YUMI DEADLY CENTRE
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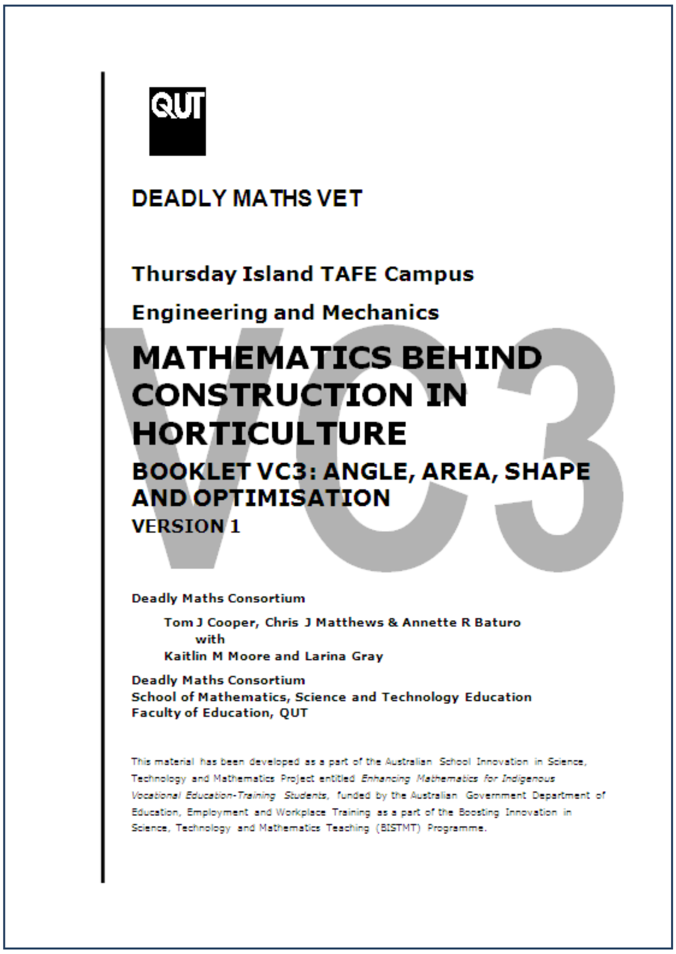
Construction and Building

Mathematics behind Construction in Horticulture

Booklet VC3: Angle, Area, Shape and Optimisation



DEADLY MATHS VET

Thursday Island TAFE Campus

Engineering and Mechanics

MATHEMATICS BEHIND CONSTRUCTION IN HORTICULTURE

BOOKLET VC3: ANGLE, AREA, SHAPE AND OPTIMISATION

VERSION 1

Deadly Maths Consortium

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with
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This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology

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BOOKLET VC3

ANGLE, AREA, SHAPE AND OPTIMISATION

08/05/09

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THIS BOOKLET

This booklet VC3 was the third booklet produced as material to support Indigenous students completing courses or certificates associated with construction at the Northern Peninsula Area College, Bamaga. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

VB1: Mathematics behind whole-number place value and operations
Booklet 1: Using bundling sticks, MAB and money

VB2: Mathematics behind whole-number numeration and operations
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

VC1: Mathematics behind dome constructions using Earthbags
Booklet 1: Circles, area, volume and domes

VC2: Mathematics behind dome constructions using Earthbags
Booklet 2: Rate, ratio, speed and mixes

VC3: Mathematics behind construction in Horticulture
Booklet 3: Angle, area, shape and optimisation

VE1: Mathematics behind small engine repair and maintenance
Booklet 1: Number systems, metric and Imperial units, and formulae

VE2: Mathematics behind small engine repair and maintenance
Booklet 2: Rate, ratio, time, fuel, gearing and compression

VE3: Mathematics behind metal fabrication
Booklet 3: Division, angle, shape, formulae and optimisation

VM1: Mathematics behind handling small boats/ships
Booklet 1: Angle, distance, direction and navigation

VM2: Mathematics behind handling small boats/ships
Booklet 2: Rate, ratio, speed, fuel and tides

VM3: Mathematics behind modelling marine environments
Booklet 3: Percentage, coverage and box models

VR1: Mathematics behind handling money
Booklet 1: Whole-number and decimal numeration, operations and computation

CONTENTS

1. INTRODUCING MEASUREMENT

The intention of this booklet is to explore the mathematics behind, and reinforce key concepts to assist VET teachers to teach, constructions behind horticulture.

The particular constructions are

(1) Outdoor furniture;

(2) Saw horse;

(3) Shade house

(4) Creek with cement and gravel lining; and

(5) Wall dividers

1.1 Mathematics

To meet the mathematics behind these constructions, the booklet will focus on

(1) Angle – particularly that associated with levelling the base of angled pieces of wood in outdoor-furniture and saw horses.

(2) Shape – particularly with respect to the shade house.

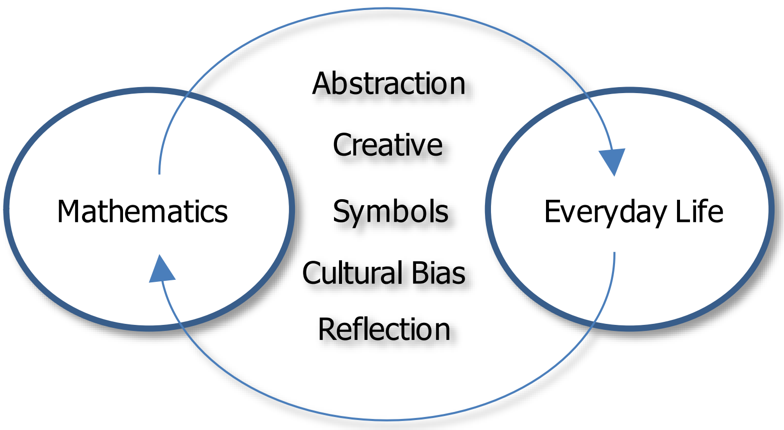
(3) Area – particularly in terms of gravel needed to cover the cemented base of the creek.

(4) Optimization – particularly in terms of efficiency in using regular shapes to cover an area.

However, before we move on to looking at the particular aspects of angle, shape, area and optimization that are part of what has to be covered in this booklet; we will spend a little time on the mathematical basis of measurement in general.

1.2 Measurement and MAST

This introduction is based on the mathematics as Story Telling (MAST) approach to mathematics learning developed by Dr Chris Matthews. This approach confronts the nature of mathematics which, as the following figure shows, is an abstraction of everyday life as invented by humans to solve the problems of that life. As the figure also illustrates, this abstraction is a creative act which generalises everyday life to symbols. However, it is also a generalisation that is affected by the cultural bias of the abstractor and therefore its learning has to be seen as an enculturation.



The MAST approach encourages students to invent their own symbols to reveal the creativity of the act and to comprehend the cultural bias in much of mathematics. It focuses on symbols, units and so on, as ways to tell stories about everyday life.

Thus, the MAST approach for measurement focuses on the students understanding of an attribute like length as an entity without numbers and encourages them to invent their own units before they are introduced to metrics.

1.3 Creating your own measurement

The initial MAST activities are to enable the student to develop their own measurement unit. Unit is the basis of measurement as it allows the continuous attribute of length to be broken up into smaller pieces to be counted. Below is an example of a lesson that attempts to do this.

(1) **Objectives:**

- Help students understanding measurement by other means.
- Help students understand, "a bit".

(2) **Materials:**

- Objects to be measured e.g. length of wood
- Objects to measured with e.g. feather

(3) **Language:**

- Measure, length, units, how many

(4) **Teacher Activity:**

- Challenge students to measure the length of an object (for example, a length of wood, table etc.)
- Give each student an object of varying sizes and tell the students that they must decide on how they are going to measure the object. Ask,l "What you use to measure the object".
- Demonstrate by suggesting the students measure this "length of wood" using a "feather". Ask, "How many feathers is this length of wood?"
- Have a set of "measuring" objects (e.g. feather) that can be used to measure out a particular object.

(5) **Reinforce the following points:**

- You can use anything, any object to measure another object.
- The object we use to measure is usually smaller than the object being measured.
- The measuring objects forms our measuring "units".
- There is a need to create a standard measuring system.

Student Activity

- Measure the same object using a particular "measuring object". 

- A written down length, e.g. 2 and "a bit" feathers or 2 and "a bit"..........
- Students should guess how much of the feather is "a bit" e.g. ¾, ½, 80% etc in any form the student is comfortable.
- If a student has difficulty with this, let them use language to express how much i.e. more than a half, less than a quarter, almost all of the feather.

1.4 Working out the "a bit"

This section looks at extending the ideas in Section 1.3. it does this by relating measures of anything to something familiar to the student, then using this to get across the metric nature of systems of units (they relate to each other by x 10 or ÷10). Below is an example of a lesson that attempts to do this.

(1) **Objectives:**

- Help students understanding measurement by other means.
- Help students understand, "a bit".
- Help students understand fractions.

(2) **Materials:**

- Objects to be measured,
- Objects to measure with

(3) **Language:**

- Measure, length, units, how many.

