YUMI DEADLY CENTRE
School of Curriculum

Enquiries: +61 7 3138 0035
Email: ydc@qut.edu.au

**http://ydc.qut.edu.au**

Construction and Building

# Mathematics behind Dome Construction Using Earthbags

## Booklet VC1: Circles, Area, Volume and Domes

**DEADLY MATHS VET**

**Northern Peninsula Area College**
**"Hands On Learning" Program**
**Construction and Building**

# MATHEMATICS BEHIND DOME CONSTRUCTION USING EARTHBAGS

**BOOKLET VC1: CIRCLES, AREA, VOLUME & DOMES**

VC1

**VERSION 1**

**Deadly Maths Consortium**

**Tom J Cooper & Annette R Baturo & Chris J Matthews**
**with**
**Matthew T Michaelson, Kaitlin M Moore & Elizabeth A Duus**

**Deadly Maths Consortium**
**School of Mathematics, Science and Technology Education**
**Faculty of Education, QUT**

This material has been developed as a part of the Australian School Innovation in Science, Technology and Mathematics Project entitled *Enhancing Mathematics for Indigenous Vocational Education-Training Students*, funded by the Australian Government Department of Education, Employment and Workplace Training as a part of the Boosting Innovation in Science, Technology and Mathematics Teaching (BISTMT) Programme.

YuMi Deadly Maths
Past Project Resource

# Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.


# YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.




School of Curriculum
QUT Faculty of Education
S Block, Room S404, Victoria Park Road
Kelvin Grove Qld 4059
Phone: +61 7 3138 0035
Fax: + 61 7 3138 3985
Email: ydc@qut.edu.au
Website: http://ydc.qut.edu.au



CRICOS No. 00213J

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Queensland University of Technology

**DEADLY MATHS VET**

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**BOOKLET VC1**

**CIRCLES, AREA, VOLUME**

**AND DOMES**

**08/05/09**

Research Team:

Tom J Cooper

Annette R Baturo

Chris J Matthews

with

Matthew T Michaelson

Kaitlin M Moore

Elizabeth A Duus

Deadly Maths Group

School of Mathematics, Science and Technology Education,
Faculty of Education, QUT

# THIS BOOKLET

This booklet VC1 was the first booklet produced as material to support Indigenous students completing courses or certificates associated with construction at the Northern Peninsula Area College, in Bamaga. It has been developed for teachers and students as part of the ASISTM Project, *Enhancing Mathematics for Indigenous Vocational Education-Training Students*. The project has been studying better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

VB1: Mathematics behind whole-number place value and operations
Booklet 1: Using bundling sticks, MAB and money

VB2: Mathematics behind whole-number numeration and operations
Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

VC1: Mathematics behind dome constructions using Earthbags
Booklet 1: Circles, area, volume and domes

VC2: Mathematics behind dome constructions using Earthbags
Booklet 2: Rate, ratio, speed and mixes

VC3: Mathematics behind construction in Horticulture
Booklet 3: Angle, area, shape and optimisation

VE1: Mathematics behind small engine repair and maintenance
Booklet 1: Number systems, metric and Imperial units, and formulae

VE2: Mathematics behind small engine repair and maintenance
Booklet 2: Rate, ratio, time, fuel, gearing and compression

VE3: Mathematics behind metal fabrication
Booklet 3: Division, angle, shape, formulae and optimisation

VM1: Mathematics behind handling small boats/ships
Booklet 1: Angle, distance, direction and navigation

VM2: Mathematics behind handling small boats/ships
Booklet 2: Rate, ratio, speed, fuel and tides

VM3: Mathematics behind modelling marine environments
Booklet 3: Percentage, coverage and box models

VR1: Mathematics behind handling money
Booklet 1: Whole-number and decimal numeration, operations and computation

# CONTENTS

# OVERVIEW

The ASISTM VET project funded in 2008 by the Australian Schools Innovation in Science, Technology and Mathematics scheme had 6 sites: Wadja Wadja High School at Woorabinda, Shalom Christian College in Townsville, Palm Island Post Year 10 Campus (run by Kirwan State High School and Barrier Reef Institute of TAFE), Tagai College Secondary Campus at Thursday Island, Thursday Island campus of Tropical North Queensland Institute of TAFE, and Northern Peninsula Area College at Bamaga. All these sites have Indigenous students and the project focused on developing instruments and materials to assist the teaching of mathematics needed for certification for Indigenous VET students with little previous mathematics success in school.

The project has been developing earthbag construction mathematics materials to assist the teaching of the mathematics required for certification for these Indigenous VET students, many of whom have had little previous success in mathematics and school.

## Earthbag Mathematics

This booklet was based on mathematics requested by the Northern Peninsula Area College (NPAC). This was with respect to a program run for students in Years 7–10 called "Hands on Learning". Indigenous students at risk of dropping out of school were given one day week learning through a construction and building project if they attended the college the other four days.

The project adopted by NPAC was for the students to use an "Earthbag" construction technique to build circular dome structures for the local council. The first structure was a 6-metre circular dome to act as an information centre. The second structure was a series of domes joined by walkways to act as a display of local history.

Earthbag construction is an inexpensive way to build structures in which cement/lime and earth/sand are mixed to wet, placed in bags which are rammed flat, and then joined together in a circular dome structure. The book *Earthbag Building* by Kaki Hunter and Donald Kiffmeyer describes how to build in this way.

