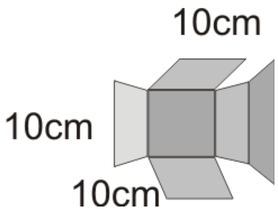
Materials: Measuring cylinders, centicubes, measuring tapes, 1cm grid paper, cardboard, plasticine, overflow trays, tape, scissors, glue and pens, things to pour (e.g., rice, macaroni, marbles, sand or water), and a variety of containers (milk cartons, bottles and jars, glasses and cups, jugs, matchboxes, cassette tapes, and other small boxes)

Direction:

(1) *Identification.* Using 1 cm grid paper, draw and cut out a net for a cube of side 1cm. Fold and tape to make the cube. Connect 1 m dowels to form a cubic metre. Fill the 1cm cube with water (or sand). Pour this into another container. This represents 1mL



(2) Use cardboard to make a net for a cube of side 10cm. Tape and fold this cardboard to make the cube. This cube is 1 L. Check this by pouring 1 L of water or sand into it. Check that this cube holds the same as a 1 L soft drink bottle.



(3) Calibrate a container into 100 mL levels in either of the following ways:

Way 1 – take a glass jar and pour 100 mL amounts into it, marking the levels with tape as you go;

Way 2 – take a 1L milk carton, cut off the top and use a ruler to divide the height into 10 equal intervals.

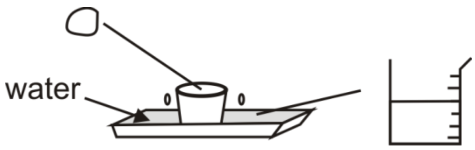
(4) *Internalisation*. Obtain a collection of small rectangular prisms (boxes – e.g., matchbox, cigarette box, cassette case) and pack these with MAB units to find their volume. Check by using the formulae V = L x B x H. Try to estimate first.

(5) Obtain a collection of jars and jugs and pour 250 mL and 500 mL into them and note levels. Try to estimate where the levels will be before pouring.

(6) *Estimation.* First estimate and then measure the volumes of objects as given by your lecturer. Estimate and measure each object before moving onto the next. ESTIMATE THE VOLUME – DO NOT ESTIMATE LENGTH, BREADTH, HEIGHT. Use a tape to measure volume (to check your estimate) and measuring cylinders for capacity.

OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Volume			
Chalk box			
Shoe box			
Cupboard			
Under the table			
…………………			
…………………			
Capacity			
Cup			
Glass			
Bottle			
Plastic container			
....			
....			

(7) Use an overflow tray and a measuring cylinder, to find the volume of objects by immersion and overflow. Estimate first.



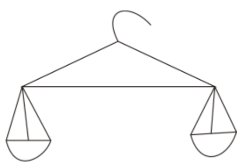
OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Lump of plasticine			
Rock			
Your fist			
....			

Activity 2.1d

Materials: Margarine containers, scissors, string and rubber bands, spring balances, beam balances or coat hangers, masses, bathroom scales, and things to weigh (e.g., rice, pasta, marbles, sawdust)

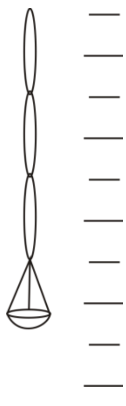
Direction:

(1) *Identification*. Construct mass measurers as follows:



<u>Method 1</u> – a wire coat hanger, string and 2 margarine containers (plus masses);

<u>Method 2</u> – a long piece of paper, 3 rubber bands, string and a margarine container (calibrate the ”spring balance”' with masses – mark lengths on the paper).



(2) Use these measurers to make up plastic bags, or other containers, containing 100 g, 250 g, 500 g and 1 kg or various materials (pasta, sand, marbles, rice, etc).

(3) *Internalisation*. Use a bathroom scale to measure your own mass in kg. Measure the mass of 1L of water.

(4) Find objects in the environment that measure approximately 1 kg, 500 g, 250 g, 100 g, 50 g and 1 g. Make up lumps of plasticine to these measures.

(5) *Estimation*. Estimate first and then measure the masses of the following objects as given by your teacher. Complete estimates and measures of objects before moving onto the next.

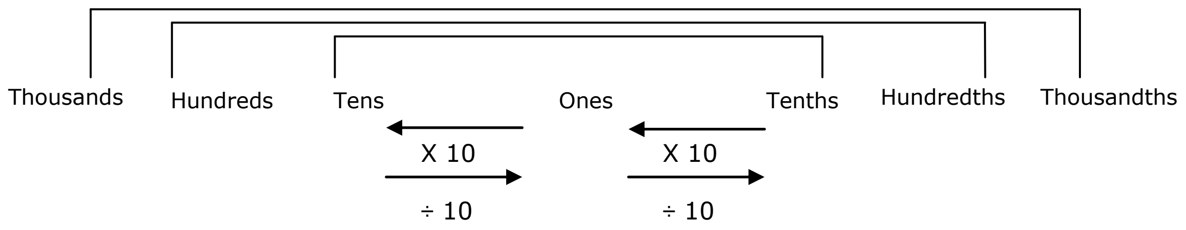
OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Duster			
Case or port			
Shoe			
Another student			
Lecturer			
………………..			

Investigation 2.1 Make up a series of activities to take children through identification, internalisation and estimation for standard units for one of the following:

Area Time Money

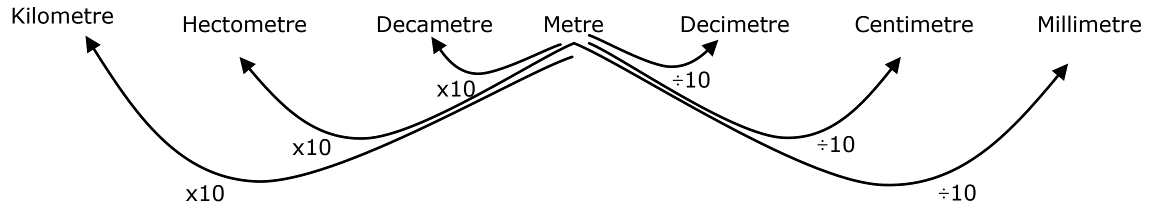
2.2 Relationship metrics and place value

The metric units in length, area, volume, and mass are designed so that different units in the same attribute relate to each other similarly to the place value positions in the number system. The full decimal system is symmetric about the ones and positions relate in terms of x and ÷ by 10, 100, 1000, etc., as shown below



Metric units are designed about a metric unit (e.g., L, f, m, m², etc.) and then other units relate to this by multiples and dividers of 10, 100, etc.

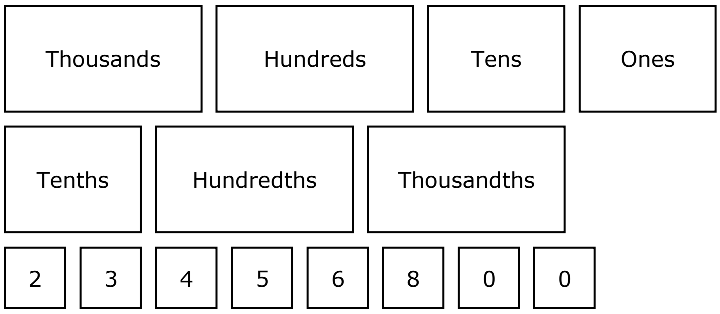
These relations are given names based on Latin names. This is best seen in length, where "milli" means 1/1000,"centi" means 1/100, "deci" means 1/10, "deka" means 10, "hecta" means 100 and "kilo" means 1000, as in the figure below.



This shows the similarity between place value and metric units.

Activities 2.2a and 2.2b cover developing decimal relationships and metric relations respectively.

Activity 2.2a



Materials: cards as on right, calculators, slide rules

Directions:

(1) Give students a calculator and slide rule that matches the cards (see below)

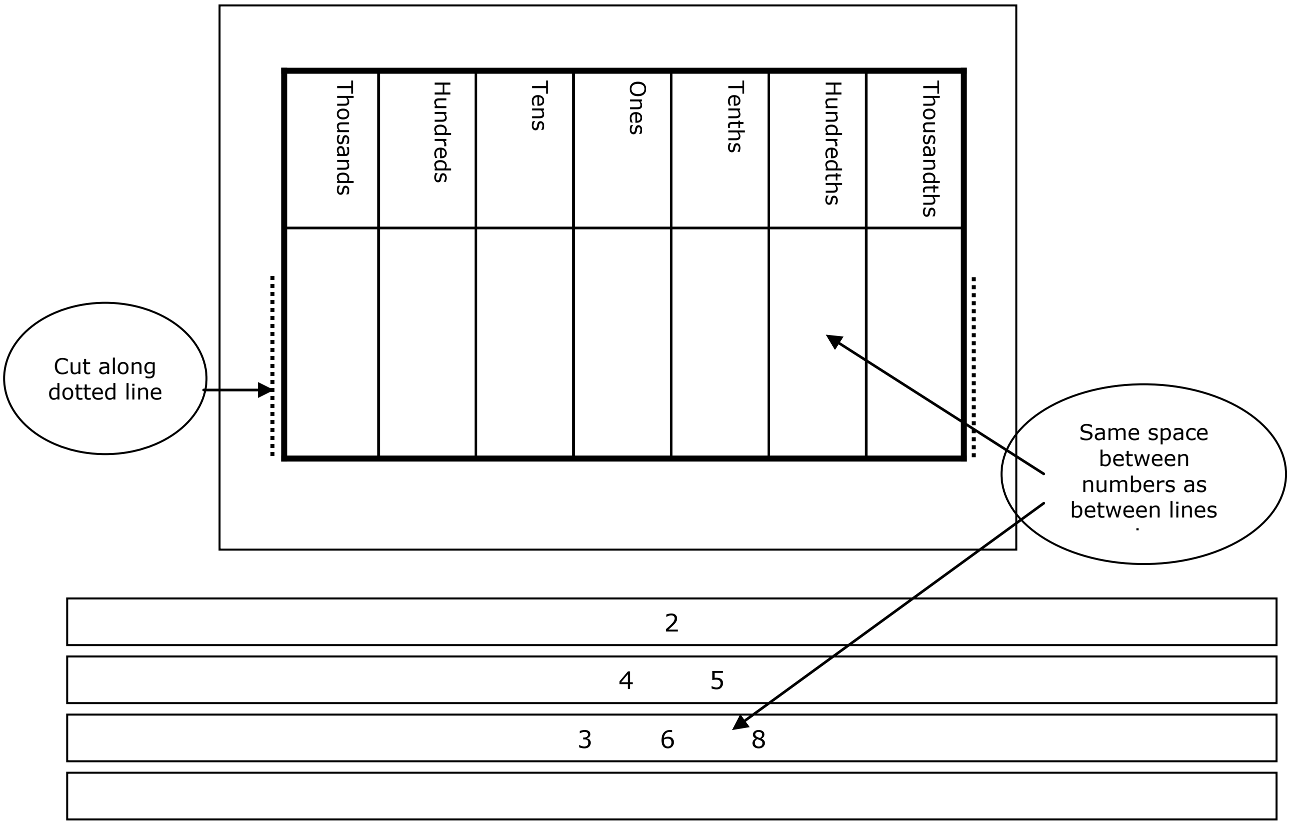
(2) Hand out place value cards and get students to stand in front of class showing the place-value positions in order. Then get students to stand in front of the place-value positions holding the number cards (the same as for the slides). As students stand at the front, ask other students to make the number shown on the slide rule and input the numbers into their calculators.

(3) Discuss numbers made. Discuss how zeros have to be added (show this with students at front). Discuss where the decimal point goes and how this relates to zeros.

(4) Stick place-value cards on front wall. Place decimal point after the ones (do not give it a space – make sure the distance between ones and tens and ones and tenths is the same). Get students holding numbers (same as for slides) to move left and right. Discuss what operations and number has to be done and calculate to follow these movements. Discuss what the rule/pattern is [x10 for 1 position left, x100 for 2 positions left and so on; ÷10 for one position right, ÷100 for 2 positions right (See next page diagram).

(5) Reverse the activity – ask what movements would be the same as ÷10, x100, and so on. Start students with numbers at any position. Copy movements with slide rulers. Discuss what the role/patter in [x100, move 2 places to left; ÷1000, move 3 places to left and so on – don’t see this in terms of adding, removing zeros or moving decimal units – these are in consequence of the above pattern].

(6) <u>Note:</u> It is better if the movement with cash is done as above. However, with one student, the slide rule may suffice.



Activity 2.2b

Materials: Calculators, cards

Directions:

Kilometre	Hectometre	Decametre	Metre
Decimetre	Centimetre	Millimetre	

(1) Look up metrics on internet and find all the names (particularly the prefixes). Discuss their meanings

(2) For length, put out metre, and place other length units on each side (put names on cards). Discuss how they relate to metre. Give multiply/divide relationship as below.