Teacher Activity:

- Recap last lesson and tell the students that we will work on measuring the "a bit" part. The teacher reinforces the need for accuracy when designing, planning and constructing a piece of work.
- Provide students with different lengths of wood and tell the students that we are going to measure the lengths of wood using any of these objects.
- Provide a few sets of cardboard lengths to measure the student's pieces of wood. These need to be three cardboard lengths in each set: a long piece, a middle size4 pieced and a small piece. Each piece needs to vbe an exact fraction of the larger piece e.g. the middle piece can be 1/10 of the largest piece and the small piece can be 1/10 of the middle piece.
- Reinforce the notion of fractions.
- Teacher in discussion with the students. Create a name for each piece of cardboard e.g. whale, dugong, fish.
- Get the students to measure their piece of wood with the larger piece of cardboard (whale) and write down it's measurement.
- E.g. Wood = 2 whales and "a bit" whale

  Wood = 2 whales + "a bit" whale

- Instruct/demonstrate students to draw a line on their piece of wood after 2 whales to isolate the "a bit" part and measure the "a bit" part with dugong (middle sized cardboard).
- Set the challenge that we want to know how many "fish" the piece of wood is without measuring the wood physically.
- Reinforce the relationship between whales, dugongs and fish starting with dugong i.e. there are ten fish to 1 dugong. What would 4 dugongs be?
- Reinforce the relationship between whales, dugong. First with 10 dugong to 1 whale and then using previous relationship between dugongs and fish. Demonstrate that 100 fish is the same as 1 whale.

Demonstrate that:

- Wood = 200 fish + 40 fish + 1 fish
- Wood = 241 fish
- Go through the above procedure writing the length of the wood in terms of dugong and also whales to get:
- Wood = 24.1 dugong and
- Wood = 2.41 whales
- Note: This exercise needs to be combined with lesson on the number system and place value.
- Introduce the idea of standardise measurements and repeat the above exercise using mm, cm and m. Reinforce the relationship of 1 cm = 10 mm, 1 m = 100 cm
- Work out 4 dugong in terms of fish.
- Wood = 2 whales + 40 fish + 1 fish
- Work out what 2 whales are in terms of fish i.e. 2 whales is 200 fish.

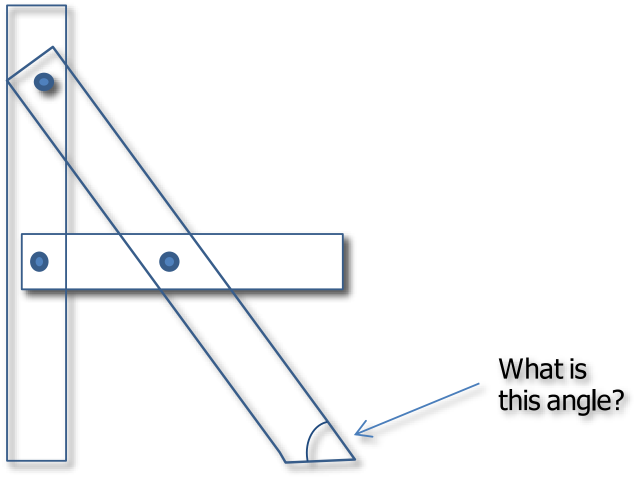
Student activity

- Students write down the measurement for the "a bit" part. "a bit" = 4 dugong + "a bit" more dugong.
- Students write down the full measurement of the piece of wood.
- Wood = 2 whales + 4 dugongs + "a bit" more dugong.
- Repeat the process with the "fish" measurement to get:

wood = 2 whales + 4 dugong + 1 fish.

2. ANGLE

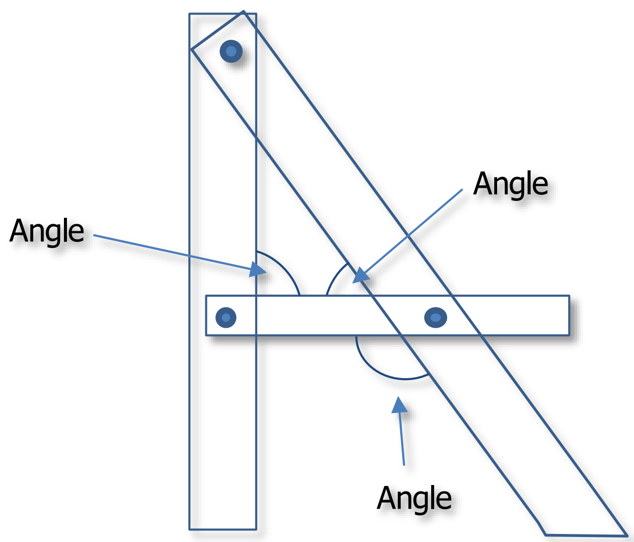
When constructing things like outdoor furniture, one of the major mathematical problems (along with measuring) is to know what angle to cut the wood so that the furniture sits flat on the ground as paving. For example:



2.1 What are angles?

Angles are the amount of turn between two directions e.g.,

In constructions, they are represented when two components meet (usually as 2 straight lines), e.g.



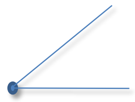
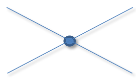
The following is a possible lesson on angle.

(1) **Objectives:**

- To understand angle
- To understand special angles
- To understand angle, angle properties of shapes

(2) **Materials:**

- Wood or cardboard hinged at one end and in the middle (call them:

A  B 

(Long thin sticks, or pieces of wood or cardboard)

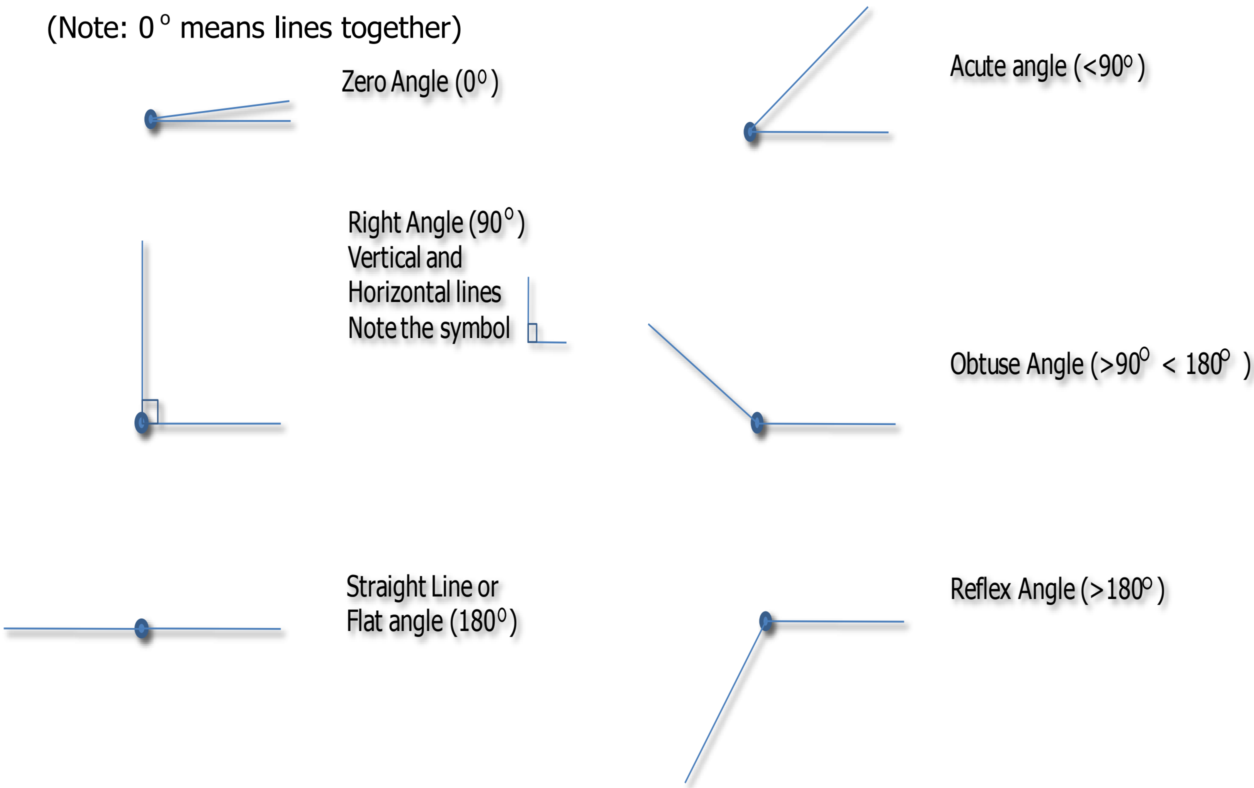
(3) **Language:**

- Measurement, angle, size of angle (bigger/smaller), right angle, acute, obtuse, zero angle, straight line angle, reflex angle.

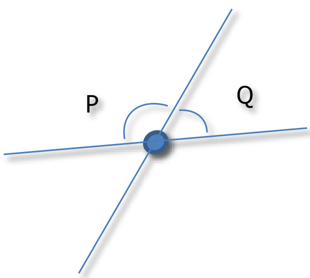
Teacher Activity

- Take hinged construction A. Start with sides together, slowly open to get to flat line then go further. Explain these positions. (Note: symbol ° means degrees.)

  (Note: 0 ° means lines together)



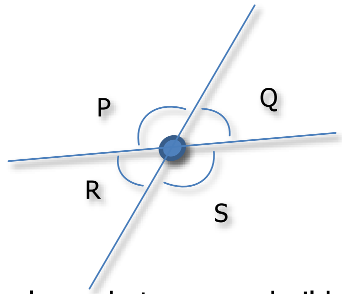
- Explore these angles on hinged construction B. What is the relation of the angles, e.g.



What happens to P as Q gets larger? [P gets smaller]

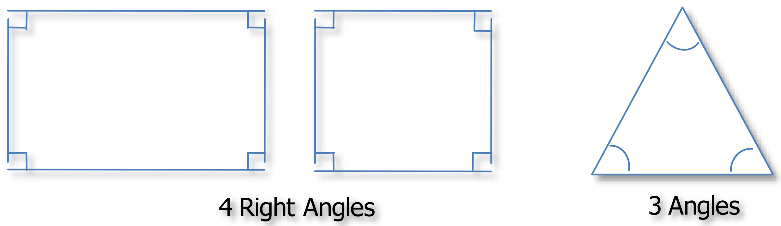
When are P & Q the same? [When right angles]

What do P and Q add to? [Straight line or 180?]