## Mathematics areas

As the building was being constructed, NPAC wished to use these opportunities to teach literacy and numeracy. The areas wanted for numeracy or mathematics included:

(1) Circles - properties of circles, including the various components of circles (e.g. diameter, sectors), symmetry, patterns, puzzles, tessellations, dissections etc.).

(2) Circle formulae - including formulae for calculating the circumference and area.

(3) Circle construction – cylinders, cones and spheres, etc., and volumes of these solids.

(4) Circle relationships – perimeter vs area, surface area vs volume, and strength.

(5) Domes – form, construction, and strength.

(6) Building rate, that is, looking at rates at which the construction takes place, using multipliers and multiple line methods for solving problems.

(7) Earthbag mixtures, including meaning of ratio and proportion, and again using multipliers and multiple line methods for solving problems.

**Mathematics in this booklet**

Booklet 1 covers circles, circle formulae, circle construction, circle relationships, and domes, while booklet 2 covers Earthbag mixtures and building construction.

Booklet 1 is based on the idea that it is important to give students a wide understanding and feeling for circles (e.g. why are the earth, moon, and sun spheres, why does spinning clay give a circular result, how is the circle related to ecology, balance, recycling, why are circles good for wheels) as formal mathematics ideas such as the formulae for circumference, area, and volume. This means that the booklet has investigations that look at circle puzzles and designs and, at what can be called circle metaphors (that is, the role of circles in culture - e.g. circle of life, circle of time, circle of friends, etc.), and other uses of circles (e.g. Ying and Yang), including investigations of uses of circles in Indigenous cultures.

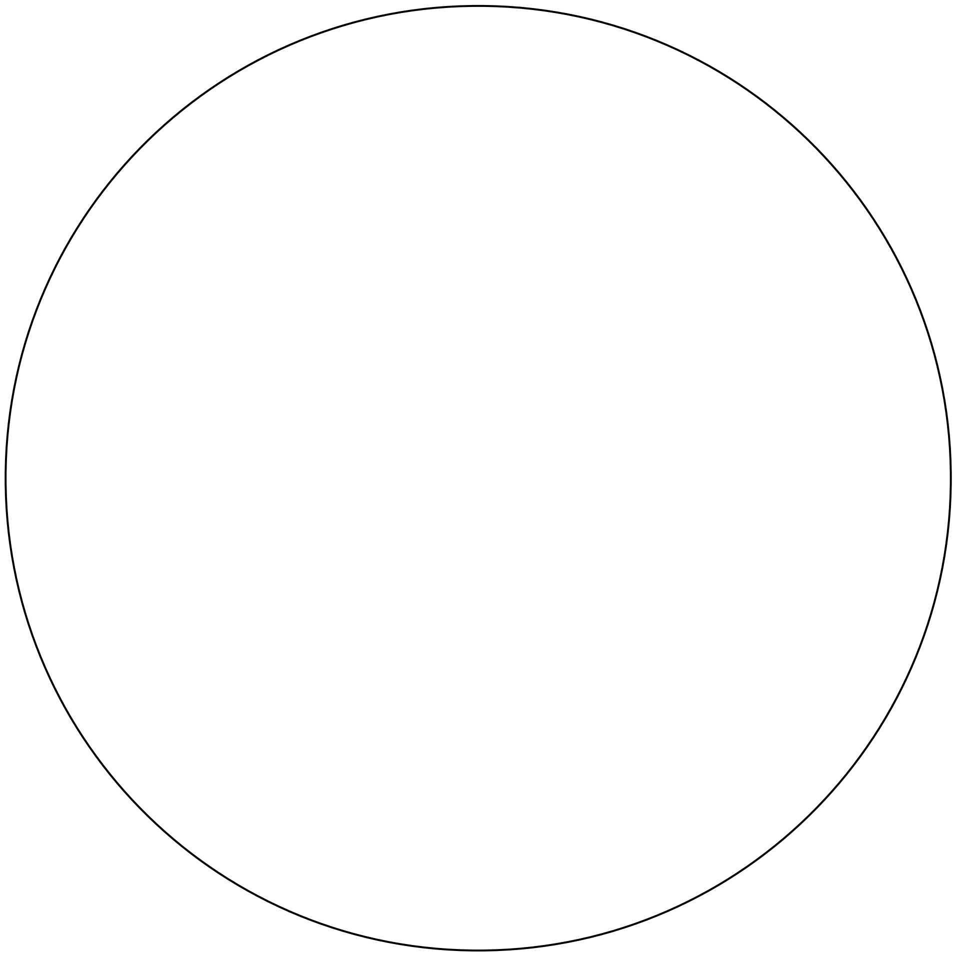
Booklet 1 contains activities and investigations. As the sections are moved through, the focus of the booklet starts to give less prominence to straightforward activities and more prominence to open investigations.

# 1. CIRCLES

## 1.1 Properties of Circles

***Activity 1.1***

1. How can you find the centre of the circle below? Find the centre and label it with a point and the word *centre*.



2. Measure the distance from the centre of the circle to any point on the edge of the circle. This distance is called the **radius**. Draw a radius and label it *r*. The radius is used to make calculations about the circle, such as its perimeter (circumference) and area.

3. The diameter is equal to the distance across the circle through the centre. How many radii make up the diameter? Calculate the diameter of the circle. Draw a diameter and label it *d*.