Kilometere	Hectametre	Decametre	Metre	Decimetre	Centimetre	Millimetre
×1000	×100	x10	1	÷10	÷100	÷1000

(3) Discuss the rotation used (e.g., cm, mm)!

(4) Discuss how to move from metre to each of the other units and from the other unit to metre in terms of x and ÷ by 10, 100, and so on. For example, how many mm in a m, how many full system are m in a km, what fraction of a m is a cm, and so on?

(5) Discuss which of the length units in the full system are normally in the system we use everyday (km, m, cm & mm). Which one is used in school that is not used in trades? (Cm).

(6) Repeat directions (1) to (4) for area, volume and mass units. What is the interesting difference in mass units? (The main unit is the kg and not the g).

2.3 Relating metric units by using place value

To enable a student to fully understand how to convert metric units, metric units need to be related to the place value and multiplicative structure of number.

For example:

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
km			m		cm	mm

Thus, as 100 hundredths equals one, 100cm = m. And, as 1000 ones is a thousand, km = 1000m.

Activities 2.3a and 2.3b cover relating place value and metrics, and looking at what happens if what is one is change to mm respectively. Since this booklet is for VET, the focus in the units in VET (e.g., m and mm).

Activity 2.3a

Materials: Cards from Activities 2.2a and 2.2b; calculators; digit cards/PVC (place value cards) for metrics and numbers (see below); slide rule (see below); extra cards

mm	m	km

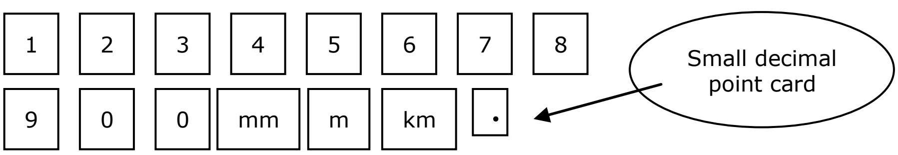
Directions:

(1) Give each student digit cards/PVC as below. Ask students to cut out cards.

(2) Stick place value cards on wall in correct sequence. Have a student (or students) with a number card (or cards) moving along in front of the place value positions. Ask students to show the movements on their charts with their small cards, and to track the movements on their calculators.

(3)

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths



(4) Ask three students to stand with the mm, m and km cards in front of the appropriate place value positions if m is ones. Ask other students to place mm, m and km cards on the second row of their chart.

(5) Discuss what would happen if mm became m (x1000), m became mm (÷1000), and so on. Put a 5 in the hundredth position. Ask students how many mm this shows (50) and how many m (0.05)? Repeat for other numbers.

(6) Build a larger place value chart and a side rule as below

Millions			Thousands			Ones			Thousandths			Millionths		
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O

Km	m	mm
KL	L	mL
Km^2	m^2	mm^2
t	kg	g

(7) Use this slide rule to show relationships when, for example, m is in one position, km is in ones position, mm is in ones position mL is in ones position, mm^2 is in ones position and so on.

Activity 2.3b

Materials: PVC and digit cards (without fixed decimal point) from activity 2.3a, calculators, and extra small cards with decimals point on it.

Directions:

(1) Ask students to put a 3 in the tens position, 5 in ones, and 7 in tenths and put the decimal point card after ones. Give number as ones [35.7]. Move the decimal point to come after tens but leave the number unchanged. Give number as tens [3.57]. Move the decimal point to come after tenths and again leave the number unchanged. Give the number as tenths [357]. Move the decimal point to come after the thousandths. Give the number as thousandths [35 700].

(2) Use other examples to show how numbers change depending on the position of the decimal point. Discuss whether we should therefore always say "23 ones" instead of just "23". There are two ways of using place value charts to look at relationships in metrics, for example, between m and mm: (1) to move decimal point around the chart (to change the "ones" in the chart); and (2) to move the position of units to ones in the chart.

(3) Ask students to place mm, m and km cards on the second row of place value chart and add the decimal point card after the ones. Put 6 in ones, 4 in tenths, and 5 in the hundreds position. Show number on calculator as metres [6.45m]. Shift the decimal

point to come after thousandths. Leave numbers (digit cards) as they are. Read the number as mm.

(4) Show how m relates to mm with other examples using digit cards and changing position of decimal point from beside.

(5) Moving *position* of units. Give each students the slide rule from activity 2.3a. Develop new slides as below. Place slide A in PVC. Make m the ones. What is the value [2.634]? Make the mm the ones. What is the value [2 634]?

A		2m	6	3	4mm
B		m			6mm

(6) Repeat for slide B [0.006m and 6mm]. Try other slides with 60mm and 600mm.

2.4 Hundredths of a mm

In some engineering situations associated with engine repair and maintenance, measures have to be taken to hundredths of mm. For example, 2.54mm would be as follows:

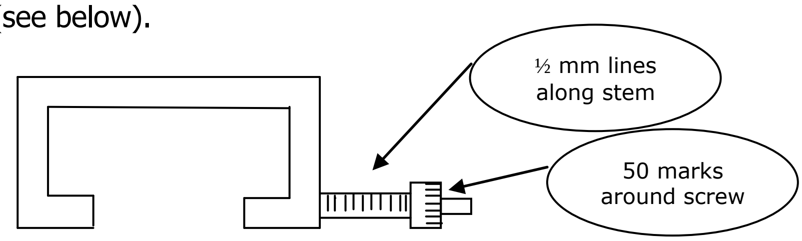
(1) using the place value chart (PVC) and the digit cards from activity 2.3a; and

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
			mm •			
			2	5	4	

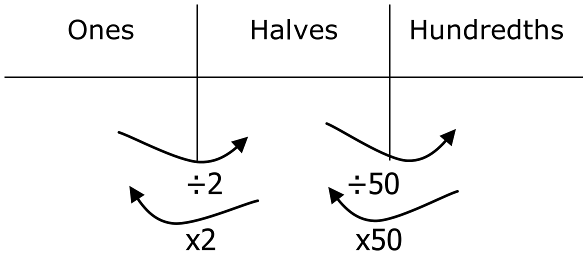
(2) using the PVC for the slide rule from activity 2.3a

Millions			Thousands			Ones			Thousandths			Millionths		
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
		Km			m			mm •						
								2	5	4				

However, the instrument that is used to measure the hundredths of mm is a micrometre screw gauge with lines marked every ½ mm along the stem and 50 marks around the screw (see below).



This would mean that the mm are broken up into positions which are based around x, ÷ 2 and ÷50 instead of x, ÷ 10 for both positions. Of course, 1/50th of a ½ is a hundredth.

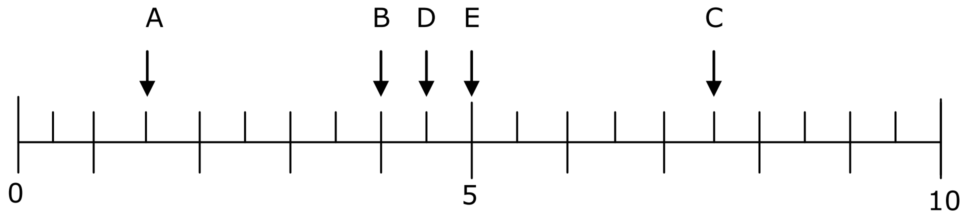


Activity 2.4a

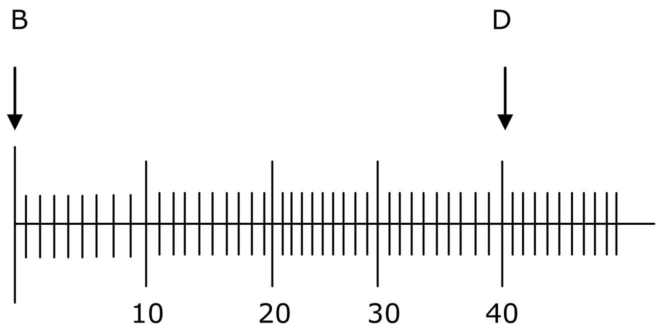
Materials: Pen & paper

Directions:

(1) Look at a scale in ½mm (see below). What is the value of A, B and D (1.5mm, 4.0mm, 7.5mm, 4.5mm, 5mm).



(2) The screw turns one revolution as gauge goes from B to D and each revolution has 50 gradations Or marks (see below). Thus, as the screw is used to measure the distance, the gauge gives three numbers: (1) a number of millimetres, (2) a ½ mm or 0, and (3) a number up to 50 for the screw (e.g., 3 - ½ - 23). What other numbers are possible [e.g., 4 – 0 – 13]?



(3) Let us look at these three components, mm, ½ mm and 50ths of ½ mm. This could be displayed on a chart as below. For example, 3 ½mm and 23 on screw.

mm	½mm	Screw marks
3	1	23

a) Put 4mm and 1 screw mark on the chart.

b) Put 15 ½mm and 48 screw marks on the chart.

(4) Discuss the relationships between positions. Discuss how to get from mm to ½mm to 50^{ths} (see below). Discuss how different this is to the normal chart as below.

mm	½mm	Screw marks

÷2 ÷50

x2 x50

Ones	Tenths	Hundredths

÷10 ÷10

x10 x10

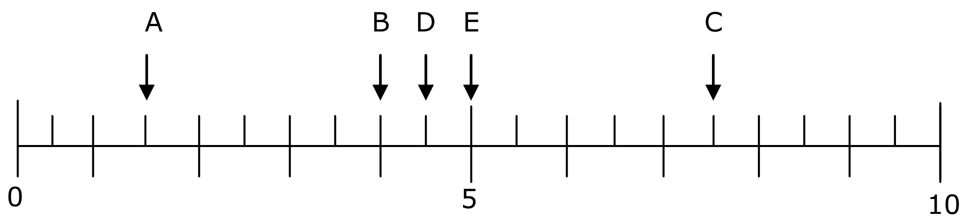
(5) Discuss what is the same about the two relationships? [50 x 2 = 100 and 10 x 10 = 100; therefore, both charts are in hundredths].

Activity 2.4b

Materials: Pen & paper

Directions:

(1) Relook at the scale in Acitivty 2.4a

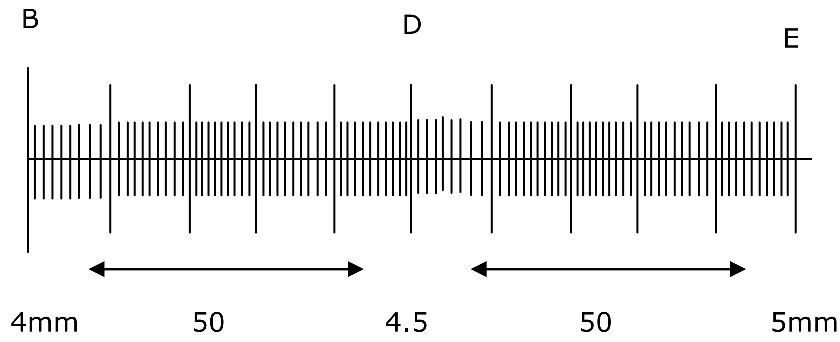


C is 7 ½mm. What is this in normal decimal place values? [7.5]. Put this on a normal chart

Ones	Tenths	Hundredths
7	5	0

Put these on the chart: (a) 4mm (b) 8 ½mm

(2) Look at B, D and the 5mm point. Enlarge this section and put in the 50 gradations from the screw as below. Note that B is 4, D is 4.5mm and E is 5mm



Discuss that there are 50 gradations Between B and D and between D and E, so B to E is 100 gradations.

Discuss that this means screw gradations are hundredths. Thus ½mm is 5 tenths and each screw gauge mark is hundredths.

(3) Relook at what the micrometer does. For example, in terms of the gauge, 3, ½, and 23 relate as follows:

mm	½mm	50ths of ½mm
3	1	23

Ones $\xrightarrow{\div 2}$ halves $\xrightarrow{\div 50}$ hundredths

Ones $\xrightarrow{\div 100}$ hundredths

Ones $\xleftarrow{\times 2}$ halves $\xleftarrow{\times 50}$ hundredths

Ones $\xleftarrow{\times 100}$ hundredths

In terms of the traditional chart, the three parts relate as follows:

	Ones	Tenths	Hundredths
3mm	3	0	0
½mm	0	5	0
23 crew marks	0	2	3
TOTAL	3	7	3

Thus, to work out what the gauge says, add the millimetres and ½ millimetres in terms of .5 and then add screw marks as hundredths .23, that is, 3.5 + 0.23 = 3.73.

(4) What is the value of:

	Stem	Screw
a)	4 ½mm	17
b)	9mm	41
C)	6 ½mm	6

3. FRACTIONS AND IMPERIAL UNITS

The basis of number systems is the one. Large numbers are developed by grouping ones into bases. In the normal number system, the base is ten, so ones are grouped into 10s, 100s, 1000s, and so on. Small numbers are partitioned into parts to form fractions. In the normal number system, the parts are 1/10ths, 1/100ths, 1/1000ths, and so on.