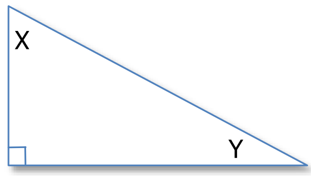


What about relationship between Q and R and between P and S? (equal)

- Explore what you can build with angles using long thin sticks



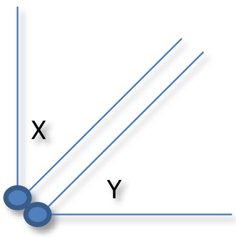
- Look particularly at the right triangle



Copy X and Y with hinged construction A.

Put them together as below on left.

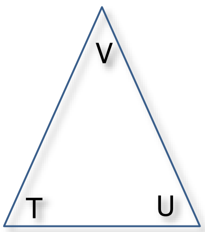
What do you notice?

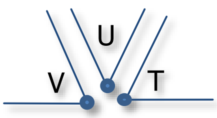


X and Y equal right angle,

i.e. X + Y = 90 degrees

- If you have time explore the 3 angles of a triangle





V + U + T = straight line angle

= 180 degrees

2.2 Angle to make the horizontal

We now return to our problem of cutting a slant piece of wood so that its bottom is flat on the ground.

(1) **Objectives**:

- To show that angle to vertical is 90 minus angle to horizontal

(2) **Materials**:

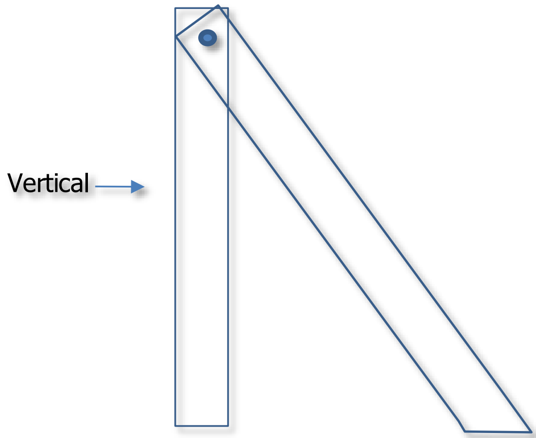
- Small pieces of wood or cardboard (long thin rectangles)

(3) **Language**:

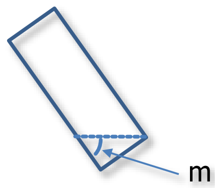
- Angle complementary, supplementary, right angle, straight angle

Teacher activity:

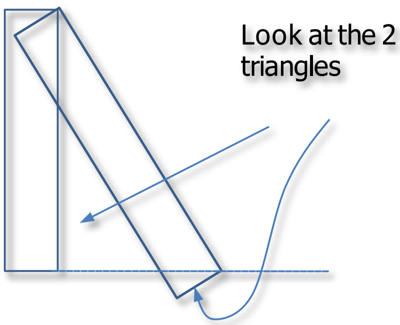
- Construct this shape with materials



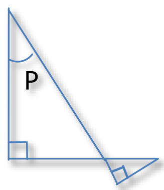
Set question, what is angle of cut m that makes horizontal?



- Draw the shape



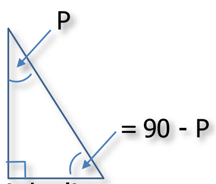
- Redraw these two triangles



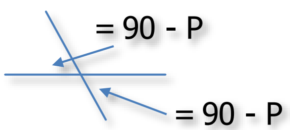
Use hinged constructions to compare angles.

Use relationships from Section 2.1 to look at how other angles relate to vertical angle P.

- In the right triangle



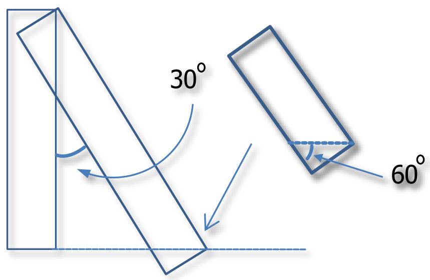
- In the straight line:



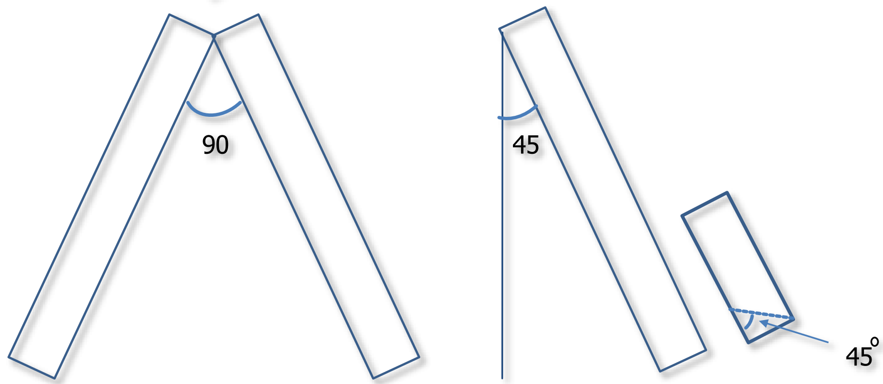
Thus the angle to cut for horizontal is 90 subtract the vertical angle or horizontal angle + vertical angle = 90 degrees.

- Work out these examples

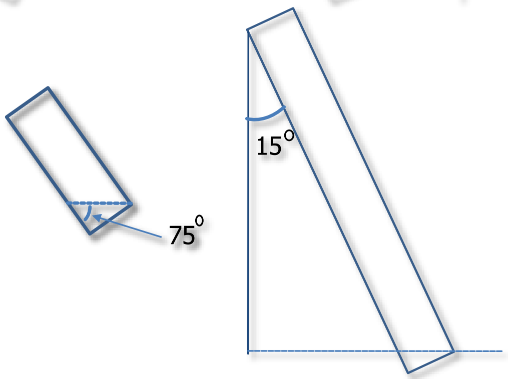
(a)



(b)



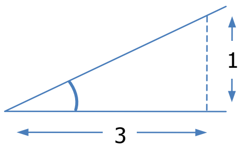
(c)



2.3 Measuring angle with ratio

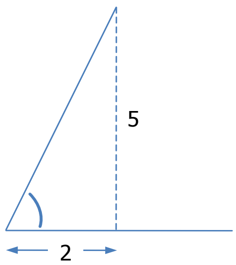
In construction, an angle is also measured as a ratio. For example:

(a)



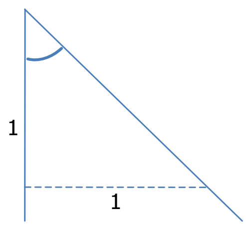
Angle is 1:3 or 1 in 3

(b)



Angle is 5:2

(c)

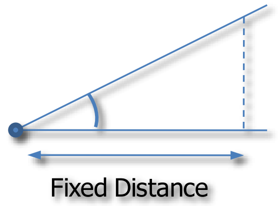


Angle is 1:1

The following lesson shows a method for teaching this idea and using it to explore similarity in right triangles (this lesson can lead to the notion of tangent).

(1) **Objectives**:

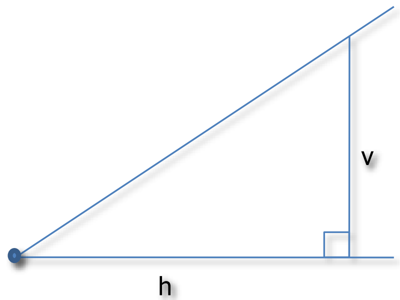
- To introduce ratio method for angles
- To use this angle approach to investigate similar right triangles
- Use hinged construction A to look at angles.



- Think of a vertical line. From one side to the other as shown Ask the students:

  What happens when angle gets longer? Smaller? [line increase/decreases]

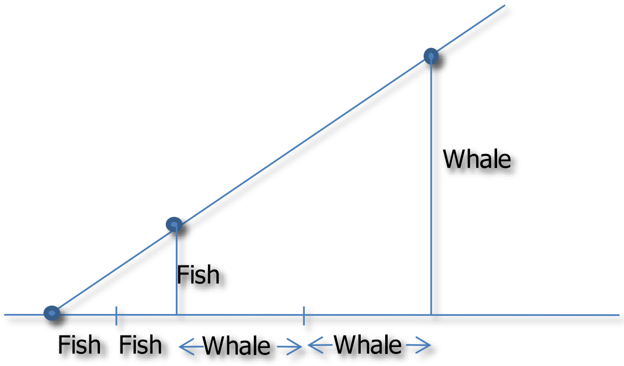
  What has this line added to the angle into? [a triangle]
- Get students to work in groups and share ideas. Introduce the idea of using a triangle from the base line to the angle line. Reinforce that the vertical line (v) must be at right angles to the horizontal line.