4. Calculate the diameter of a circle whose radius is 65 cm. Calculate the radius of a circle whose diameter is 530mm.

***Investigation 1.1***

1. Look up circles on Internet or in a book. Describe and draw a semicircle, a sector, a quadrant, a segment, and an arc

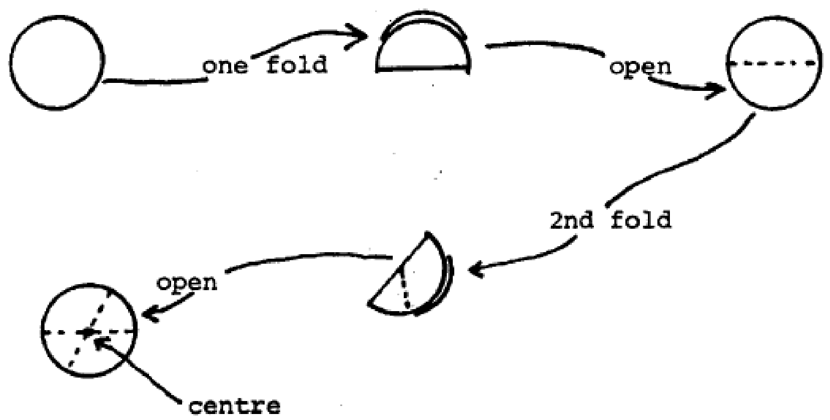
## 1.2 Circle Patterns

Activity 1.2

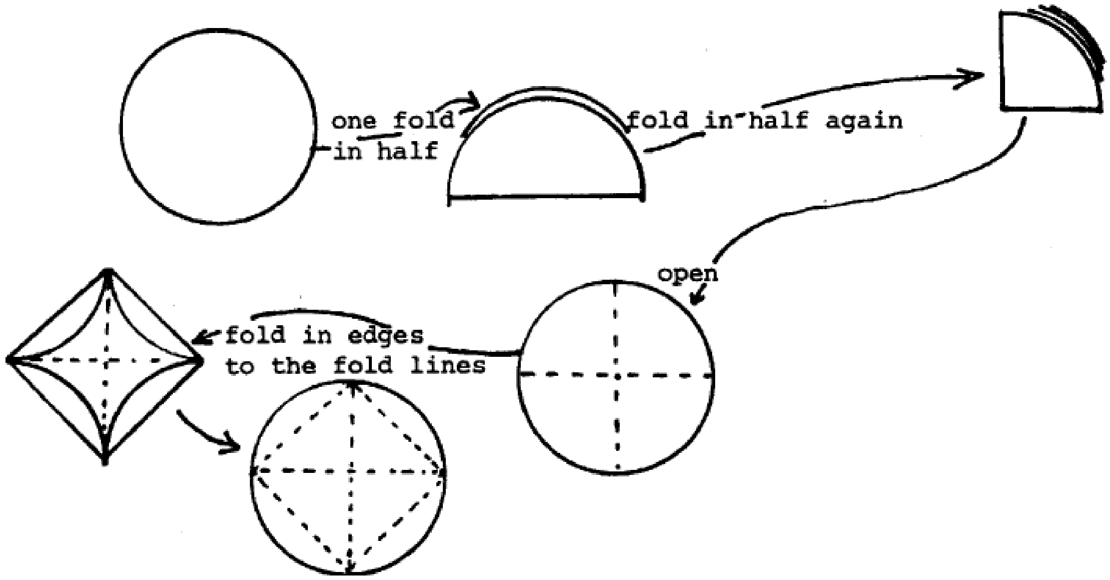
1. From a piece of A4 paper, cut out a circle that is about 25 cm across.

2. Fold the circle, as shown below, to find the diameter of the circle.


3. Fold the circle, as shown below, to find the centre and radius of the circle.



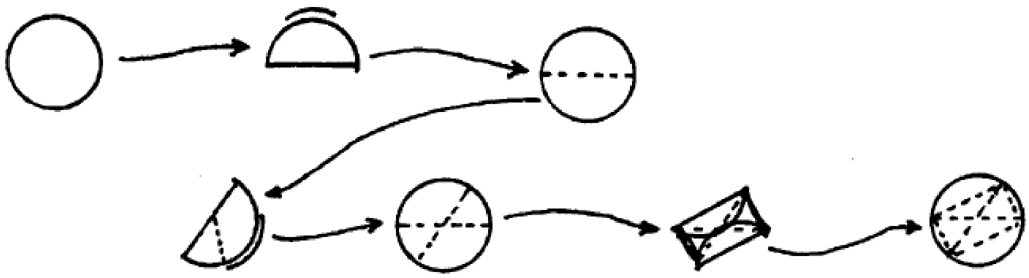

4. (a) Fold the circle, as shown below, to inscribe a square in the circle.




(b) What properties of a square does this show? (Hint: Look at the diagonals!)