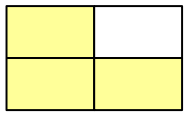
In metric units, the one was chosen to be mm, a small length, so that factions are unnecessary. In imperial units it was different. The unit chosen was an inch which is large enough to require fractions.

The imperial units were developed across history as a measurement system based on body measurements and cultural and traditional relationships. For example, the length measures inch, foot, and yard are related to the body and convert as follows: 12 inches = 1 foot and 3 feet = 1 yard. The mile was developed from 1000 paces (left foot to left foot) and set at 1760 yards or 5280 feet.

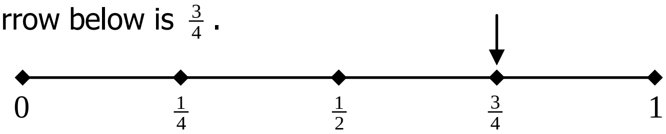
The inch is large enough that it requires an understanding of fractions and equivalent fractions.

Fractions have five meanings:

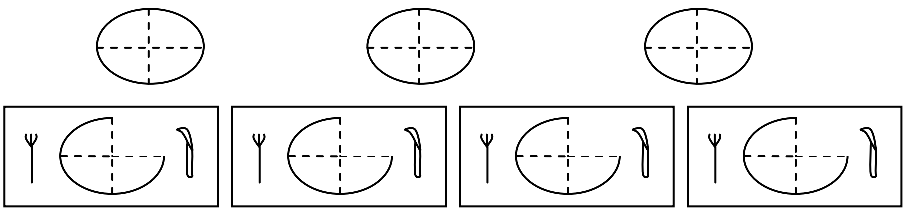
(1) *Fractions as part of a whole*. Start with a whole, divide it into equal parts – The nature of the fraction comes from the number of constituent parts. Taking more than one of these parts gives rise to fractions with numerators larger than one. For example, in the figure at right, 3 out of 4 parts are shaded, or $\frac{3}{4}$ of the area is shaded.



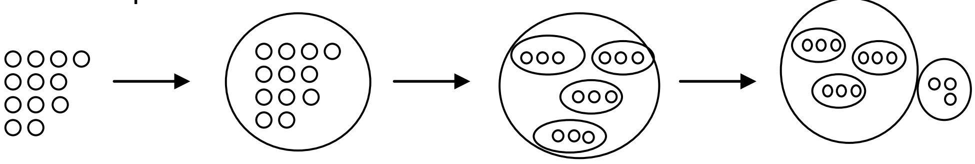
(2) *Fractions as a point on a number line*. Start with a length that is designated as one whole that is divided into a number of equal parts. The ends of these parts are parts are fractional numbers and give the position at that point. For example, the position indicated by the arrow below is $\frac{3}{4}$.



(3) *Fractions are division*. If three cakes are shared among 4 people, then each person gets $\frac{3}{4}$ of a cake (as below).



(4) *Fractions as part of a set*. This is similar to the first meaning but the set first has to be reconsidered as one whole and then this whole divided equally into parts. For example, for $\frac{3}{4}$ of 12 items, consider 12 as one whole, break it into 4 equal parts, and take 3 of these parts.

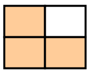
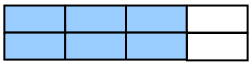


Fractions as multipliers. This relates to the actions of fractions and says that a fraction $\frac{p}{q}$ is multiplying by p and then dividing by q. For example, $\frac{3}{4}$ of 12 is 12×3 ÷ 4 = 9.

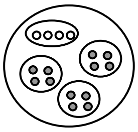
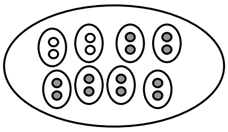
Equivalent fractions are fractions with different numerators and denominators that are of the same size. For example, $\frac{3}{4} = \frac{6}{8}$ in which the numerators are 3 and 6 and the denominators are 4 and 8.

Equivalent fractions can be understood in one of three ways:

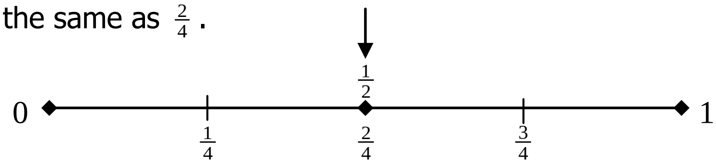
(1) *Area diagrams*. For example, in the two diagrams below, $\frac{3}{4}$ is represented by the shaded areas in the left diagram, whilst $\frac{6}{8}$ is represented by the shaded areas in the right diagram. The shaded areas in both diagrams represent the same fraction of each whole.

$\frac{3}{4}$  $\frac{6}{8}$ 

(2) *Set diagrams*.

$\frac{3}{4}$  $\frac{6}{8}$ 

(3) *Number line diagrams*. For example, in the number line below, the arrow is pointing to $\frac{1}{2}$ which is also the same as $\frac{2}{4}$.



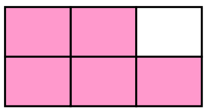
3.1 Meaning of common fractions

For being a mechanic, the two understandings needed are part of a whole and number line. Activities 3.1a and 3.1b cover how to teach the part of a whole and number line meanings of fractions respectively.

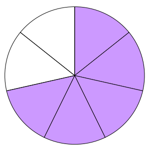
Activity 3.1a

Materials: Cardboard shapes, scissors, paper, and pens.

(1) Take a shape, say a rectangle, and partition it into equal parts, say 6, as shown in the figure at right. These parts are called sixths. Take 5 of them. The fraction of the whole is 5 sixths, which is written as $\frac{5}{6}$.



(2) Repeat this for other shapes and other partitions, such as:

$\frac{2}{3}$  $\frac{5}{7}$ 

(3) Discuss how the number of parts gives the name:

2 – halves	5 – fifths
3 – thirds	6 – sixths
4 – quarters or fourths	7 – sevenths and so on.

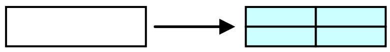
Discuss how the number of these parts you take is what gives the *other* part of the fraction name, e.g. 4 sevenths or $\frac{4}{7}$

(4) Discuss how the total number of parts is the denominator and the number of these parts being considered is the numerator.

Remind students that for fractions:

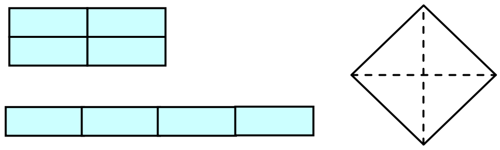
a) retain the whole;

b) realise that the below diagram is not 1 to 4 but is 1 to 4 parts as 1 whole

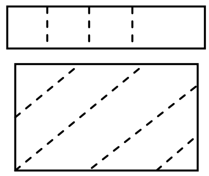


c) have equal parts, as below;

¼



Not ¼



(5) Reverse the process by making sure students also know how to go from part to whole.

For example, this is ¼  draw the whole.

Or, this is ¾,  draw the whole.

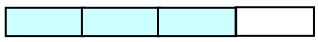
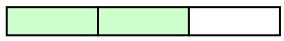
Note: Because spanners and wrenches fit the distance across a nut, then their nomenclature can be best understood as a number line.

Activity 3.1b

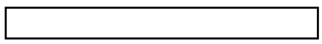
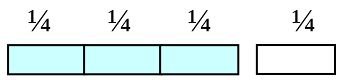
Materials: Paper strips

Directions:

(1) Repeat (1) from Activity 3.1a with paper strips. E.g.

3/4 is  2/3 is 

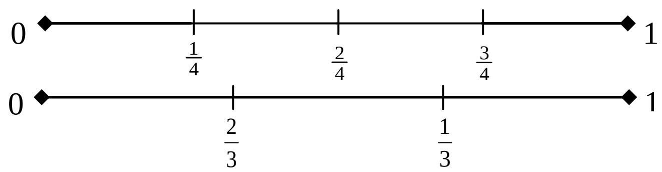
(2) Reverse this process

¾ is  1 Whole = 

(3) Extend this to number lines

3/4 is

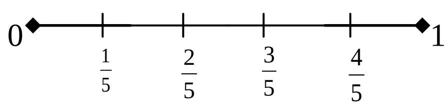
2/3 is



It also helps to walk a line – going half way etc. Also can get a rope, pegs, and fraction cards - have someone hold each end of the rope, then peg the fraction cards on the rope.

(4) Reverse this process

2/5 is 

therefore, 1 whole is 

3.2 Equivalent fractions

The number line and area models are good for equivalent fractions. The number line model easily shows whether or not fractions are equivalent or not. The area model is good to show the pattern for when fractions are equivalent. Activities 3.2a and 3.2b cover meaning of equivalent fractions and patterns for determining equivalence respectively.

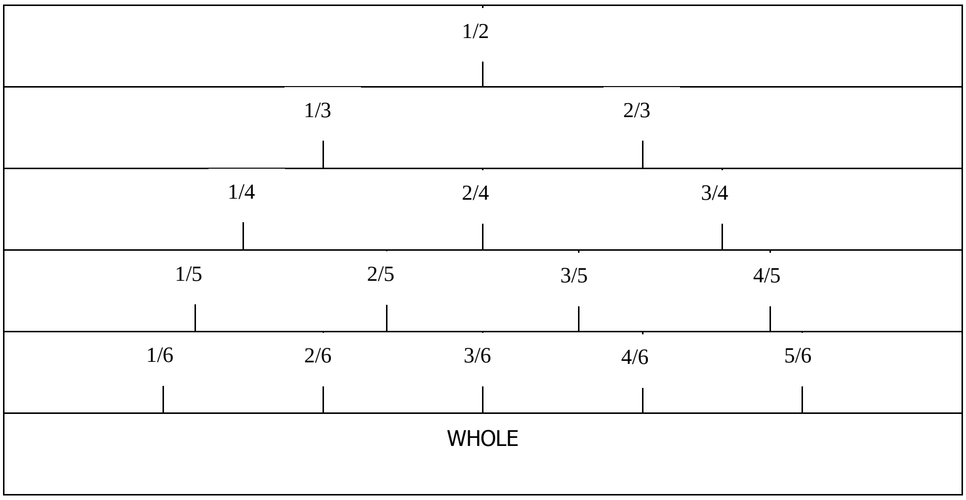
Activity 3.2a

Materials required: Lined paper, pen and scrap paper.

Directions:

(1) Draw a series of lines.

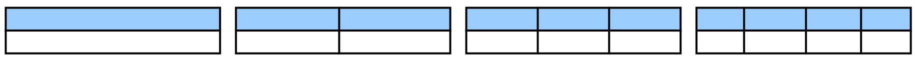
(2) Divide them by different fractions, as shown in the figure below.



(3) A vertical line through ½ will show that $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$

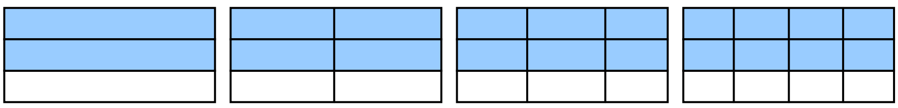
Find other equivalent fractions using this table.

(4) Obtain many rectangles. Fold length-wise and shade $\frac{1}{2}$, then fold in opposite directions as below.



This shows that $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\ldots$

Do the same for $\frac{2}{3}$, as shown in the figure below.



This shows that $\frac{2}{3}=\frac{4}{6}=\frac{6}{8}=\ldots$ and so on.

Discuss how it is important to realise that $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}\ldots$ and so on because $\frac{2}{4}=\frac{1}{2}\times\frac{2}{2}$ and $\frac{3}{6}=\frac{1}{2}\times\frac{3}{3}$ and so on. Discuss how $\frac{2}{4}=\frac{1}{2}$ because $\frac{2}{4}=\frac{1}{2}\times\frac{2}{2}$ is the same as $\frac{1}{2}\times 1$

Activity 3.2b

Materials: Fraction sticks, pen, paper

Directions:

(1) Make up a set of fraction sticks.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**
3	**6**	**9**	**12**	**15**	**18**	**21**	**24**	**27**	**30**
4	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**	**40**
5	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
6	**12**	**18**	**24**	**30**	**36**	**42**	**48**	**54**	**60**
7	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**
8	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**	**80**
9	**18**	**27**	**36**	**45**	**54**	**63**	**72**	**81**	**90**
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**

(2) Put 2 with 5 to make 2/5, then you can see a sequence of equivalent fractions.

2	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**
5	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**

What is the pattern? Top up by 2, bottom up by 5?

Multiplying both numbers by the same $2\times 2 \equiv 4$, $5\times 2 \equiv 10$, $2\times 3 \equiv 6$, $5\times 3 \equiv 15$?

(3) Discuss the $\frac{2}{5} = \frac{2}{2} = \frac{4}{10}$, $\frac{2}{5}\times\frac{3}{3} = \frac{6}{15}$ pattern and show this is the same as x1 which means the two fractions are the same.