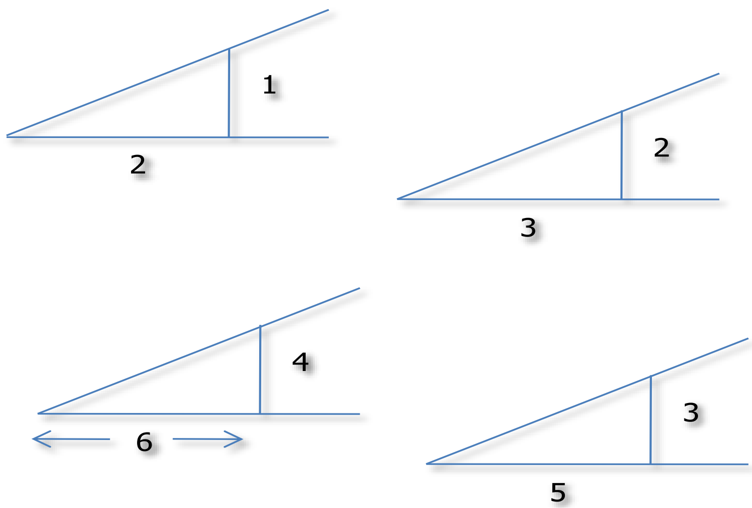
- Demonstrate that angles are measured by measuring the length of the horizontal line (h) and the length of the vertical line (v)
- Reinforce the findings from Construction A. Ask the students, *"If I increase the angle but keep "h" the same length, what happens to the "v", the vertical line?" Ask the students if I decrease the angle keeping "h" the same, what happens to the "v"?*
- Demonstrate measuring an angle and how it is stated. Give students a series of angles and ask the students to:
  - Mark the angle they are interested in;
  - Measure the distance along the horizontal (h);
  - Draw a vertical line (must be at right angles to "h") at the end of "h";
  - Measure the vertical line; and
  - State what the angle is.
- Demonstrate how to create an angle given its measurement i.e. draw a 3:1 angle or a 1:5 angle. Step through this by:
  - Drawing a vertical line;
  - Measuring the horizontal length;

- Measuring the vertical length (need set square); and
- Connecting the line to form the correct angle.

- Give a worksheet for students to create their own angles. Demonstrate that no matter what unit of measurement is used, you will still get the same angle for given ratio. You could use the "whales" and "fish" idea from Section 1.



- Highlight the two triangles using different coloured pens.
- Instruct students to draw a 1:3 angle using different measurements on paper (e.g. cm, mm, dugong) For each measurement (or triangle) the student can colour them in and cut them out and show that the angle is the same by lying them on top of each other.
- Explore when angles are the same and different e.g.



Can you find a rule? (the ratios are equivalent)

2.4 Making outdoor furniture

(1) **Objectives**:

- Show how formula is made using ratio approved to angle
- To find rules for horizontal (vertical cuts of slant pieces)

(2) **Materials**:

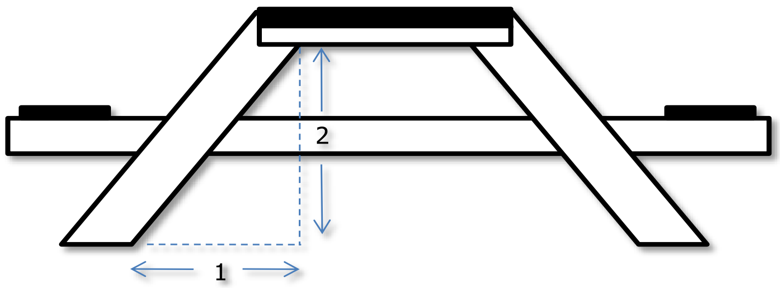
- Pen, paper, wood or strips of cardboard pieces to model constructions, (or real constructions)

(3) **Language**:

- Measure, length, units, how many

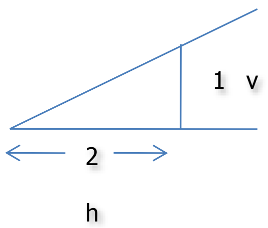
Teacher activity:

- Show the students plans for making an outdoor table setting where legs make an angle of 1:2, e.g.

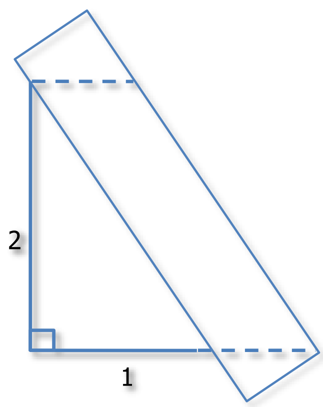


Get students to create the angle for the table Ie.g., Students are asked to do a large triangle for their angle.

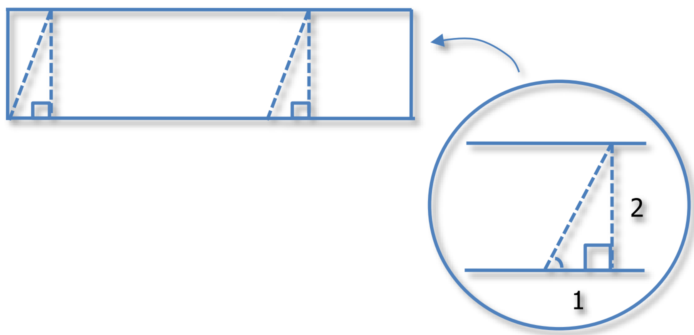
- Inform students that the most difficult aspect of building this table is the legs since they are at an angle of 1:2 and the bottom of the leg must be horizontal to sit flat on the ground.
- Demonstrate this angle on the board, reinforcing that the "v" is ½ "h"



- Hand out strips of cardboard/small pieces of wood and tell the students these are going to be our legs for the table. Inform students not to worry about the length of the leg yet and that we are concentrating on making sure the legs stand flat. Lay a strip of cardboard/wood on the angle as follows:



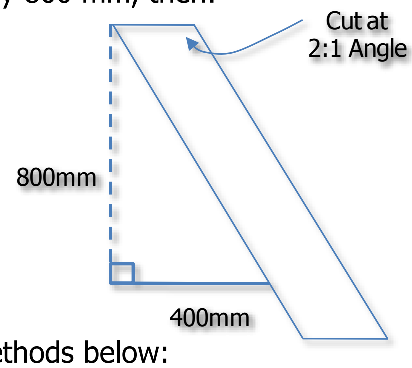
- Demonstrate by drawing a line on the cardboard that we have to get a flat or horizontal edge at the top and bottom.
- Lay cardboard down to (i.e. away from the triangle) to discuss the angle of the cardboard.
- Demonstrate to the students the smaller triangles that represent the angle on the cardboard by drawing a vertical line from the edge as follows. Get the students to do this on their cardboard.



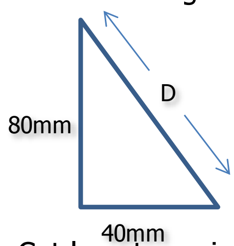
- Instruct students to measure the vertical and horizontal lines and determine the angle of the line as a ratio (use a calculator) Relate these measurements to the 1:2 ratio of the table leg. [They should get a ratio of 2:1]
- To find <u>where</u> to cut, decide on height of table, say 800 mm, then:

(a) Cut angle at top of leg.

(Note: the measures 400mm and 800mm mean that the angle of the table leg is 1:2.)

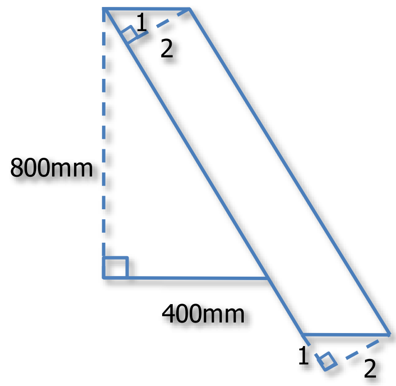


(b) Calculate length of leg by one of the two methods below:



- Pythagoras theorem ($\sqrt{800^2 + 400^2}$)
- Draw a small diagram (800mm is 80mm, 400mm is 40mm) accurately with a ruler as on left. Measure D. If it is 96mm, the length on leg is 960mm.

(c) Cut leg at required length at the 2:1 angle



- Repeat this for

(1) Table where leg angle is 2:3

(2) A saw horse where leg angle is 1:6

3. SHAPE, AREA AND VOLUME

There are many constructions in horticulture that utilise a variety of shapes and which rely on calculation of area and volume.

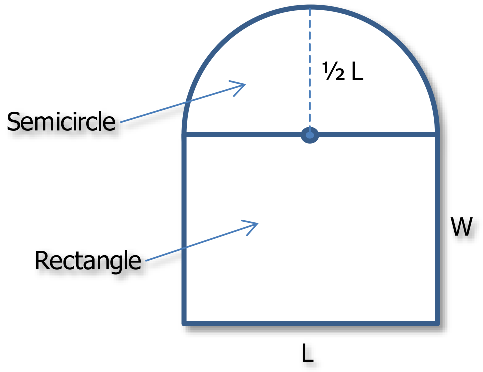
3.1 Shade Houses

Growing certain plants in certain areas requires the climate to be cooler or hotter. Shade or plastic houses enable this to happen.