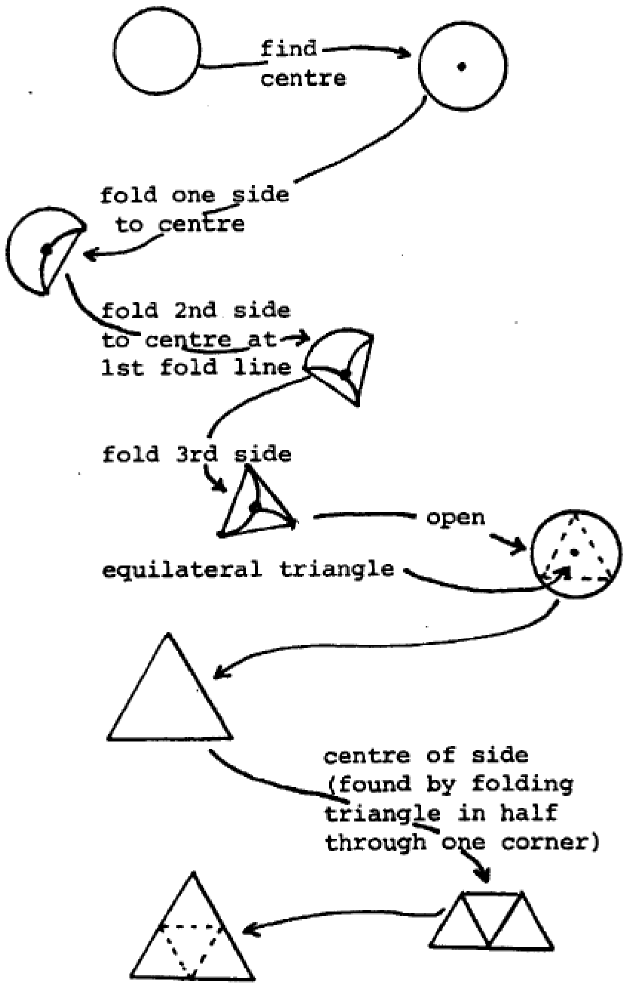
Activity 1.2 (cont.)

5. (a) Fold the circle, as shown below, to inscribe a rectangle in the circle.



(b) What properties of a rectangle does this show? (Hint look at 4(b)?)

6. (a) Fold the circle, as shown below, to inscribe a triangle in the circle.




(b) What are the properties of an equilateral triangle?

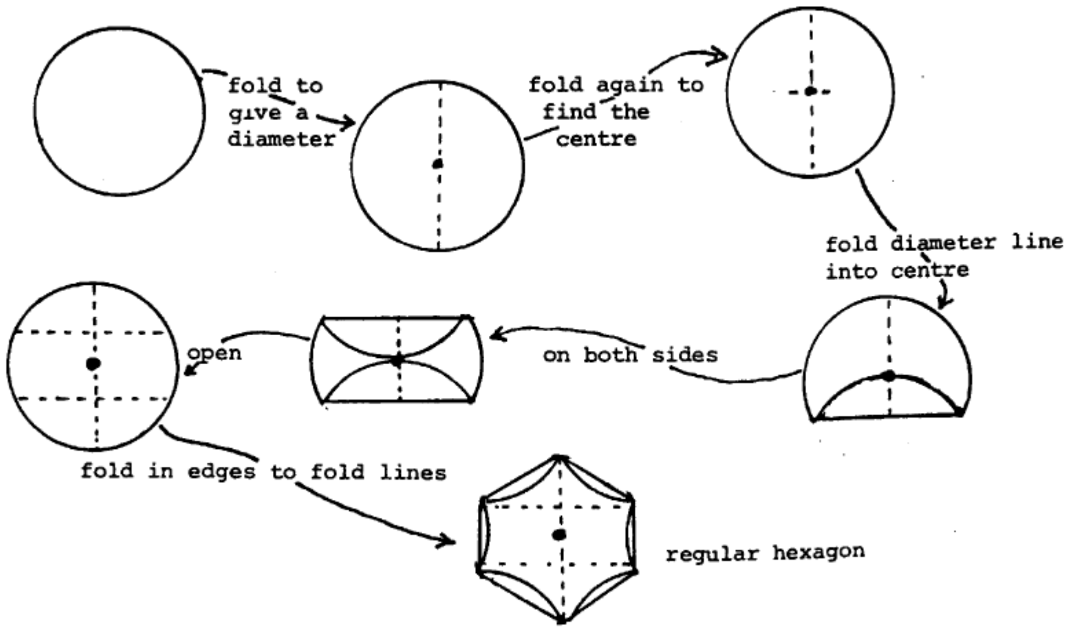
(c) Show all three lines of symmetry in the triangle.

(d) Fold the equilateral triangle into four smaller equilateral triangles.

(e) Construct a tetrahedron from the folded triangle.

Activity 1.2 (cont.)

7. (a) Fold the circle, as shown below, to inscribe a regular hexagon in the circle. (Construct the hexagon by folding the corners of the triangle into the centre.)




(b) What can we say about the diagonals of a regular hexagon?

Investigation 1.2

1. There are wonderful designs that can be made from a circle using a compass and pen. Look up Internet and books and find some of these designs. Make it and colour it. Make a large poster of your design.

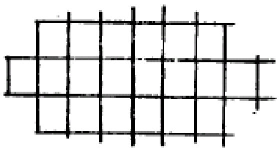
2. Circles are the basis of designs in Indigenous art. Approach local Indigenous artists and learn about the role of circle in art. Construct your own Indigenous circle design.

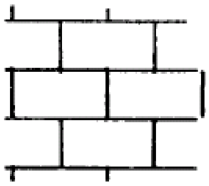
3. Why are circles good for wheels? Does this relate to line and rotational symmetry? What are the line symmetries and rotational symmetries for a circle?