(4) Show how $\frac{2}{5}\times\frac{2}{2} = \frac{4}{10}$ means:

$$\frac{\cancel{4}^{\,2}}{\cancel{10}_{\,5}} = \frac{2}{5}$$ by cancellation

Show that $\frac{8}{20} = \frac{14}{35}$ because: $$\frac{\cancel{8}^{\,2}}{\cancel{20}_{\,5}} = \frac{2}{5}$$ (divide top and bottom by 4)

And also equals: $$\frac{14}{35} = \frac{2}{5}$$ (divide top and bottom by 7)

Discuss this meaning – that two fractions are equivalent is they cancel down to the same starting fraction.

3.3 Equivalent fractions in Imperial units

Imperial units are based on fractions of 1 inch. In Engineering they are those that relate to halving, (e.g. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and so on) and those that, measure really small lengths (e.g. 1/1000ths).

Activities 3.3a and 3.3b cover the fractions used in Imperial units and their equivalences, calculating equivalent or near equivalent fractions respectively.

Activity 3.3a

Materials: Paper strips.

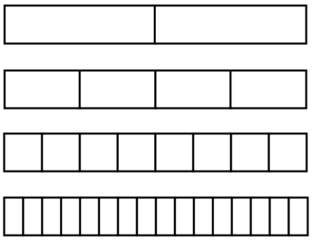
(1) Take stripes of the same length and do the following, as shown in the figure at right:

(a) Divide in $\frac{1}{2}$ gives halves.

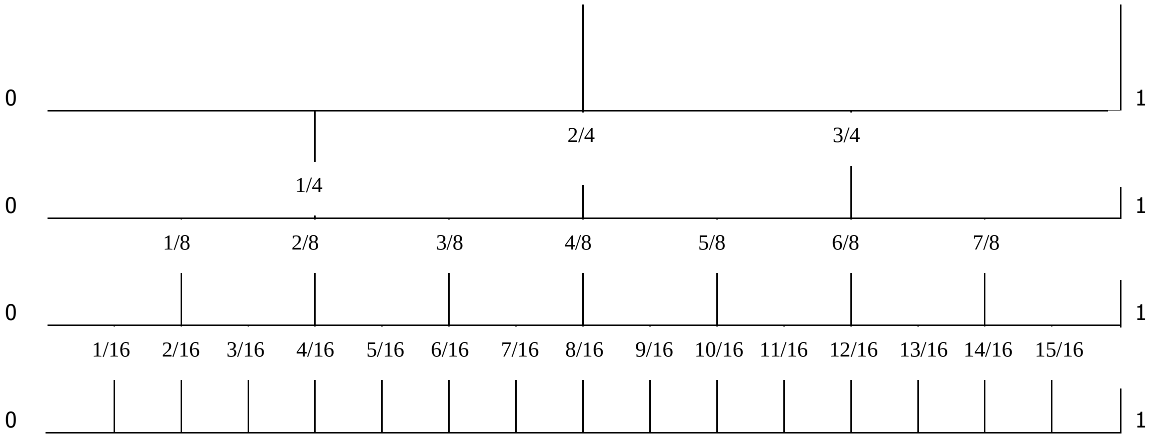
(b) Divide in $\frac{1}{2}$ twice gives fourths.

(c) Divide in $\frac{1}{2}$ three times gives eighths.

(d) Divide in $\frac{1}{2}$ four times gives sixteenths.



(2) Use a series of four number lines to record this information.



(3) Relate all the sixteenths to all other fractions, using strips and number lines to check.

0,	5/16,	11/16,
1/16,	6/16 = 3/8, 7/16,	12/16 = 6/8 = 3/4,
2/16 = 1/8,	8/16 = 4/8 = 2/4 = 1/2,	13/16,
3/16,	9/16,	14/16 = 7/8,
4/16 = 2/8 = 1/4,	10/16 = 5/8,	15/16

Thus wrenches and bolts in imperial are named by their fraction and are $\frac{7}{16}$, $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{5}{8}$. They can also go one more halving to $\frac{15}{32}$.

Activity 3.3b

Materials: None

Directions:

(1) To understand wrenches in imperial requires remembering the following order

1/16, 1/8, 3/16, 1/4, 5/16, 3/8, 7/16, 1/2, 9/16, 5/8, 11/16, ¾, 13/16, 7/8, 15/16

Or, understanding how all the sizes are changed to 16ths and vice versa.

(2) Changing a fraction to 16ths relies upon the denominator and numerator changing by the same multiplier; for example:

$$\frac{3}{8} \xrightarrow[\text{x 2}]{\text{x 2}} \frac{?}{16} \quad = \quad \frac{6}{16} \quad \text{or} \quad \frac{3}{8} = \frac{6}{16}\,(\text{x2})$$

$$\frac{1}{4} \xrightarrow[\text{x 4}]{\text{x 4}} \frac{?}{16} \quad = \quad \frac{4}{16} \quad \text{or} \quad \frac{1}{4} = \frac{2}{8}\,(\text{x2}) = \frac{4}{16}\,(\text{x2})$$

(3) Changing 16ths to other fractions requires using cancellation; for example:

Keep dividing by 2….

$$\frac{8}{16} \overset{\div 2}{=} \frac{4}{8} \overset{\div 2}{=} \frac{2}{4} \overset{\div 2}{=} \frac{1}{2} \qquad \text{OR} \qquad \frac{12}{16} \overset{\div 2}{=} \frac{7}{8} \overset{\div 2}{=} \frac{3}{4}$$

(4) Use activity (2) and (3) to answer the following questions:

a) What wrench is slightly larger than $\frac{9}{16}$″ ?

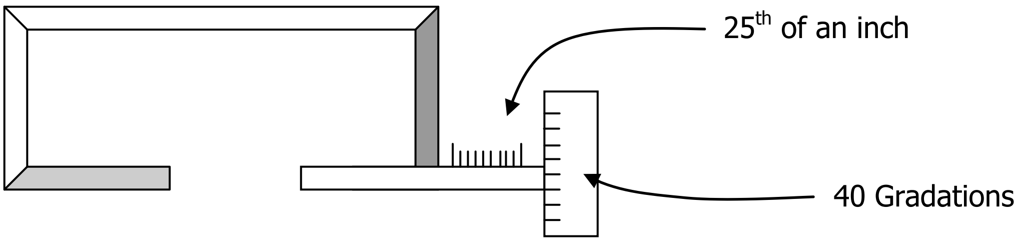
$$\frac{9}{16} \longrightarrow \frac{10}{16} = \frac{5}{8}$$

b) What Wrench is slightly smaller than $\frac{3}{4}$″ ?

$$\frac{3}{4} = \frac{6}{8} = \frac{12}{16} \longrightarrow \frac{11}{16}$$

3.4 Thousandths of an Inch

Similar to hundredth of a mm, micrometer screw-gauges are used to find thicknesses to a thousandth of an inch. The micrometre does not do this in the normal manner through tenths and hundredths to thousandths, but by dividing each inch on the stem into 25ths and then halving 40 marks (gradations on the screw as below).



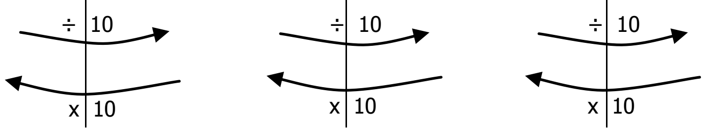
This means that three measures with this micrometer have 3 numbers (1) inches, (2) 25ths of an inch, and (3) 40 gradations. An example could e 1, 17, 23 as below:.

Inches	25ths	40 Gradations
1	17	23

$$1 \underset{\text{x } 25}{\overset{\div\, 25}{\rightleftarrows}} 17 \underset{\text{x } 40}{\overset{\div\, 40}{\rightleftarrows}} 23$$

The objective is to transfer these numbers to a normal chart:

Ones	Tenths	Hundredths	Thousandths



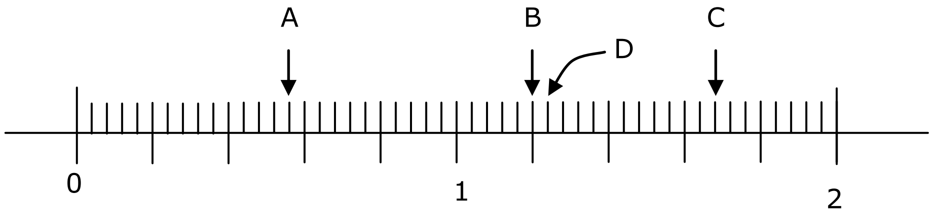
Activities 3.4a and 6.4b cover meaning of the micrometre screw-gauges' numbers, and how to use the numbers to calculate the number of thousandths of an inch respectively.

Activity 3.4a

Materials: A micrometer screw-gauge for thousandths of an inch, pen, paper.

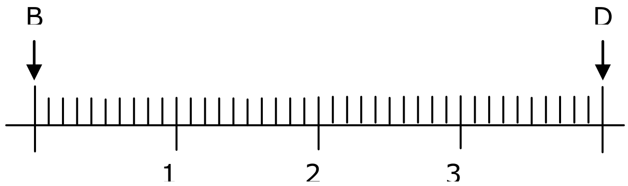
Directions:

(1) Look at a scale where inches have been divided into 25 equal parts as below:



Discuss what A, B and C and D are. (14/25 of an inch, 1 1/5 Inches, 16/25 inches, 1 17/25 inches).

(2) The screw turns one revolution as gauge goes from B to D and each revolution has 40 gradations or marks (see right):



Thus, as the screw is turned to measure distance, the gauge gives us three numbers

a) the number of inches

b) the number (up to 24) of 25ths of an inch, and

c) the number (up to 39) on the screw.

An example is 1 – 17 – 23. Discuss what other numbers are possible?

(3) Place the 3 numbers on a chart as below. For example 1 – 17 – 23, this is:

Inches	25ths	Screw
1	17	23

Put the following on the chart:

a) 2 – 6 – 28,

b) 0 – 24 – 39Discuss why 1 – 31 – 17 is not a suitable number.

(4) Discuss the relationships between the positions. Look at how we get from screw to inches and vice versa.

Inches	25ths	Screw
1	17	23
÷ 25 → / ← x 25	÷ 40 → / ← x 40	

(5) Discuss how this is different to the normal chart:

Ones	Tenths	Hundredths	Thousandths
÷ 10 → / ← x 10	÷ 10 → / ← x 10	÷ 10 → / ← x 10	

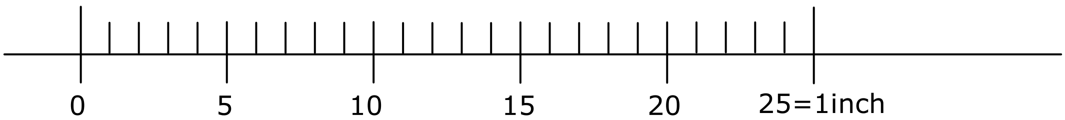
Look for similarities (10 x 10 x 10 = 1000 and 40 x 25 = 1000), this means the screw gradations are 1/1000 of an inch.

Activity 3.4b

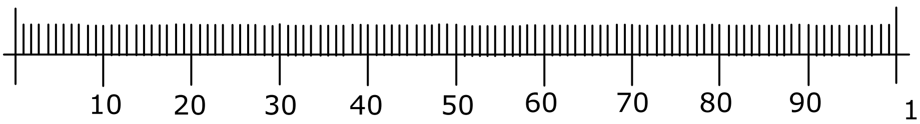
Materials: Pen, paper.

Directions:

(1) Re-examine the scale in Activity 3.4a for 1 inch.



If the inch had been divided into 100 parts (as below), each gradation would be one hundredth or 0.01



Discuss how, if each 25th was broken into four parts, this would give hundredths. Explain and discuss how this means that each 25th is 4 hundredths or 0.04. Thus, for example, 11/25ths is 11 x 0.04 - .44 (See Below)

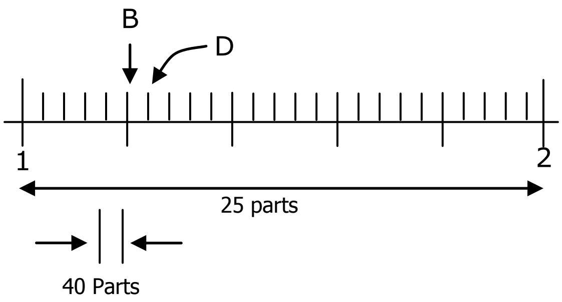
25ths	Ones	Tenths	Hundredths	Numbers
1/25th	(1 x 4)		4	0.04
11/25ths	(11 x 4)	4	4	0.44
23/25ths	(23 x 4)	9	2	0.92

Put these on the chart and find numbers:

(a) 16/25

(b) 19/25

(2) Re-examine the distance from B to D. This is 40 gradations. B to D is 1/25th of an inch. Discuss what therefore makes on gradation.