Shade House Shapes

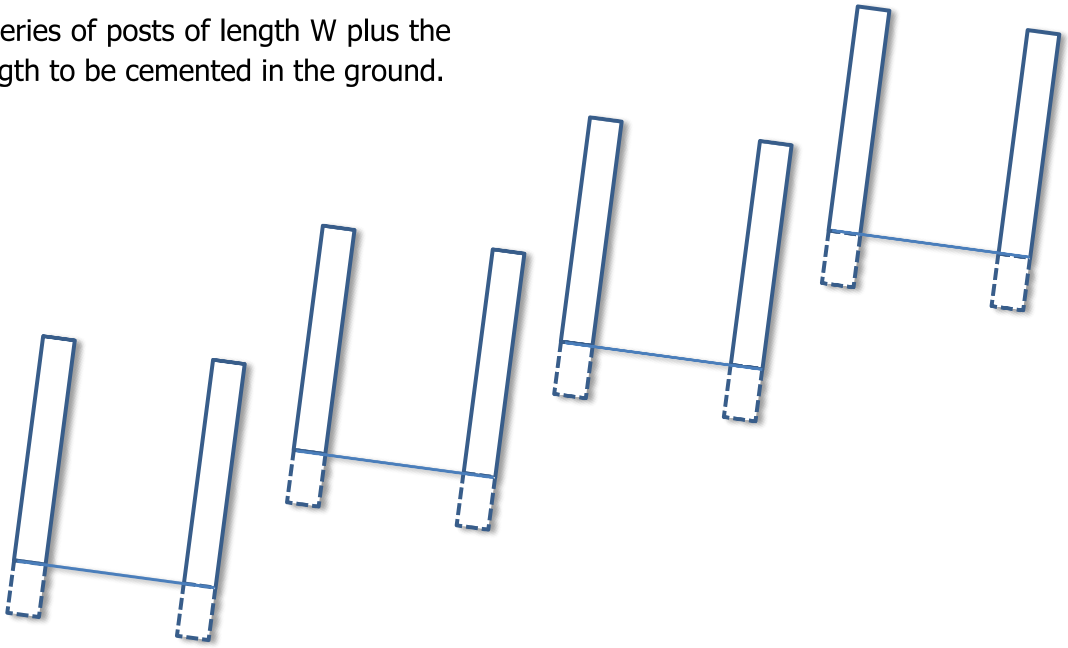
The most common shade house is rectangular or semicircular or a combination of both as on right.

If the rectangle is L wide and W high, the semi circle has a diameter of L and a radius of ½ L. Since the circumference of a circle is 2 π R, the circular length of a semicircle is π R, or in this case ½ π L.

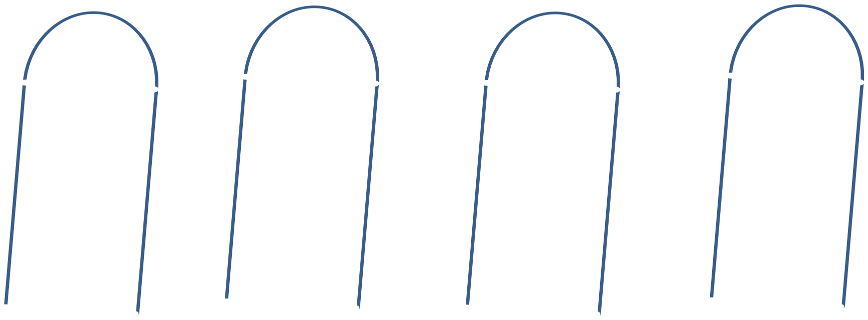


Thus the material for this type of shade house is

(a) A series of posts of length W plus the length to be cemented in the ground.

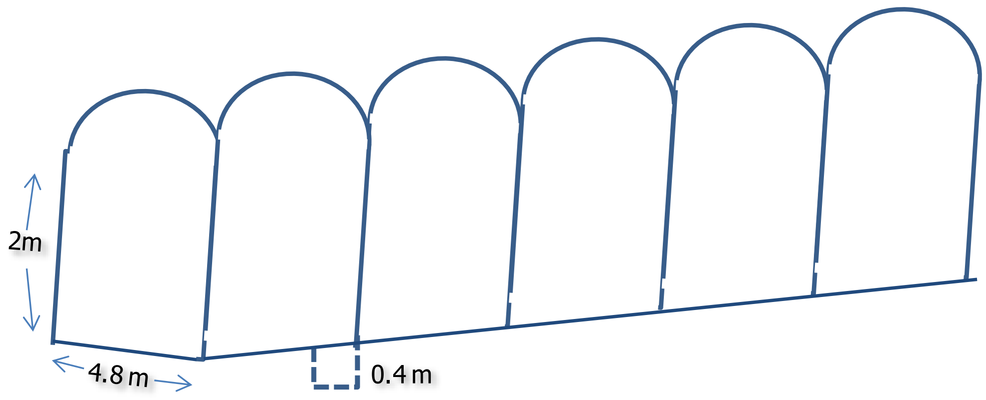


(b) A series of bendable plastic or steel piping that forms the semicircular top (circular length of each ½ π L)



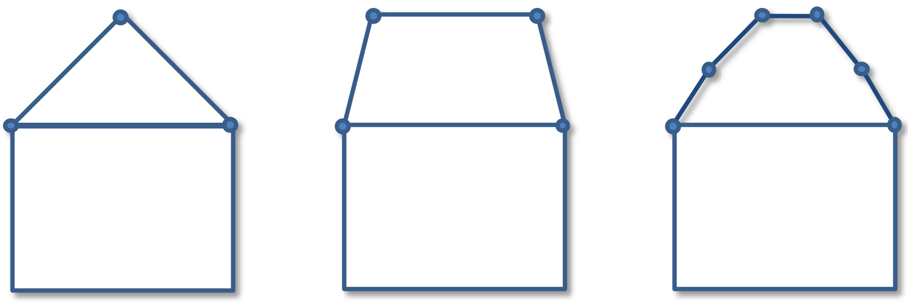
Shade house shapes activity:

(1) Calculate the length of posts and length of piping for a shade house of sides 2 m high, 0.4 m cemented in the ground, width 4.8 m, 6 sets of material.



(2) What other shapes can be put together for a shade house.

e.g. rectangle and triangle rectangle and trapezium rectangle and hexagon

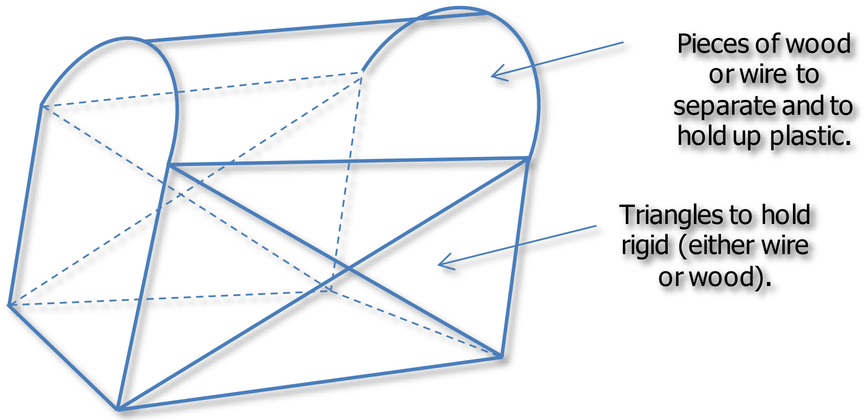


Calculate the material needed for the rectangle and trapezium version when the trapezium is 1/3 its base width at the top and 1/3 its base width in vertical height and there are 5 sets of uprights. Draw a diagram of the resulting shade house.

Shade House strength:

The strength of the traditional shade house lies in the semi circle. This needs no cross pieces to hold it up.

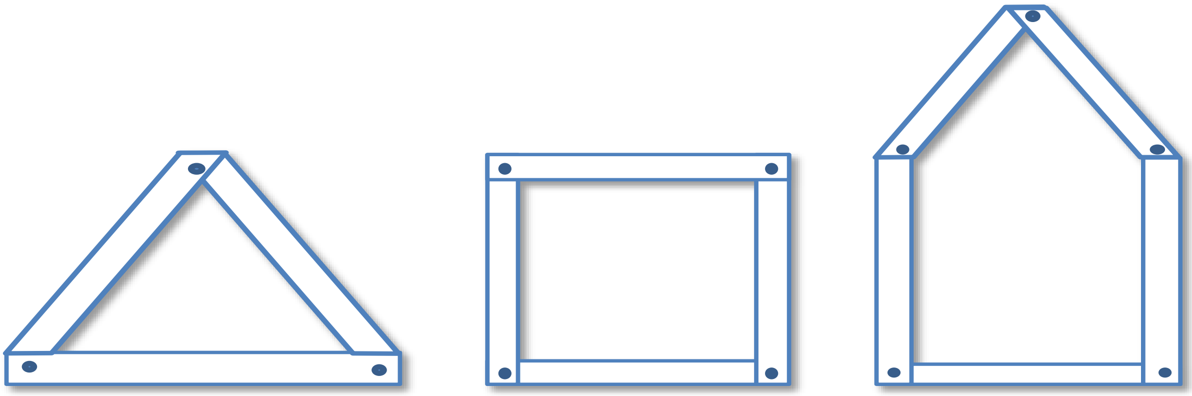
However, the uprights can begin to sag if not braced with triangles and separated by lengths of wood. Thus the following may be needed.



Look up a manual on shade house construction – what form of bracing may be required according to these manuals.

Shade house strength activity:

(1) Make shapes out of wood and screws or geostrips as follows



Which is rigid? Which flop? How can we make them all rigid?

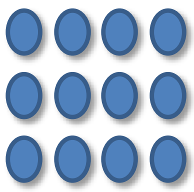
(2) What bracing would you require for a rectangle and trapezium shade house?

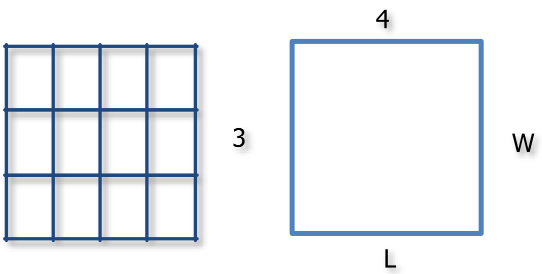
3.2 Gravel creek beds

One common construction in wet areas is artificial "creeks" - to prevent water from rain going under and in buildings. Often gravel is placed on the creek bed; how do we work out how much gravel is needed is the question being considered?