## 1.3 Tessellations

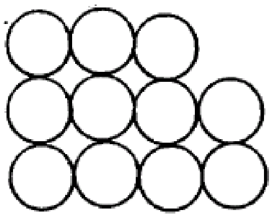
Activity 1.3

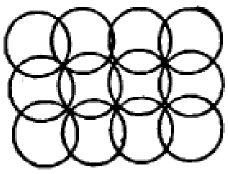
In a society that puts shapes together to build and cover and that packs shapes together to carry them around, shapes that fit and pack well are important. Shapes that fit together without gaps or overlaps are called **tessellations**. Squares and rectangles tessellate, as shown in the figures below.



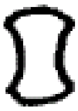


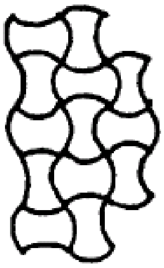
However, circles do not tessellate, as shown in the figures below.



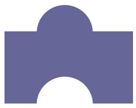


When attempting to pack circles, there will always be either a gap or an overlap. However, circles can be the starting point for shapes that do tessellate. For example, the shape tessellates into the pattern at right.





1. Construct tessellation for the shape on the right (based on a rectangle and two semi circles.



2. Construct a second tessellation for this second shape.

**3.** Can you make another tessellating shape out of semicircles and rectangles? Out of circles/semicircles and some other shape?

Investigation 1.3

1. A **double tessellation** is the name when two different shapes are packed together. Investigate and find a shape that will tessellate with a circle. Draw the tessellation.
2. Draw a double tessellation when one shape is a semi-circle.
3. Can you think of any more double tessellations that include a circle or part of a circle?

## 1.4 Sectors

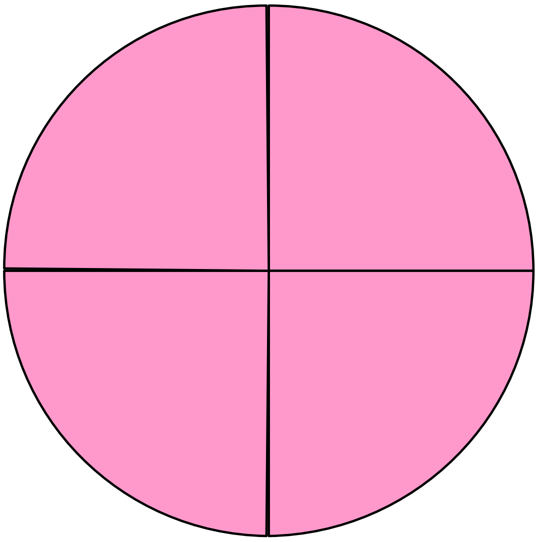
Activity 1.4 a

1. Cut out the circle on right.

2. Cut along the lines to form equal-area sectors of the circle. How many sectors do you have? How do you know?

3. One sector represents what fraction of the whole? What percentage area does one sector cover?

4. How is the fraction in Question 4. related to the percentage found in Question 5.?



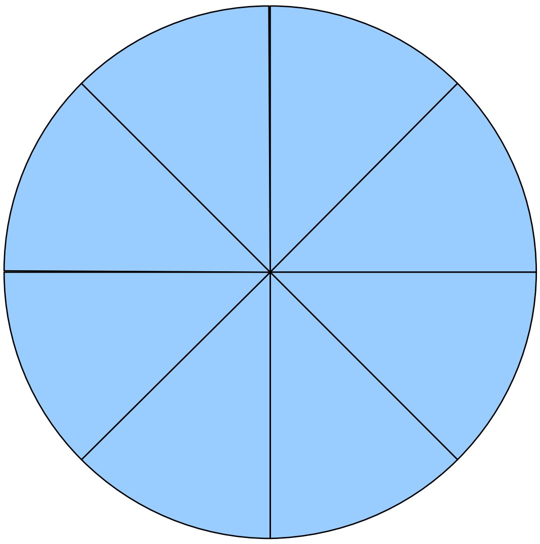
Activity 1.4 b

1. Cut out the circle on right

2. Cut along the lines to form equal-area sectors of the circle. How many sectors do you have? How do you know?

3. One sector represents what fraction of the whole? What percentage area does one sector cover?

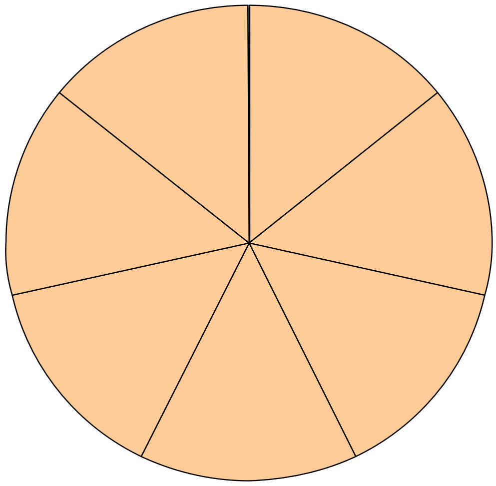
4. How is the fraction in Question 4. related to the percentage found in Question 5.?



5. How many sectors equal the same area as one sector from the circle in Activity 2.4 a?

Activity 1.4 c



1. Cut out the circle on right.

2. Cut along the lines to form equal-area sectors of the circle.How many sectors do you have? How do you know?

4. One sector represents what fraction of the whole? What percentage area does one sector cover?

6. How is the fraction in Question 4. related to the percentage found in Question 5.?

Investigation 1.4

1. Cutting up circles into different numbers of equal sectors can enable you to find equivalent fractions. Look up what equivalent fractions are.