Discuss that there are 25 lots of 40 in the inch so this means the screw divides the inch into 25 x 40 = 1000 parts. Discuss that this means each gradation is 1/1000 inch.

Discuss how to translate this to a normal chart (see below)

Chart

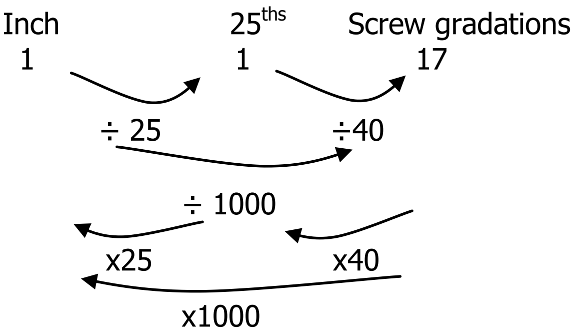
Gradations	Ones	Tenths	Hundredths	Thousandths	Number
23			2	3	0.023
37			3	7	0.037

Put the following on chart:

(a) 7gradations,

(b) 16 gradations

(3) Relook at what the micrometer does: for example, 1 – 11 – 17 does the following



Thus, in terms of the traditional chart the 3 parts of 1 – 1 – 17 relate as follows

Chart

Screw measurement	Ones	Tenths	Hundredths	Thousandths	Number
1 inch	1				1.000
11/25 inch		4	4		0.440
17 gradations			1	7	0.017
					1.457

It gives 1.457 inches

Thus, to work out what an imperial micrometer screw gauge says, add the inches to the stem number x .04 to the gradations x .001. That is 2 – 16 – 31 is 2 + 16 x .04 + 31 x .001 = 2 + .64 + .031 = 2.671

(4) What is the value of the following:

<u>Screw measurement</u> | <u>Chart</u> | <u>Calculation</u>

(a) 1 – 7 – 28

O	T	H	Th
1			
	2	8	
		2	8
1	3	0	8

1 + 7 x .04 + 28 x .001

= 1 + 0.28 + 0.028

= 1.308

(b) 0 – 14 – 7

O	T	H	Th

(c) 2 – 24 – 38 -

O	T	H	Th

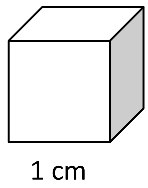
(d) 1 – 21 – 16

O	T	H	Th

4. PERIMETER, AREA, AND VOLUME FORMULAE

Small engines are described in terms of cubic centimetres of their cylinders, although length measurement is in terms of millimetres. We use the relationship 1000 mm³ = 1 cm³ which comes from the volume formulae: $V = l \times w \times h$ where l is the length, w is the width, and h is the height.

Consider the cube at right with side lengths of 1 cm, which is equivalent to 10 mm. Then, in both units, the volume is:



$V = l \times w \times h$	$V = l \times w \times h$
V = 1 cm × 1 cm × 1 cm	V = 10 mm × 10 mm × 10 mm
V = 1 cm³	V = 1000 mm³

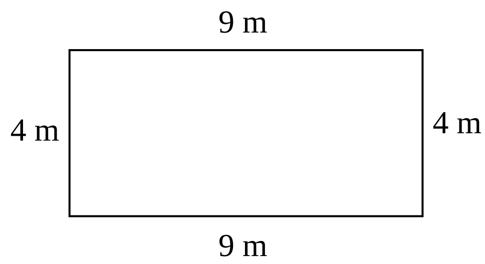
Hence, 1000 mm³ = 1 cm³.

Of course, cylinders are not rectangular prisms. The formula for the volume of a cylinder is $\pi r^2 h$ where r is the radius of the circular base and h is the height of the cylinder. However, in order to understand this formula, a knowledge of prior formula in perimeter (circumference), area, and volume of rectangles and circles is required.

4.1 Perimeter

The **perimeter** of a shape is the total distance around it. In other words, the perimeter is the distance over which a fence or wall would need to be erected to fully enclose the shape. For most shapes, calculating the perimeter is easy, because the only operation involved is addition.

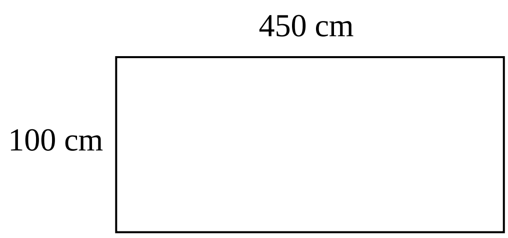
For example, have a look at the rectangle at right.



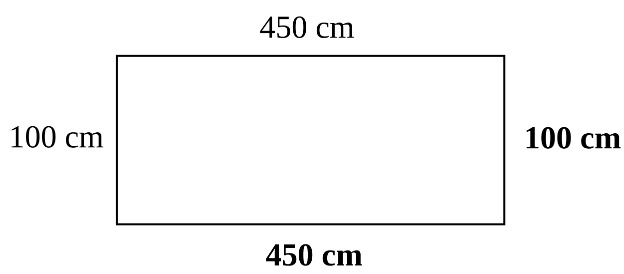
The total distance around the rectangle is simply the result of adding all of the sides together:

9 m + 4 m + 9 m + 4 m = <u>26 m</u>.

Remember that a rectangle will always have opposite sides that are of the same length. So, if given a rectangle with the lengths of only two sides marked, we automatically know the lengths of the opposite sides.



For example, have a look at the rectangle at right. We know what the lengths of the opposite sides are: the side length opposite the 450 cm must be 450 cm, and the side length opposite the 100 cm must be 100 cm.



So, we can add these lengths to the drawing so that we do not forget what they are.

Now, the total distance of this rectangle is simply:

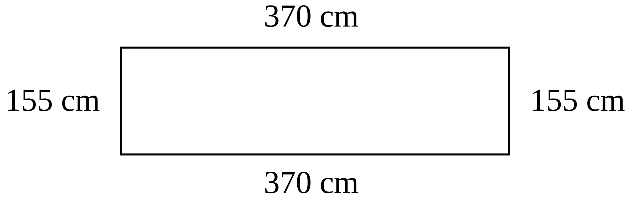
450 cm + 100 cm + 450 cm + 100 cm = <u>1100 cm</u>.

Note that, because you can add numbers in whatever order you like, you can start adding from any side of the rectangle that you choose.

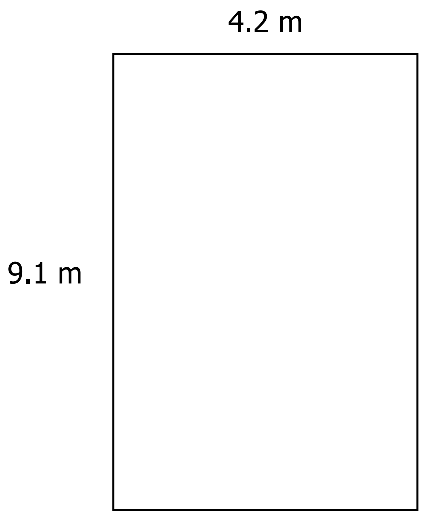
Activity 4.1a

Find the perimeter of each of the following rectangles. Make sure that all of your answers include the correct units of length (e.g., m for metres, cm for centimetres).

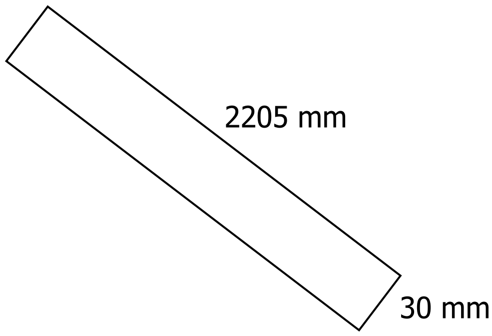
(1)



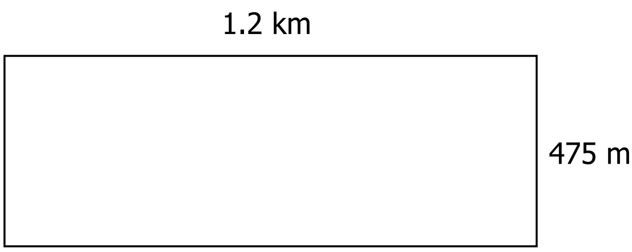
(2)



(3)



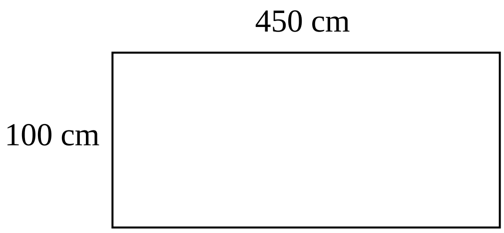
(4)



Remember that, in order to be able to add quantities together, they **must** be in the same units.

Because the lengths of opposite sides of a rectangle are always the same, we can find a quicker way to calculate the perimeter of rectangles.

Have another look at the rectangle at right.



Recall that the solution to finding the perimeter of this rectangle was:

450 cm + 100 cm + 450 cm + 100 cm = <u>1100 cm</u>.

If we rearrange the order in which we add the side lengths of the rectangle, we get:

450 cm + 450 cm + 100 cm + 100 cm = <u>1100 cm</u>.

Remember that we can add numbers in whatever order we like and we get the same answer.

Now, if we group our addition like this,

(450 cm + 450 cm) + (100 cm + 100 cm)

= 900 cm + 200 cm,

= 1100 cm

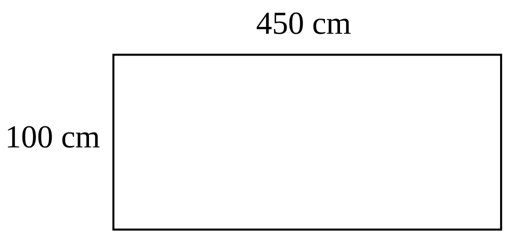
then we can see that the result of adding each group is the same as doubling each length and then adding them together.

So, we have now found another way to calculate the perimeter, P, of rectangles:

$P = 2 \times a + 2 \times b$

where a and b are the lengths of the rectangle.

Back to this rectangle again. If we use the formula, then we see that a = 450 cm and b = 100 cm (or a = 100 cm and b = 450 cm, because the order of addition does not matter). So,



$P = 2 \times a + 2 \times b$

$P = 2 \times 450 + 2 \times 100$

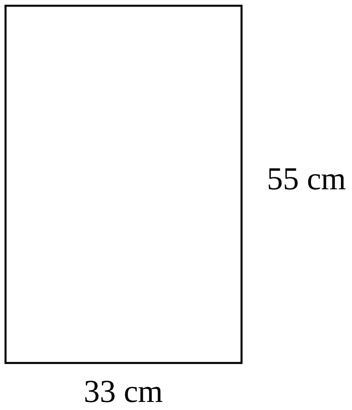
$P = 900 + 200$

P = 1100 cm.

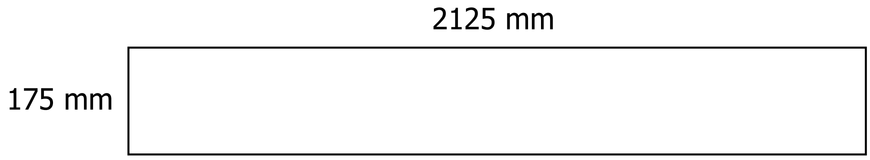
Activity 4.1b

Use the new perimeter formula to find the perimeter of each of the following rectangles. Remember to show all of your working, as in the example on the previous page.

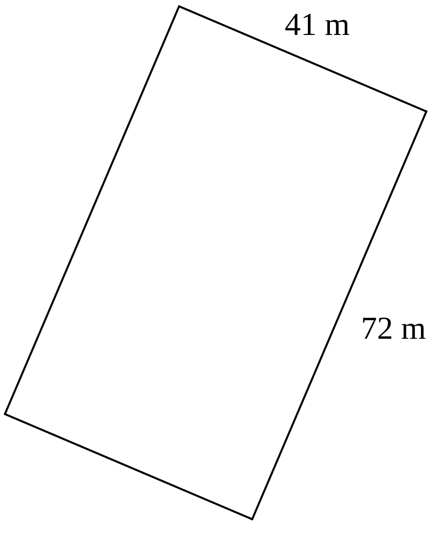
(1)



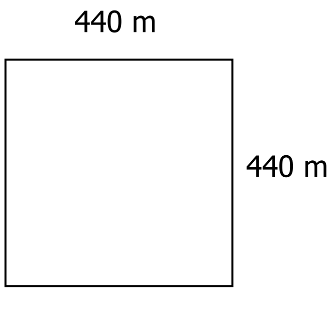
(2)



(3)



(4)



The perimeter of a circle has a special name, the **circumference**. The circumference of a circle is still the distance around the circle or the distance over which a fence would need to be erected to fully enclose the circle. The reason why a special name is used for a circle is because calculating the circumference of a circle requires a special formula. Let's find this formula.