<u>Area</u>

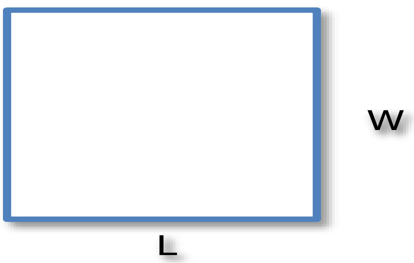
(1) Multiplication is always 3 x 4 in 3 rows of 4 as on right



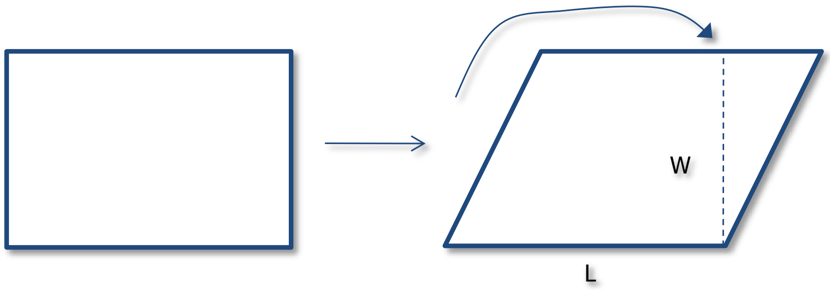


This can be considered as 3 rows of 4 squares and then a 3 by 4 area.

(2) Thus area of a length L and width W rectangle is L x W

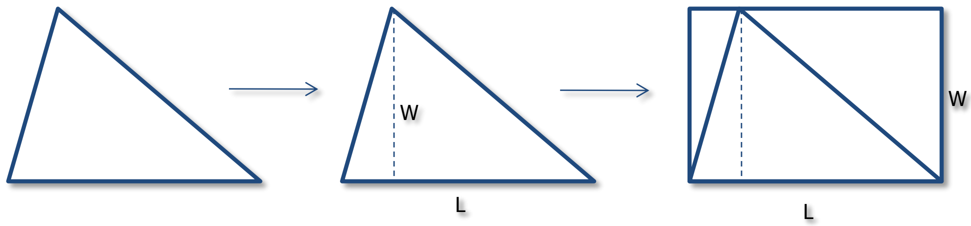


(3) As shown below, a parallelogram is the same area as a rectangle and a triangle is the ½ the area of a rectangle (because 2 triangles equal the rectangle)



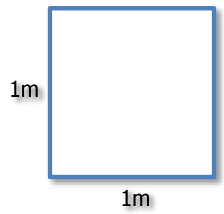
Thus the area of a parallelogram is L x W.

And the area of a triangle is ½ L x W



Coverage

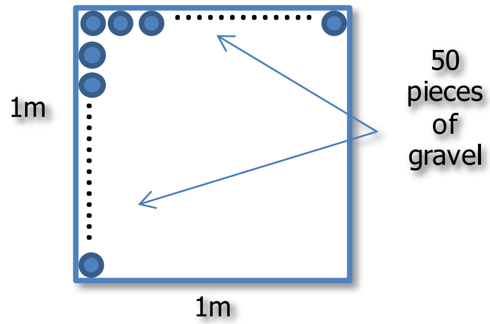
Suppose we have a square m of river bed as on right. How much 40 mm gravel will we need to cover it?



There are two ways:

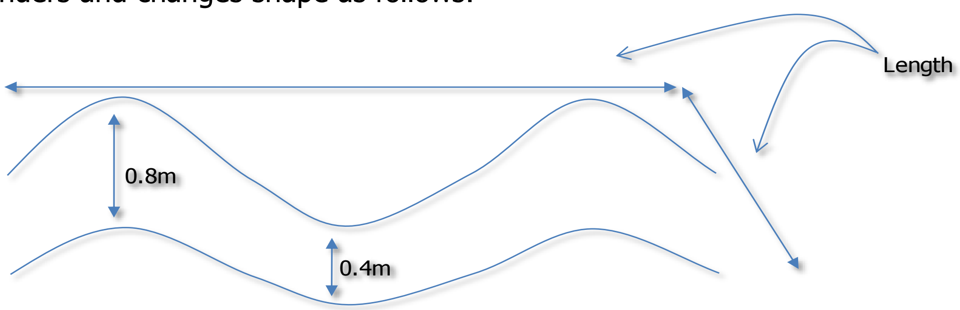
(1) Work it out by experiment – buy some gravel, draw 1 square metre on the ground and cover it, pick up and weigh the gravel (or weigh what is left in the bag and subtract from original weight); or

(2) Model the coverage – 40 mm gravel will fit in a square m as on right. 25 x 40 mm = 1000 mm = 1 m so there will be 25 x 25 = 685 gravel pieces will cover 1 square m. Weigh 100 pieces of gravel and multiply by 6.25.



Estimation

A river meanders and changes shape as follows:



(1) To work out area

(a) Measure along edge of river in roughly straight lines

(b) Measure across the river at regular intervals

(c) Average the across length, and

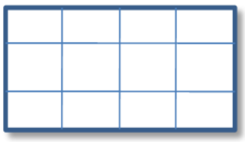
(d) Multiply length by average across river distance.

(2) To calculate gravel: Multiply the area by the gravel 1 square metre.

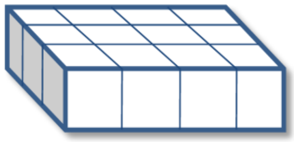
<u>Note</u>: Need to reduce amount because gravel does not fall perfectly into rows. Ask a tradesman what their reduction would be?

3.3 Cement bases

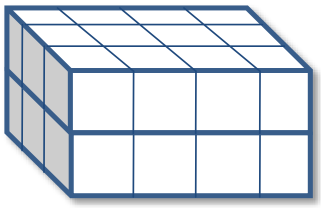
If you have the area of a shape, you have the volume for a height of 1 unit as follows



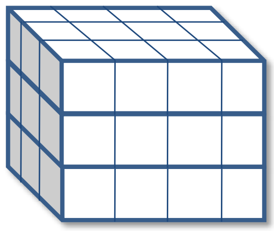
Area is
12 square metres



Volume is
12 Cubic metres



If you have 2 layers, volume = 24 = 2 x 12 cubic units

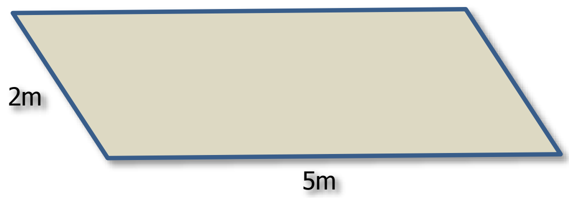


If you have 3 layers and so on, volume = 36 = 3 x 12 cubic units, that is number of layers x 12 cubic units. Thus, volume = area of base x height. This holds for any base, e.g., circle, triangle and so on.

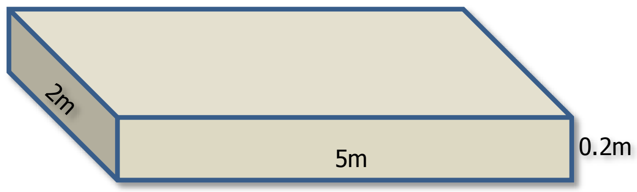
Volume of concrete

The question is how much concrete for a cement base?

(1) Work out the dimensions, e.g.



(2) Work out the depth, e.g.



(3) Calculate the volume
= area of base x height

= 2 x 5 x 0.2

= 2 cubic metres of concrete

***Student Activity*:**

(1) Calculate the volume of concrete for the following:

(a) Right triangle 3 m by 4 m by 5 m, 0.4 m deep.

(b) Walk way around rectangular garden 2 m wide by 0.2 m deep, garden is 8 m by 13 m.

(c) A trench 0.8 m wide by 0.6 m deep by 10 m long with a 0.15 m diameter pipe through its middle.

4. OPTIMIZATION

Some jobs are very straight forward but are made mathematically complex by requiring minimization of waste. In these cases, the question is what is the optimal method of completing the job for the minimum material use?

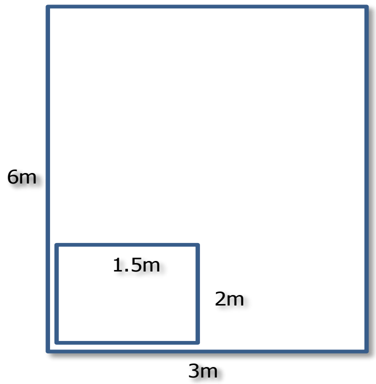
4.1 Rectangles on rectangles

Boards for completing walls come in set shapes. Yet the wall to be completed may not match these shapes. How do we complete with minimum wastage? How do we limit the amount of time saving?

Both board dimensions are divisions of wall dimensions

(1) Look at wall 6 m by 3 m and look at board 1.5 m by7 2 m. How do we place boards to minimise waste? And with minimum saving.

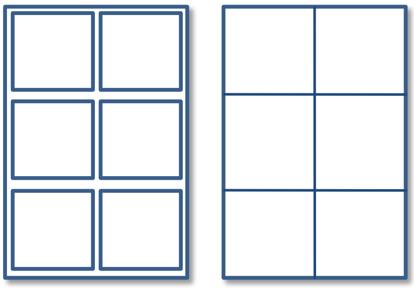
Pedagogy: Give students 600 x 300 mm piece of paper to cover with 150 mm x 0 mm smaller paper rectangles. Discuss how they did it?



(2) Place board so dimensions divide (are factors). For example, board is considered as 2 m x 1.5 m and placed as on right.