2. Look up how to use pie charts in Excel (or ask your teacher). Do a pie chart of the numbers 1, 1, 1, 1. What do you get. How could you get 1/2s, 1/4s, 1/6s, and so on. Show ½ = ¼ = 1/6 = and so on with the pie charts.

3. Shwo 2/3 = 4/6 = 6/9 = and so on with pie charts.

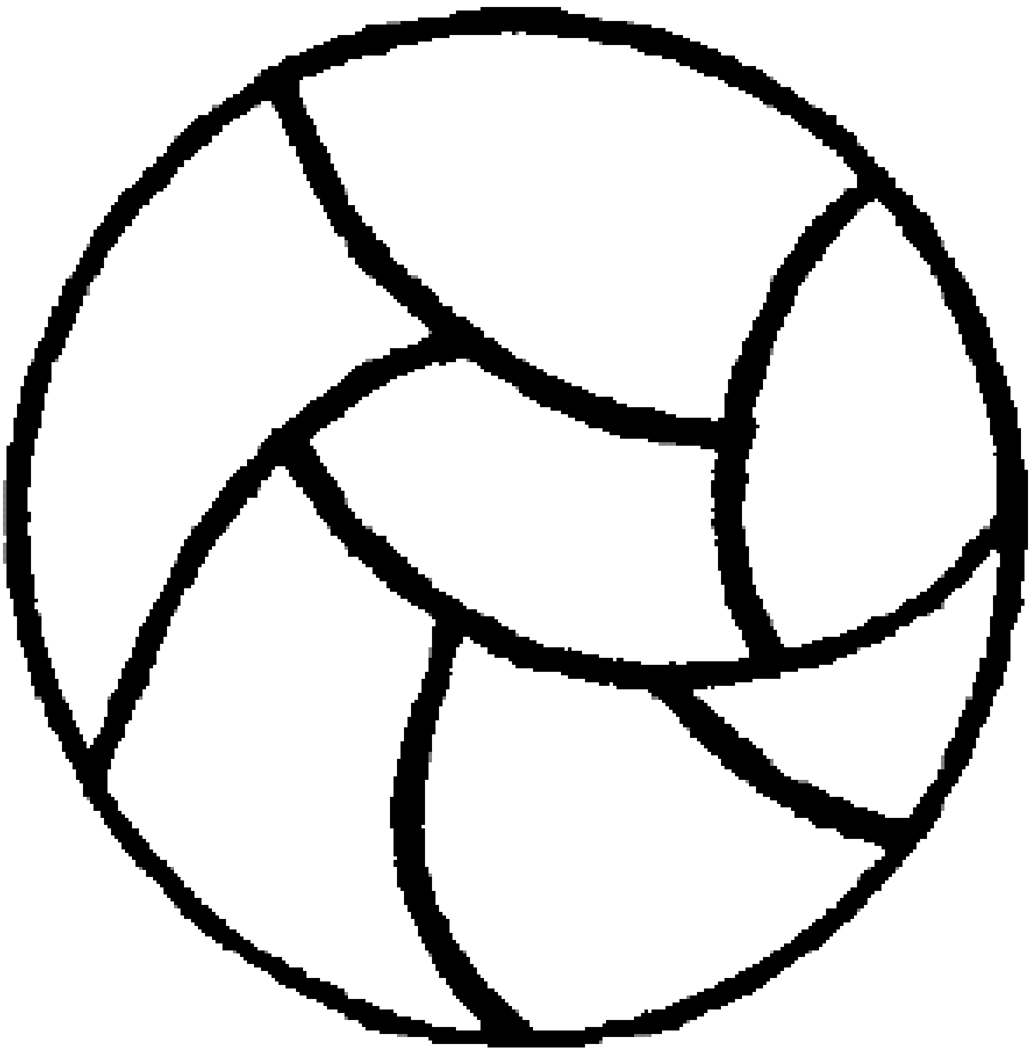
4. Cutting up circles into equal sectors and then joining where they meet the edge of the circle enables you to construct regular polygons (look these up). Make an eight sided shape and a 12 sided shape. What are these called?

5. As the number of sectors increases, the area of the polygons gets closer and closer to the area of the circle. How did Archimedes use this to find the area of circle? Look this upon the Internet or in a book.