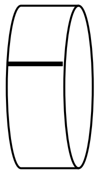
Activity 4.1c

To find the formula for calculating the circumference of a circle, follow the procedure below:

(1) Find a sheet of paper, three different-sized lids, a ruler, and a pen. Use a ruler to draw a straight line across a sheet of paper.

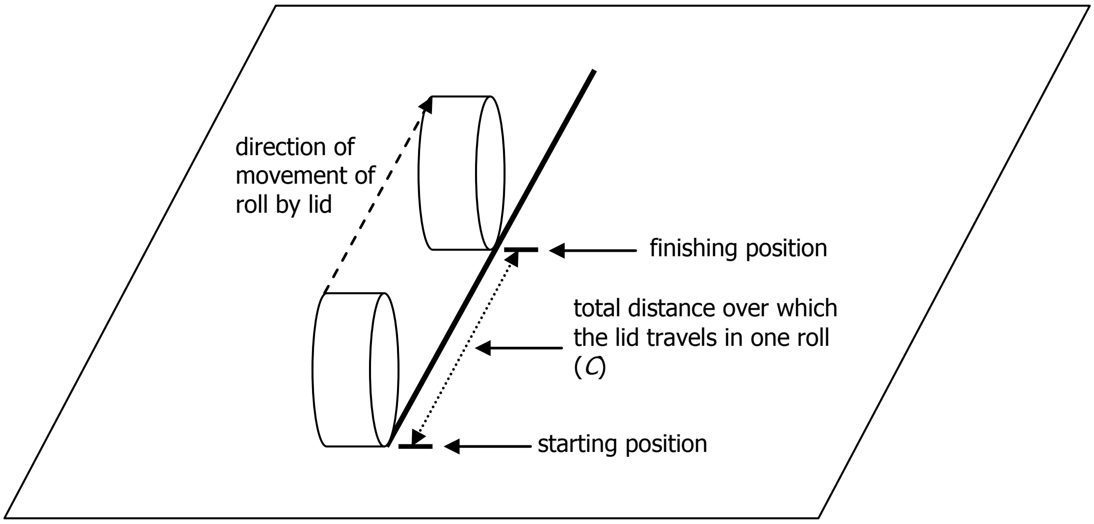
(2) Label each lid small, medium, and large, according to their relative sizes.

(3) Select one lid, and mark a line on it in one place on its side (as shown in the
figure at right). Turn the lid on its side, and position the lid at one end of the line you drew on the paper so that the line you drew on the lid touches the paper. Roll the lid one time along the paper. Mark the position where it stopped on the paper.



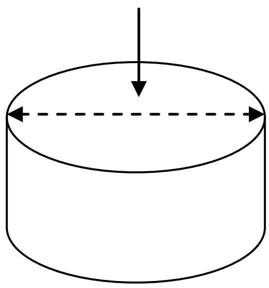
(4) Measure this distance, and call it C (see figure below). Make sure you label the lid for which C is calculated.

(5) Repeat Steps 3–8 for each of the remaining lids.



(6) Select one lid, and place it on its top or bottom.

(7) Measure the distance directly across the centre of the lid from edge to edge (as shown in the figure at right), and call it *d*. Make sure you label the lid for which *d* is calculated.



(8) Repeat Steps (6)–(7) for each of the remaining lids.

(9) Write the data you collected in Steps (4) and (7) the table below (remember to include the units of length that you are using) and then divide *C* by *d* for the third column:

Lid	*C*	*d*	*C* ÷ *d*
Small			
Medium			
Large			

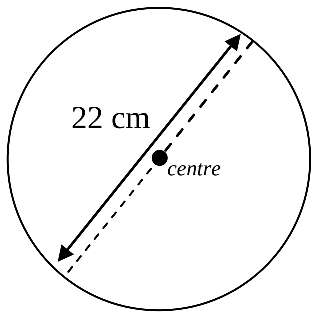
(10) What did you get in the third column for each of the lids?

You should have observed that the value for $C \div d$ is equal to 3-and-a-bit. This value represents the ratio of the circumference (C) of a circle to the **diameter** (d) of the circle. A better approximation of this ratio is known to be 3.14 and is more formally referred to as **pi** and is symbolised using the Greek letter π.

So, we have developed the formula $C \div d = \pi$. Rearranging the formula, we obtain $C = \pi \times d$. Hence, the formula for calculating the circumference of a circle is $C = \pi \times d$. That is, to find the circumference of a circle, we multiply the diameter of the circle by π.

By the way, your calculator may have a π button. If so, then you can use this button to calculate the circumference of a circle. If not, then you can just use 3.14 in place of π.

Have a look at the circle at right.



The diameter *(d)* is 22 cm. To find the circumference of the circle, we use the formula:

$C = \pi \times d$

$C = \pi \times 22$ cm

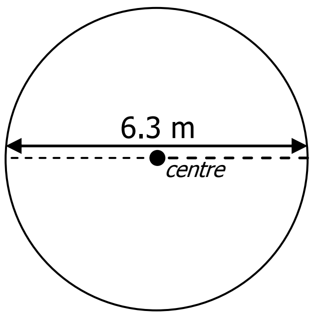
$C = \underline{69.1 \text{ cm}}$

So, the circumference of the circle is 69.1 cm.

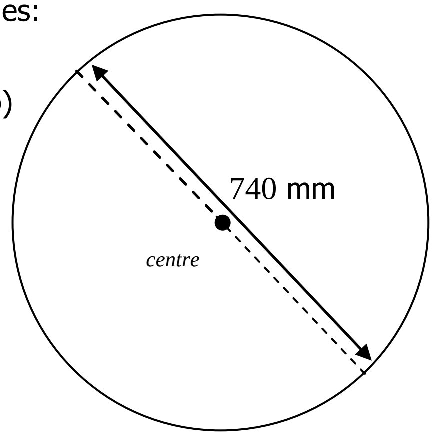
Activity 4.1d

Find the circumference of each of the following circles:

(a)

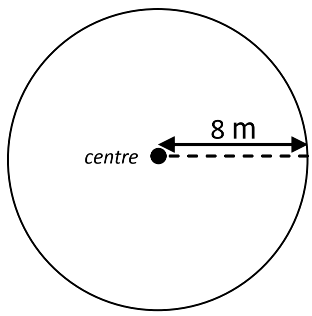


(b)



The **radius** (r) of a circle is the distance from the centre of the circle to any point on the edge of the circle. Because the diameter is the distance across the circle, the radius is equal to half the diameter. That is, $r = d \div 2$ which implies that $d = 2 \times r$. If we substitute $2 \times r$ for d in the formula for finding the circumference of a circle, we obtain $C = \pi \times (2 \times r)$. Because we can multiply numbers in any order we like, let us write the formula is this way $C = 2 \times \pi \times r$.

Have a look at the circle at right.



The radius is 8 m. To find circumference of a circle, we use:

$C = 2 \times \pi \times r$

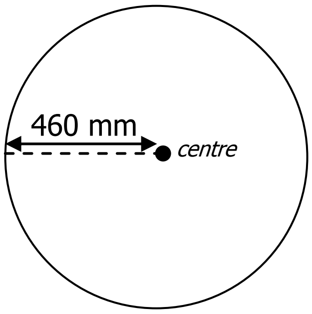
$C = 2 \times \pi \times 8$ m

$C = \underline{50.3 \text{ m}}$ So, the circumference of the circle is 50.3 m.

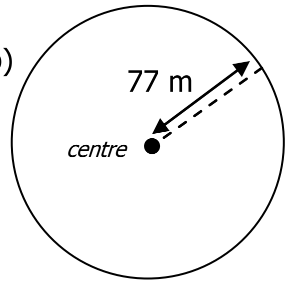
Activity 4.1e

Find the circumference of each of the following circles:

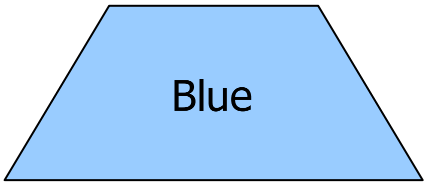
(a)



(b)



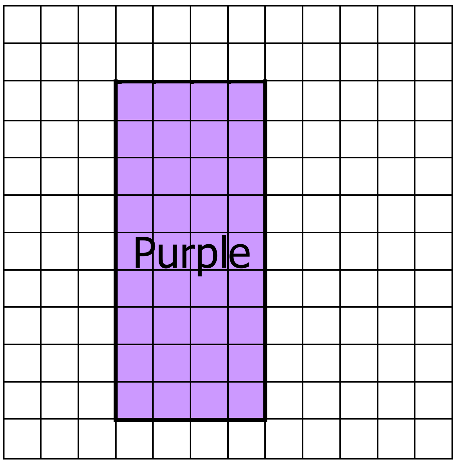
4.2 Area



The **area** of a two-dimensional shape is a measure of the size of the surface contained inside the shape. For the shape at right, the area is the blue shaded part.

Finding the area of a shape depends on the shape itself. Different shapes have different formulae for finding the area.

Have a look at the rectangle on the grid at right.

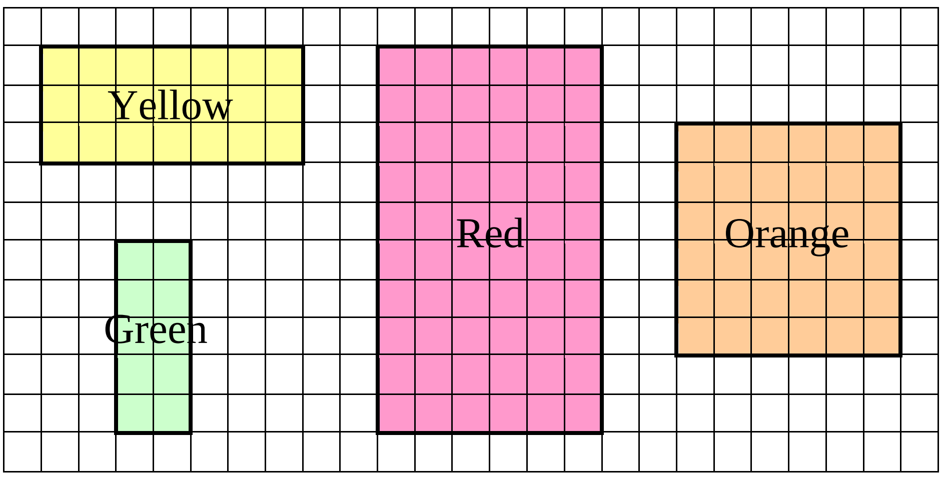


The area is the shaded purple region. The shaded purple region contains a total of 36 squares. Therefore, the area of the shaded purple region is equal to 36 squares.

Note that the rectangle is 4 squares wide and 9 squares long. Is there an operation that can be used to get 36 from 4 and 9?

Activity 4.2a

(1) Find the area of each of the following rectangles by counting the number of squares inside it.



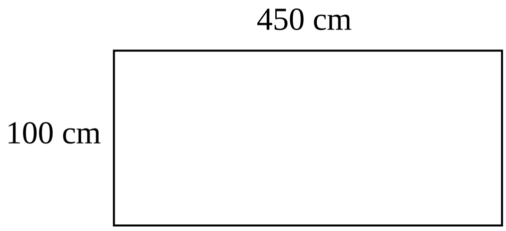
(2) Complete the table below with the length (*a*), width (*b*), and area (*A*) of each of the rectangles in Question 1.

Rectangle	Length (*a*)	Width (*b*)	Area (*A*)
Yellow			
Green			
Red			
Orange			

(3) What is the relationship among length (*a*), width (*b*), and Area (*A*) for each rectangle?

From the previous activity, you should have realised that, instead of counting the squares inside a rectangle, we can find the rectangle's area (*A*) by multiplying its length (*a*) by its width (*b*). Hence,

$A = a \times b$.



Have a look at the rectangle at right. We can see that a = 100 cm and b = 450 cm (or a = 450 cm and b = 100 cm, because the order of multiplication does not matter). So,

$A = a \times b$

A = 100 cm × 450 cm

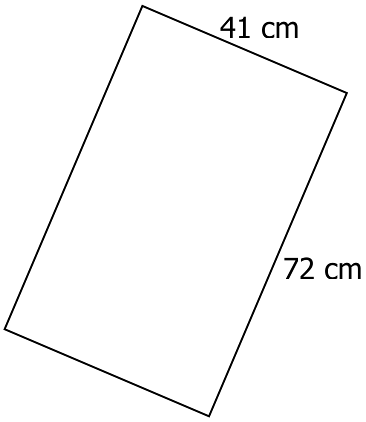
A = <u>450 000 cm²</u>.