(3) In this way, 6 boards will cover the wall (as on right)

and there is no wastage



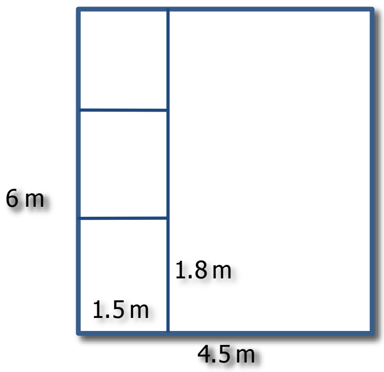
Note: The boards will have to be nailed on to rails (or noggins). We have to minimize their use. Here the rails would be as on right and fairly well optimized.

One board dimension is not a divisor

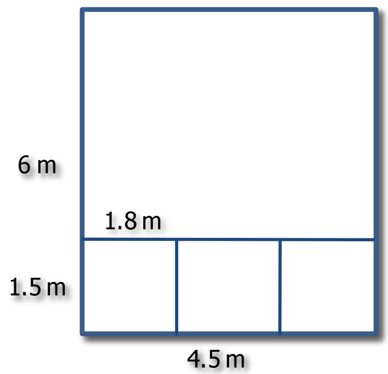
(1) Look at wall 6 m by 4.5 m and boards 2.2 m x 1.5 m

(2) Explore by placing 220 mm x 150 cm pieces of paper on a large 60 mm by 45 mm piece of paper.

(3) Will find have to base placement on the one dimension that does divide, but which is best?

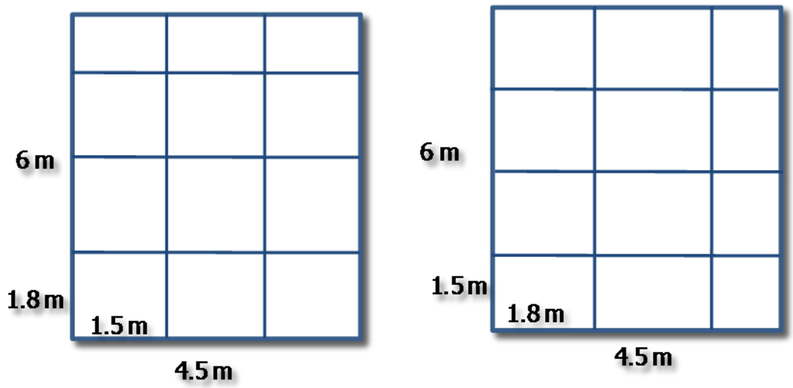


OR

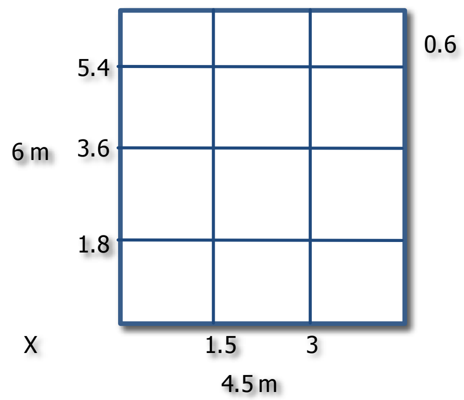


(4) Have to take account of rails?

(5) The rails appear to be the same in terms of length use – optimization will be decided by the amount of left over and saving.

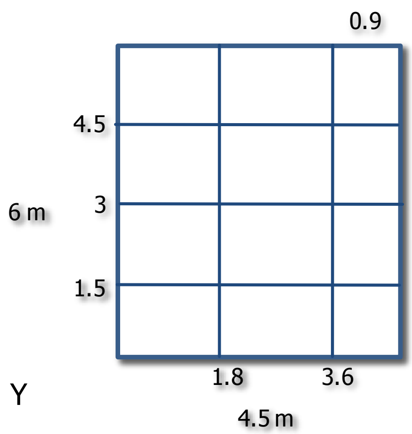


(6) However, leftovers can be used for unfinished pieces (As below).



The top 3 smaller pieces can be cut out of one board exactly (0.6 x 3 = 1.8) – no wastage.

So total no. of boards = 3 x 3.41 = 10 boards



The 2 of the 0.9 pieces left over can be cut from one board (0.9 x 2 = 1.8). So the 4 pieces on right can be cut from 2 boards exactly (no wastage). So total no. of boards = 2 x 4 + 2 = 10 boards

(7) However, way X uses 3 saw cuts and way Y only 2. So Y is marginally better!

Questions

(a) If one dimension only divides, do we always get the same number of boards for both directions (if 2 directions are possible) – in the above the 1.5 divides into both of the large dimensions.

(b) Is it possible to have a dead heat?

No board dimension divides into wall dimension

(1) Look at 6.2 m by 4.8 m wall and 1.5 m x 1.8 m board

(2) Use smaller pieces of paper to try to work out best result. Take account of rails and sawing.

(3) Is there a rule to follow? Ask a tradesman. Look it up on the internet.

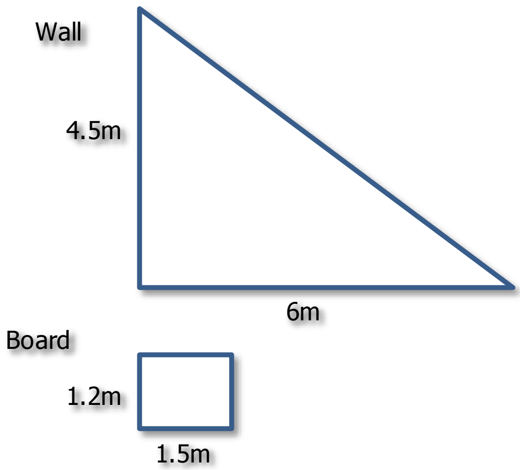
4.2 Rectangles on triangles

Sometimes the wall to be covered is right triangles. Again, how well you can optimize and reduce waste and time saving is to look at factors.

Both board dimension factors of triangle wall dimensions

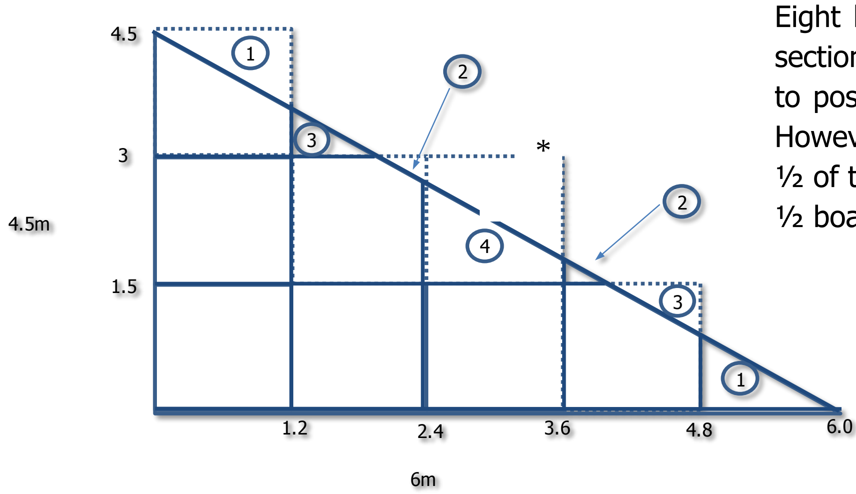
(1) Suppose the wall is a right triangle 4.5 m by 6 m and the board is 1.2 m by 1.5 m. then 1.2 is divisor of 6 m and 1.5 is a divisor of 4.5.

(2) Place the rectangles on the triangle. When they do not fit cut to fit then see if the cut off piece can be placed elsewhere. Try to minimise waste.



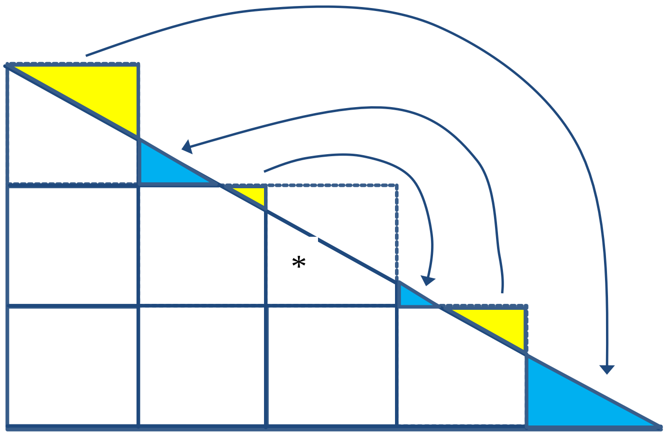
(3) Waste minimisation occurs when left over bits from the placement of one board can be used elsewhere, when sawing is a minimum and when rails (noggins) are not over used.

(4) When board dimensions are factors of wall dimensions, then there should be no waste and saving is low. The following is one arrangement that works.



Eight boards are used and the protruding sections are cut off. Cut off section 1 goes to position 1, 2 to position 2, and 3 to 3. However, * has no where to go and thus ½ of this board has to be wasted. Thus, 7 ½ boards cover the triangle.