## 1.5 Dissection puzzles

Activity 1.5 a

1. Cut along the lines to form a jigsaw puzzle (called a dissection in mathematics).



2. Mix the shapes.

3. Try to fit the pieces together to form the original shape.

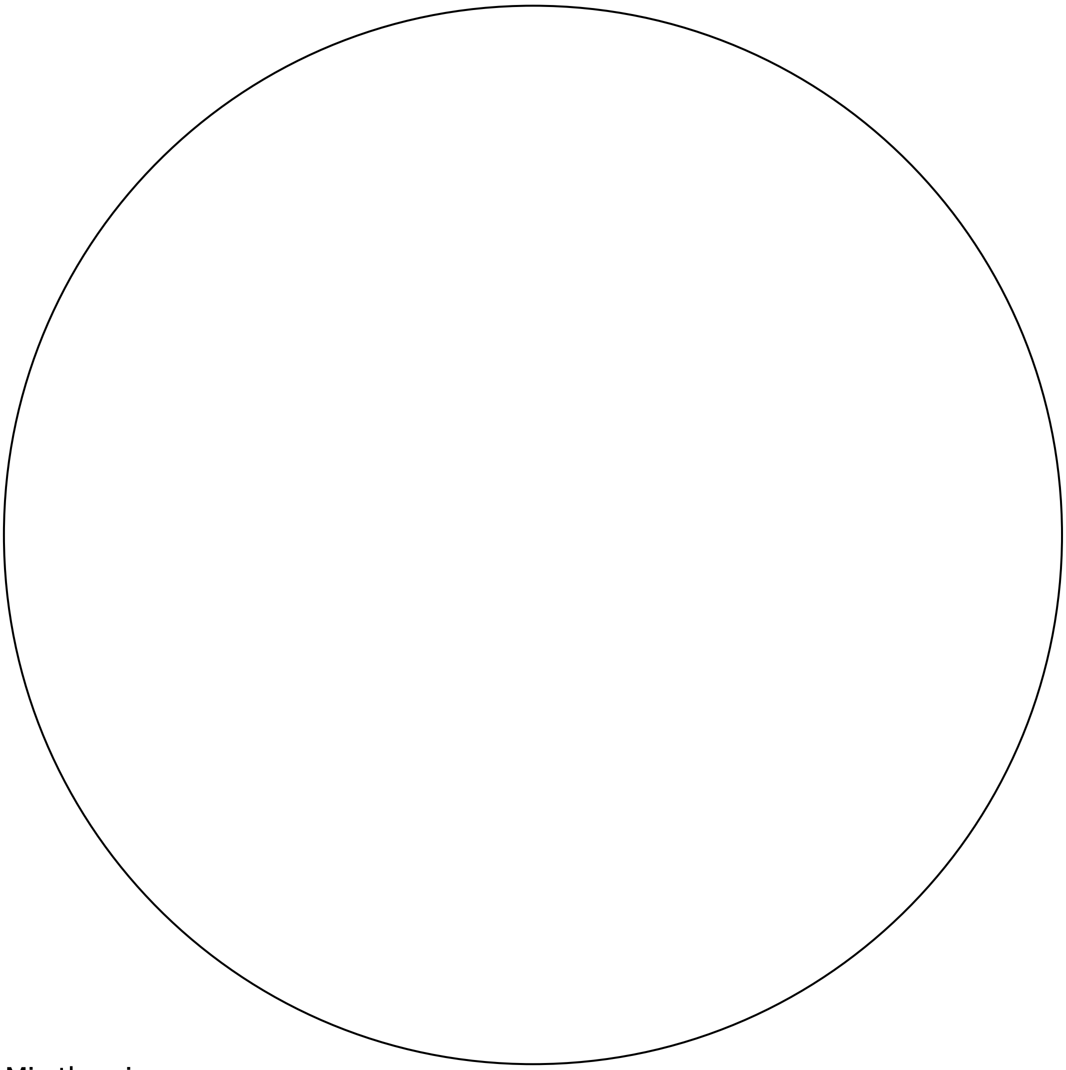
4. Did you find easy or difficult to put the pieces together to form the original shape?

5. If it was difficult, why was it?

6. Would you find it easier or more difficult to do the jigsaw puzzle if there were more pieces?

Activity 1.5 b

1. Cut the circle below into 8–10 pieces to make a jigsaw puzzle.



2. Mix the pieces.

3. Exchange jigsaw puzzles with classmates. Try to solve their puzzle.

4. Who solves the puzzle first: you or your classmate? Was it easy or difficult to solve the puzzle?

5. What aspects of any of the puzzles did you find easy or difficult?

Investigation 1.5

1. Look on the internet for circle puzzles. When you find one, make a large copy of it onto cardboard and cut it into a puzzle for others to solve.

# 2. CIRCLE FORMULAE

## 2.1 Circumference

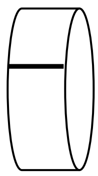
Activity 2.1 a

To find the formula for calculating the circumference of a circle, follow the procedure below:

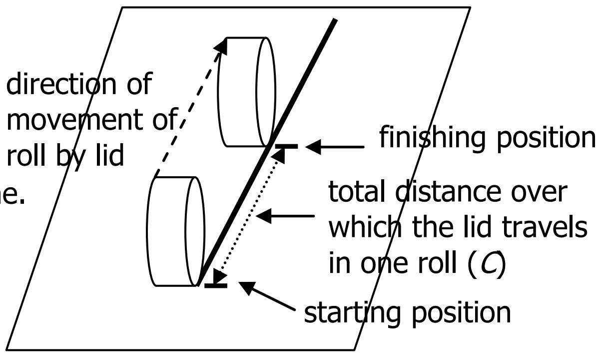
1. Find a sheet of paper, three different-sized lids, a ruler, and a pen. Label each lid **small**, **medium**, and **large**, according to their relative sizes.

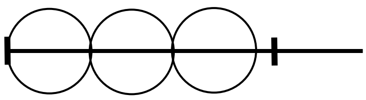
2. Use a ruler to draw three straight lines across a sheet of paper.

3. For each of the three lids in turn, mark a line on it in one place on its side (as shown in the figure at right). Turn the lid on its side, and position the lid at one end of one of the lines you drew on the paper so that the line you drew on the lid touches the paper. Roll the lid one time along the paper (as shown in Figure on right below). Mark the position where it stopped on the paper.