That means that the area of the rectangle is 450 000 cm². But, what does the ² mean in cm²? It means "squared." That is, cm² stands for "centimetres squared" or "square centimetres". Because area is a measure of the dimensions of two sides, its units must be squared. So, when finding the area of a shape, remember to put in the squared symbol ² (e.g. mm², cm², m², km²).

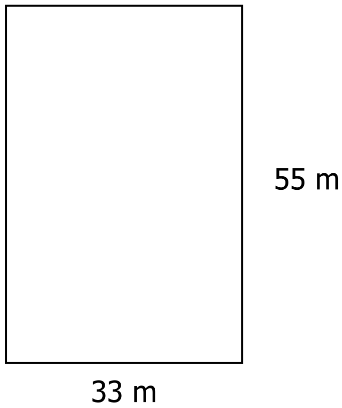
Activity 4.2b

Find the area of each of the following rectangles by using the formula $A = a \times b$. Remember that your answer will be in square units (e.g. mm², cm², m², km²).

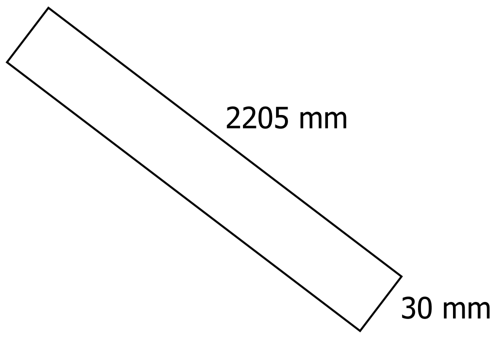
(1)



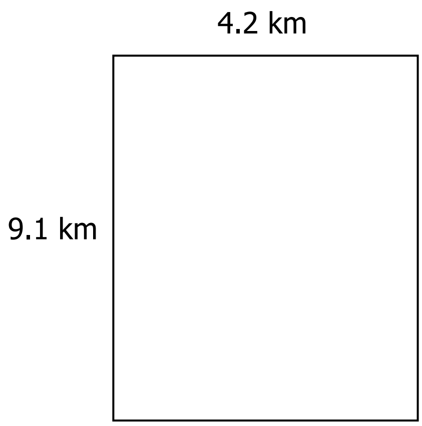
(2)



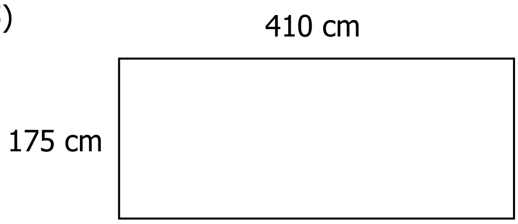
(3)



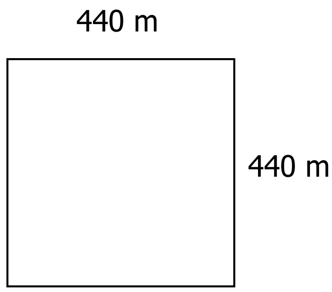
(4)



(5)



(6)



Now, we will investigate how to find the area of a circle.

Activity 4.2c

(1) Find a piece of paper, a large lid (at least 12 cm in diameter), a ruler, and scissors.

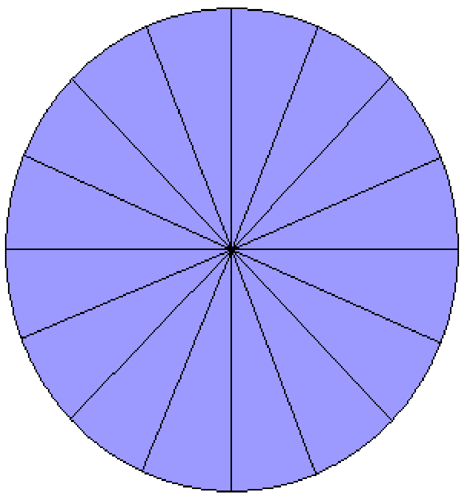
(2) Measure the diameter of the lid, and calculate the radius.

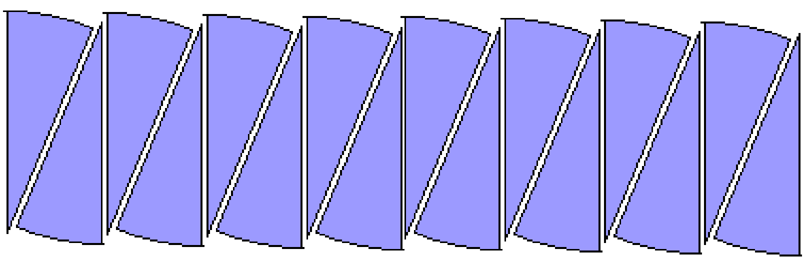
(3) Place the lid firmly on the piece of the paper, and draw a circle around it.

(4) Draw lines across the circle to form 16 sectors that are approximately equal in area, as shown in the figure at right.

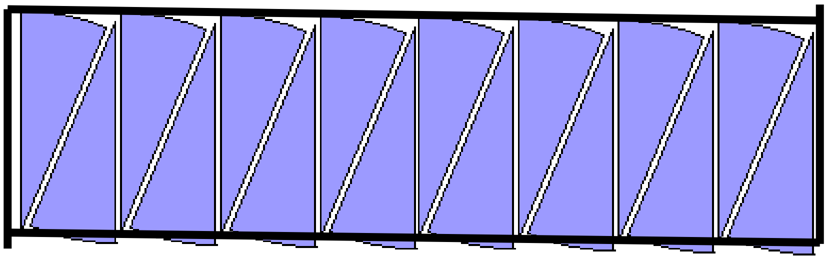
(5) Cut out the 16 sectors, as shown at right.

(6) Arrange the sectors in the configuration shown below.





(7) Note that this configuration looks very much like a rectangle.

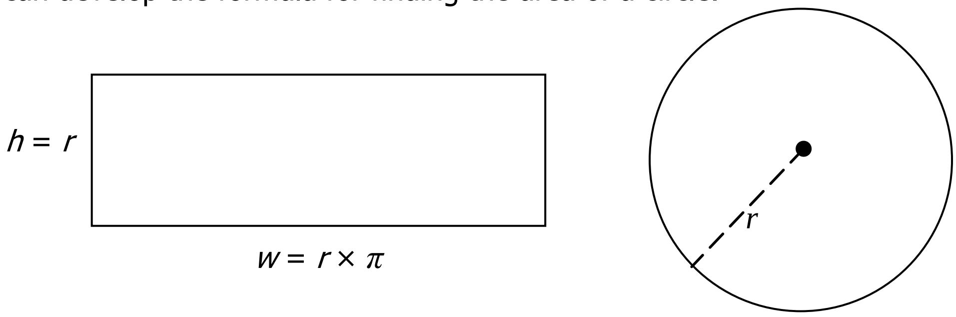


(8) Measure the height and width of the rectangle. Calculate the area of the rectangle.

(9) You should have recognised that the height of the rectangle is the same as the radius of the lid.

(10) Divide the width of the rectangle by the radius. Did you get an answer that is close to π? Remember that π is approximately equal to 3.14.

Since the height (h) of the rectangle is equal to the radius (r) of the circle and the width (w) of the rectangle is (approximately) equal to the radius (r) of the circle times π (see figures below), we can develop the formula for finding the area of a circle.



To calculate the area of the rectangle, you would have used $A = h \times w$. Since $h = r$ and $w = r \times \pi$, we can substitute

$A = h \times w$

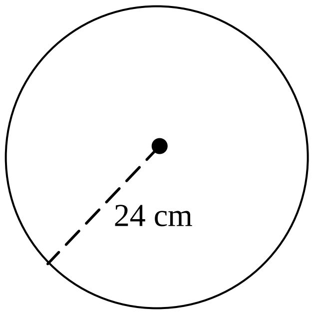
$A = r \times r \times \pi$

$A = r^2 \times \pi$.

Remember that $r^2 = r \times r$.

So, we have developed the formula for finding the area of circle, $A = r^2 \times \pi$ where r is the radius of the circle. Given the circle's radius, all we have to do is multiply the radius by itself and then multiply the result by π.

Have a look at the circle at right. We can see that $r = 24$ cm. So, to calculate the area of the circle, we have



$A = r^2 \times \pi$

$A = (24\text{ cm})^2 \times \pi = 24\text{ cm} \times 24\text{ cm} \times \pi$

$A = \underline{1809.6\text{ cm}^2}$

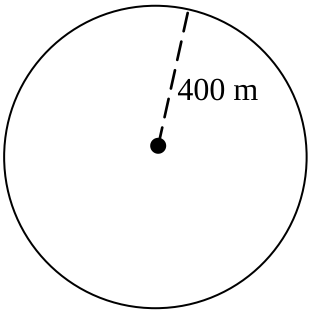
That is, the area of the circle is approximately equal to 1809.6 cm^2.

Remember that, if given the diameter of a circle instead, you must divide it by 2 to find the radius before using the formula for calculating the area of a circle.

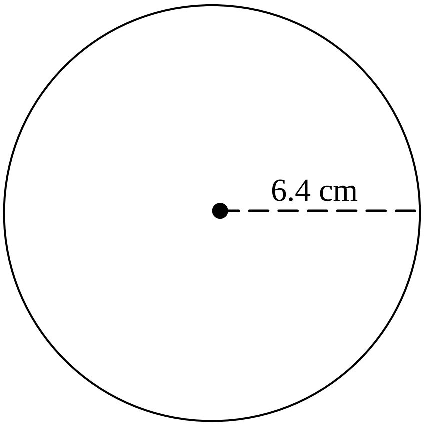
Activity 4.2d

Find the area of each of the following circles.

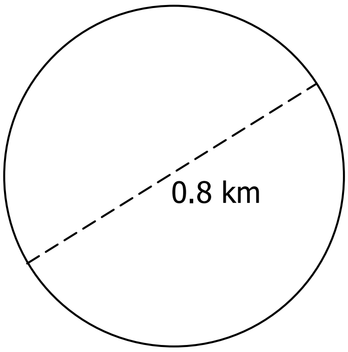
(1)



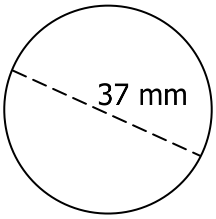
(2)



(3)



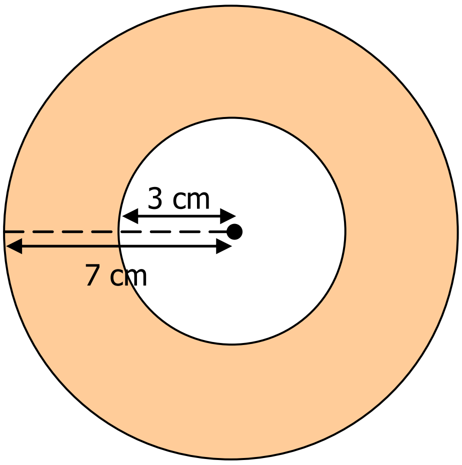
(4)



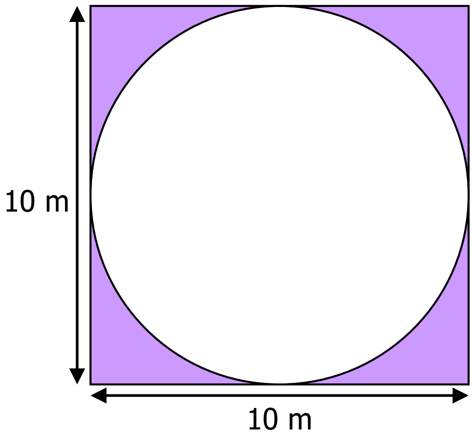
Activity 4.2e

Find the area of the shaded region in each of the following figures.

(1)



(2)

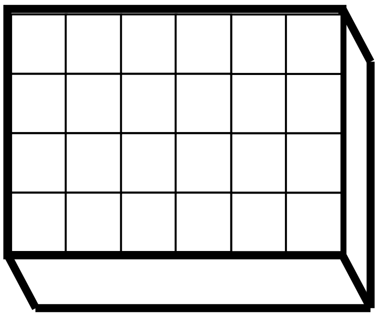


4.3 Volume

A **rectangular prism** is the formal name for a rectangular box.

A rectangular prism looks like the figure at right when shown in two dimensions.

The volume of the rectangular prism is the amount of space enclosed inside it.



Have a look at the block at right.

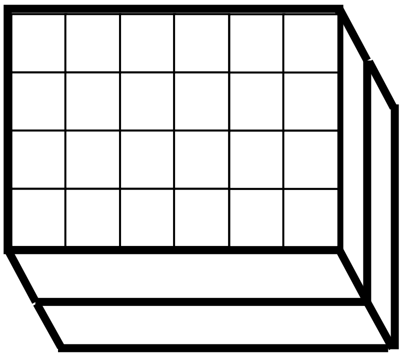
Note that the face of the block is 6 units wide and 4 units long.

Therefore, the area of the face of the block is 6 units × 4 units = 24 square units.