A second way is to use 7 boards and see what is left over



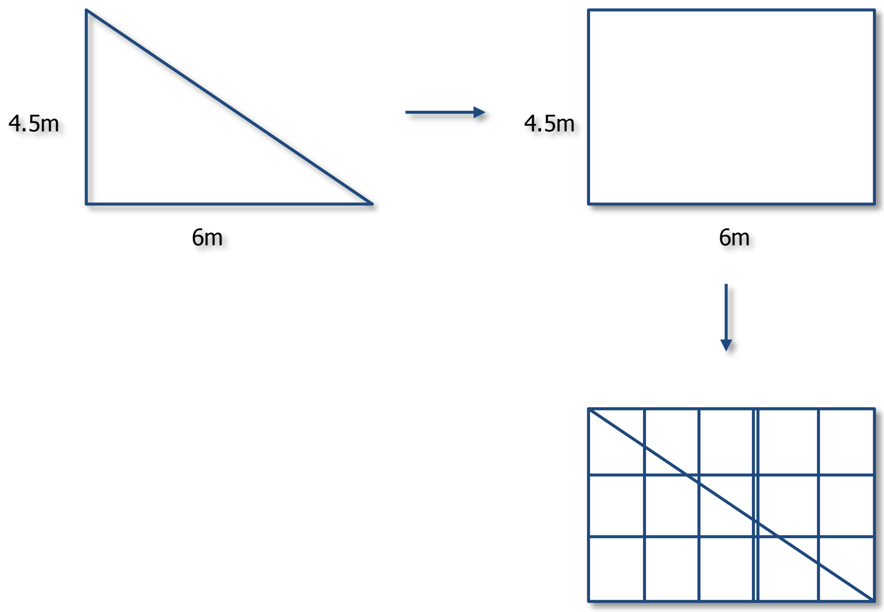
The Yellow parts fill in the Blue areas. Notice the symmetry in the movement of left overs.

The area * is left over (nothing is available to fill it) and another board has to be ½ used to cover it.

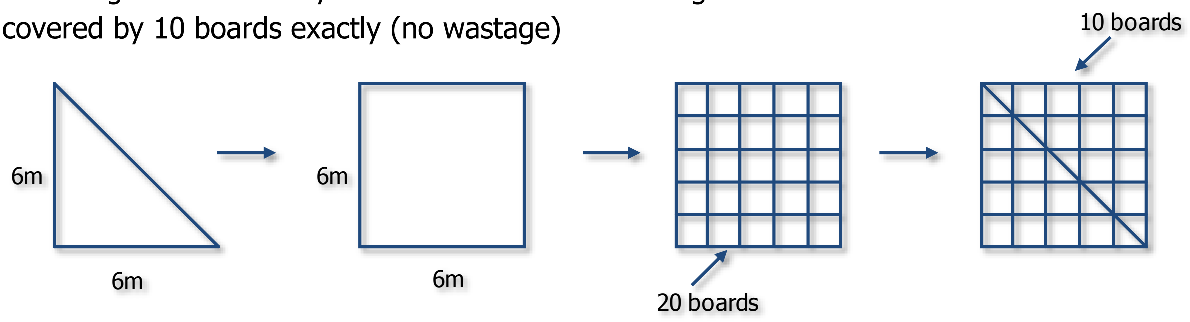
(5) Another way it can be understood is to consider the triangle wall is ½ a rectangular wall

Put the boards in the rectangular wall.

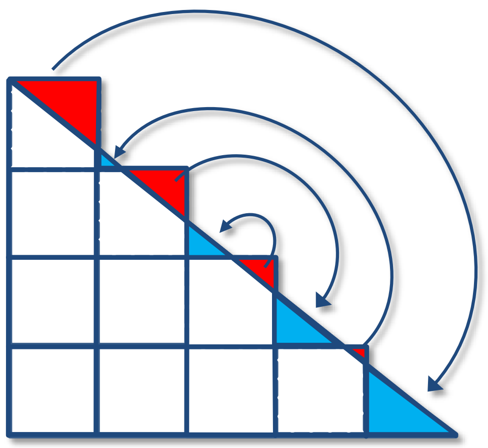
Fifteen boards will cover the rectangular wall. This means that7 ½ boards will cover the triangular wall. This gives the same result – have to use 8 boards and have to waste a ½ a board.



(6) Consider a second example (see below): Triangular wall is 6 m by 6 m and the board remains 1.5 m by 1.2 m. Now the rectangle is covered by 20 boards and so the triangle is covered by 10 boards exactly (no wastage)



(7) The question remains, we know 10 boards is possible, but how do we cut the 10 boards to have no wastage. The following is one way.



Once again, we should notice the symmetry of the movement of the cut offs.

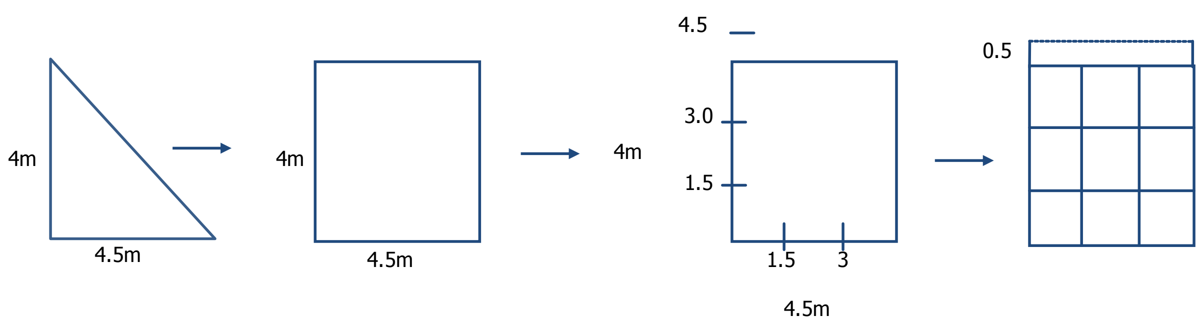
There is also efficiency in the cuts and rails – this construction only needs 4 saw cuts and 7 rails.

(8) What this means is that if divisors are such that give an even number of boards on a rectangle, can cover with no wastage, otherwise ½ board wastage.

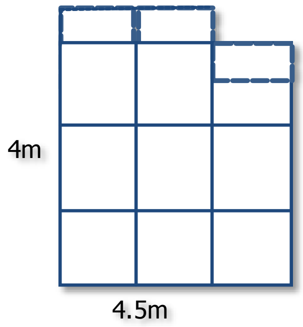
(9) <u>The pedagogy here is important</u> – Don't tell the students these results – Give them small paper versions of boards and walls and get them to experiment!

One board dimension is a factor of triangle dimensions

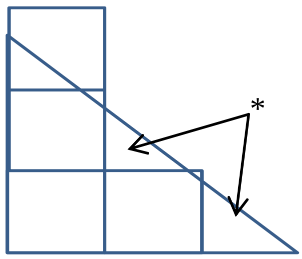
(1) Consider right triangle 4 by 4.5 m and board 1.5 m by 1.5 m. Now only one dimension divides (4.5 ÷ 1.5 = 3). Again use the rectangle approach, as below.



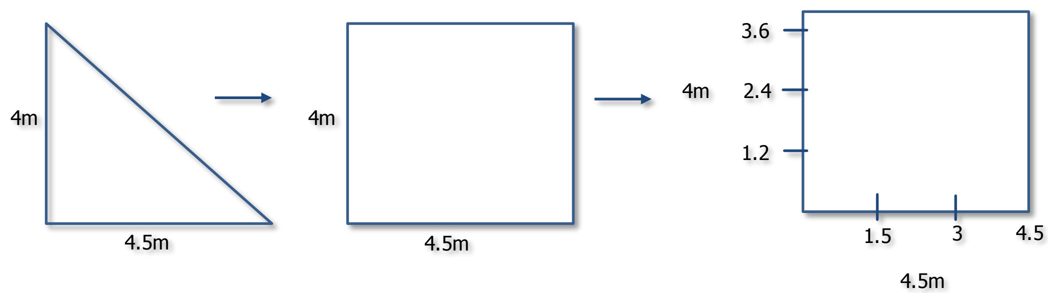
(2) Thus simply filling the rectangle uses 9 boards and wastes 1/3 of each of 3 boards. Looking at it again, 2 wastages could be used to replace one board as on right. In this way, 8 boards covers the rectangle.

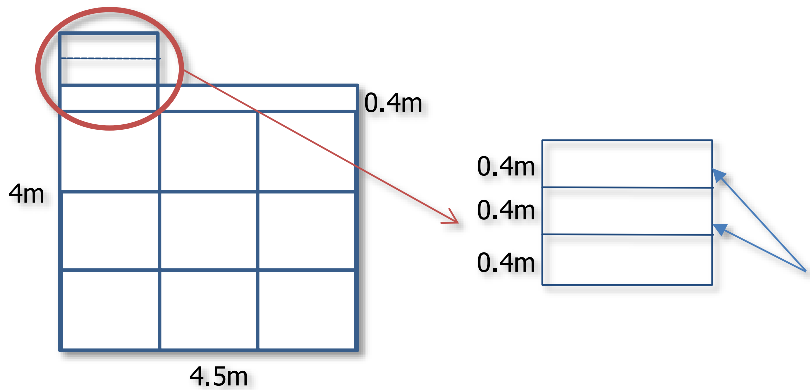


(3) However, the question is, since the triangle is ½ the rectangle, can we have 4 squares with no wastage for the triangle? So we cover the triangle with 4 squares. The question now is, will the left over bits fill the areas *?



(4) What about if the board is 1.2 m by 1.5 m?





These two can be moved to fill in the tops of columns 2 and 3.

(5) Thus, the rectangle is covered by 10 boards. However, can the triangle be covered by 5 boards. What are the moves for the cut off pieces to make this possible?

(6) Make up pieces of paper for the boards and the triangular wall and try to find the cuts.

(7) Try to do the 4 m by 4.5 m triangle wall covered by 5 boards. Can it be done? How close can you get? Remember – you do not want many sawcuts and fancy rail structures. Again, make pieces of paper for the boards and the triangular wall and try to find the cuts

No board dimensions are factors of wall triangle dimensions

(1) This is too complicated to be worked, so make up paper copies and try a few cases

(a) 1.2 m by 1.5 m boards and 4 m by 5.5 m right triangle

(b) 0.8 m by 1.2 m boards and 3 m by 4.5 m right triangle?