Note that the block is 1 unit height.

Therefore, the volume of the block is 24 cubic units.

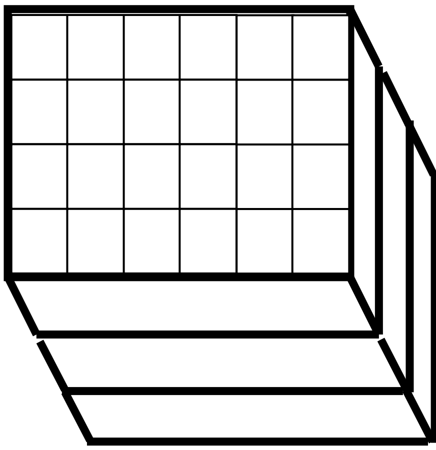
That is, a measure of the amount of space inside the block is 24 cubic units.



If we put another block of the same dimensions against the first one, as shown at right, then you will see that the prism now has a height of 2 units.

The volume, therefore, is twice that of the first block.

So, the volume of this prism is 48 cubic units.



If we add a third block to the prism from before, then the prism will now be 3 units high (see the figure below right).

Therefore, the volume of the prism will be 24 × 3 = 72 cubic units.

Hence, the volume (*V*) of a rectangular prism is equal to the area (*A*) of the face of the prism times the height (*c*) of the prism.

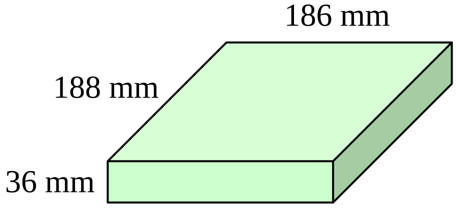
Since the area (A) of the face is simply length (*a*) times width (*b*), we have the formula for calculating the volume of a rectangular prism:

$V = a \times b \times c$.

Because we are multiplying three quantities together, it makes sense that units of volume must be cubic (e.g. mm^3, cm^3, m^3, km^3).

Have a look at the rectangular prism at right.

We see that the a = 186 mm, b = 188 mm, and c = 36 mm. (Remember that, because we can multiply numbers in any order and get the same result, we can assign a, b, and c to any of the quantities and obtain the same result.) The volume of the rectangular prism is



$V = a \times b \times c$

V = 186 mm × 188 mm × 36 mm

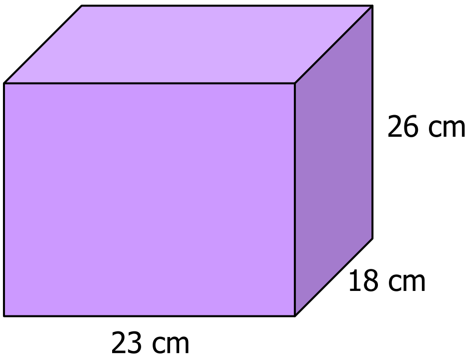
V = <u>1 258 848 mm³</u>.

So, the volume of the rectangular prism is 1 258 848 mm³, which is the measure of how much space is enclosed inside the rectangular prism.

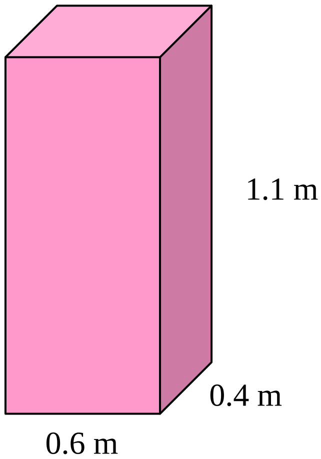
Activity 4.3a

Find the volume of each of the following rectangular prisms.

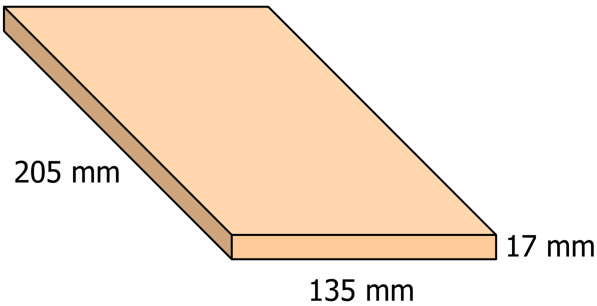
(1)



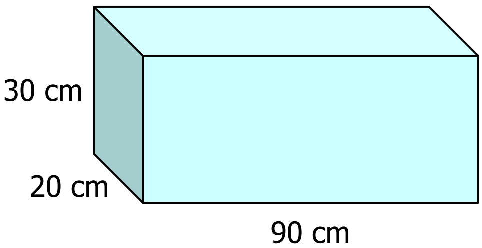
(2)



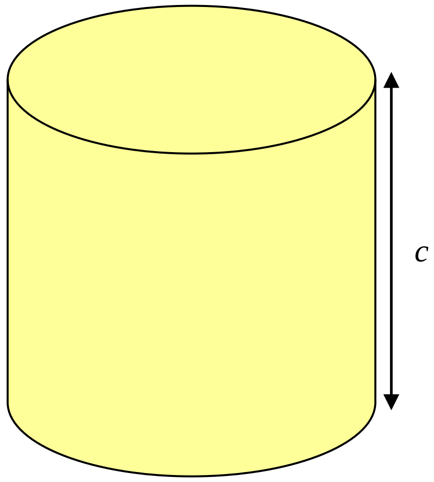
(3)



(4)



A **cylinder** is just like a prism, except that the shape on the base is a circle, as shown in the figure at right.



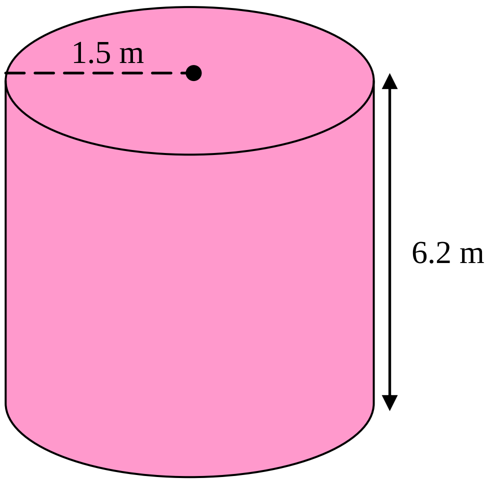
You should have recognised that finding the volume of a cylinder is the same as finding the volume of a prism. First, you find the area of the face. Then, you multiply the area by the length of the cylinder.

Since we have already learned that the area of a circle is $A = r^2 \times \pi$ where r is the radius of the circle, we can derive the formula for finding the volume of a cylinder:

$$V = A \times c$$

$$V = r^2 \times \pi \times c.$$

Have a look at the cylinder at right. We can see that the radius (r) is 1.5 m and the length (c) is 6.2 m. The volume of the cylinder is



$$V = r^2 \times \pi \times c$$

$$V = (1.5\text{ m})^2 \times \pi \times 6.2\text{ m}$$

$$V = 1.5\text{ m} \times 1.5\text{ m} \times \pi \times 6.2\text{ m}$$

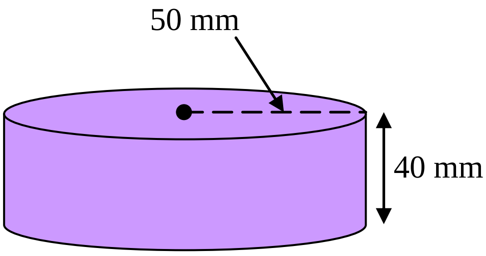
$$V = \underline{43.8\text{ m}^3}.$$

So, the volume of the cylinder is 43.8 m³.

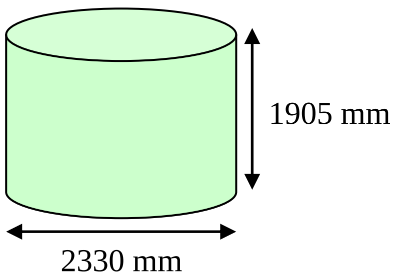
Activity 4.3b

Find the volume of each of the following cylinders.

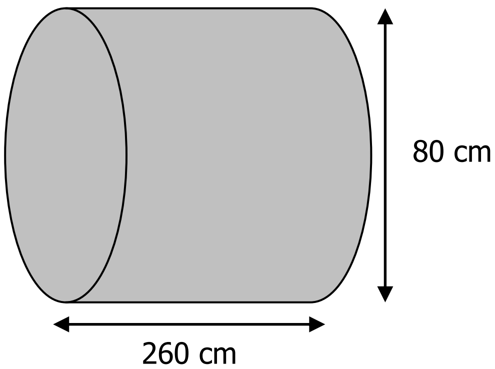
(1)



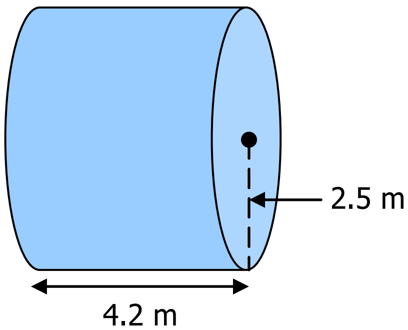
(2)



(3)



(4)

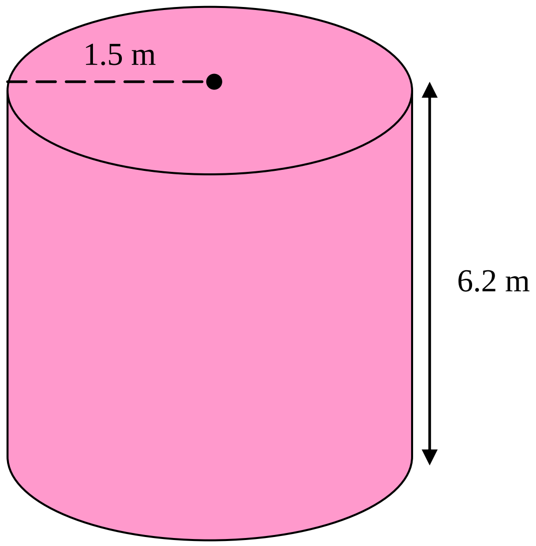


As we learned before, volume is the amount of space inside a prism or cylinder. Volume can also be viewed as the maximum amount of solid material that can be used to fill up the prism or cylinder. On the other hand, when we are interested in finding the amount of liquid that can fill up a prism or cylinder, we need to find the capacity of the prism or cylinder. **Capacity** is the volume of liquid.

Finding the capacity of a prism or cylinder is just like finding the volume. However, once we have the volume, we must convert it to capacity, which is usually measured in litres (L).

The conversion is 1 m^3 = 1000 L.

Have another look at the cylinder at right from before.



We already know that the volume of this cylinder is 43.8 m^3. If we need to calculate the capacity (C) of this cylinder, then all we have to do is multiply the volume (V) by 1000 L:

$C = V \times 1000$

$C = 43.8\ m^3 \times 1000$

$C = \underline{43\ 800\ L}$.

So, the capacity of the cylinder is 43 800 L. That is, a maximum of 43 800 L of liquid can fill up the cylinder.

The conversion factor of 1000 works only when you have the volume in m^3.

If you have the volume in units other than m^3, then the conversion factor to capacity is different. The formula below shows the conversion factors for different units of volume:

1 km^3 = 1 000 000 000 000 L

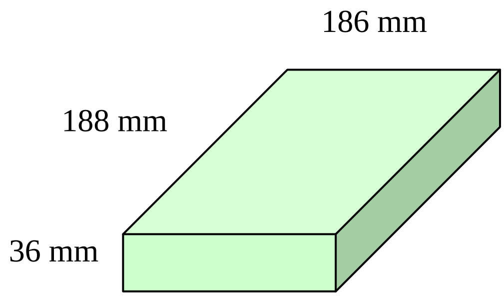
1 m^3 = 1000 L

1 cm^3 = 0.001 L = 1 mL

1 mm^3 = 0.000 001 L = 0.001 mL

We always want to make sure that we use the correct conversion factor given the units of volume that we have.

Have another look at the rectangular prism at right from before.



We already know that its volume is 1 258 848 mm³. If we need to calculate the capacity (C) of the rectangular prism, then all we need to do is multiply the volume (V) by 0.000 001:

$C = V \times 0.000\,001$

$C = 1\,258\,848 \text{ mm}^3 \times 0.000\,001$

$C = \underline{1.26 \text{ L}}$.

Alternatively, we could have multiplied the volume (V) by 0.001 in order to obtain the capacity in mL. In this case,

$C = V \times 0.001$

$C = 1\,258\,848 \text{ mm}^3 \times 0.001$

$C = \underline{1258.8 \text{ mL}}$.

So, the capacity of the rectangular prism is 1.26 L or 1258.8 mL.

Activity 4.3c

Find the capacity of each of the prisms and cylinders (in either mL or L) from Activities 4.3a and 4.3b.