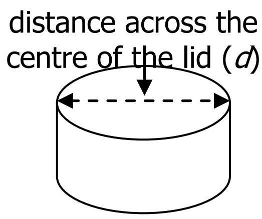
4. For each of the three lids, lay the lid flat, and see how many fit on the line (as in figure immediately below). Note there are 3-and-a-bit circles in the line.



5. Is it the same 3-and-a-bit for all the lids? What does this mean?

6. For each lid in turn, measure this distance called *C* on the line (see figure above right) and measure the distance called *d* directly across the centre of the lid from edge to edge (as shown in the figure at right). Make sure you label the lid for which *C* and *d* are calculated.




7. Write the data you collected in Step 6 on the table below (remember to include the units of length that you are using) and then divide *C* by *d* for the third column:

| **Lid** | ***C*** | ***d*** | ***C* ÷ *d*** |
|---|---|---|---|
| Small | | | |
| Medium | | | |
| Large | | | |

8. What did you get in the third column for each of the lids? Was it close to 3-and-a-bit? What does this mean?

Activity 2.1 b

From Activity 3.1 a, you should have observed that the value for $C \div d$ is equal to 3-and-a-bit. This value represents the ratio of the **circumference** ($C$) of a circle to the **diameter** ($d$) of the circle. A better approximation of this ratio is known to be 3.14 and is more formally referred to as **pi** and is symbolised using the Greek letter $\pi$.

So, we have developed the formula $C \div d = \pi$. Rearranging the formula, we obtain $C = \pi \times d$. Hence, the formula for calculating the circumference of a circle is $C = \pi \times d$. That is, to find the circumference of a circle, we multiply the diameter of the circle by $\pi$. By the way, your calculator may have a $\pi$ button. If so, then you can use this button to calculate the circumference of a circle. If not, then you can just use 3.14 in place of $\pi$.

Match each of the expressions on the left with its approximated calculation on the right. Draw a line to connect the expression with its calculation.

| | | |
|---|---|---|
| 1. | $2\pi$ | 0.52 |
| 2. | $\pi + 2$ | 1.27 |
| 3. | $\pi \div 6$ | 1.86 |
| 4. | $5 - \pi$ | 2.37 |
| 5. | $3\pi \div 4$ | 3.57 |
| 6. | $4\pi - 9$ | 5.14 |
| 7. | $\pi^2$ | 6.28 |
| 8. | $4 \div \pi$ | 9.87 |

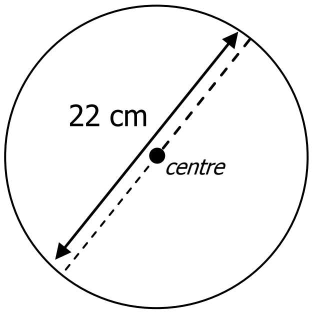
Use the formulae $d = C \div \pi$ and $C = \pi \times d$ to complete the tables below. Round your answers to the same number of places as given.

| **C** | **d** |
|---|---|
| 48 cm | |
| 3.5 m | |
| | 718 mm |
| 86 cm | |
| | 0.19 km |

| **C** | **d** |
|---|---|
| | 2.75 m |
| | 41 cm |
| 777 mm | |
| | 21 m |
| 34.8 cm | |

Activity 2.1 c




Have a look at the circle on the right.

The diameter (*d*) is 22 cm. To find the circumference of the circle, we use the formula:

$$C = \pi \times d$$

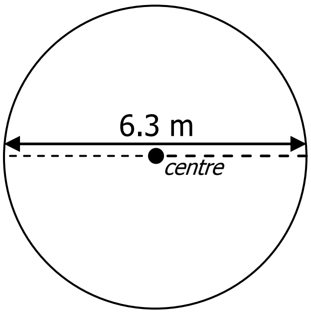
$$C = \pi \times 22 \text{ cm}$$

$$C = \underline{69.1 \text{ cm}}$$

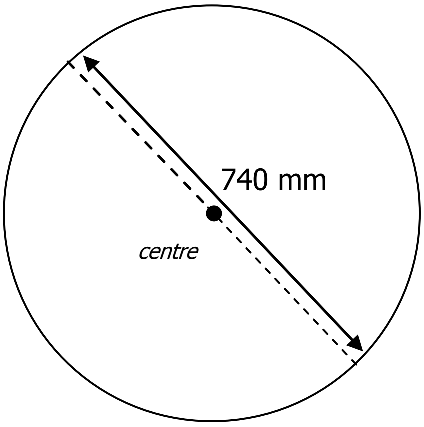
So, the circumference of the circle is 69.1 cm.

Find the circumference of each of the following circles:

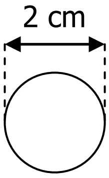
1.



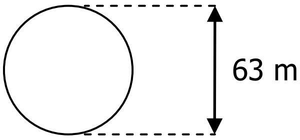

2.




3.




4.

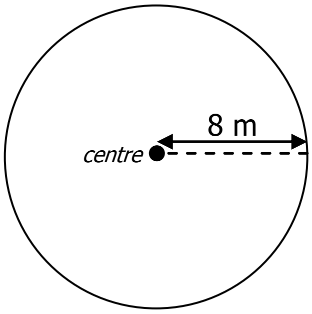



5. Make up your own circle!

Activity 2.1d

The **radius** ($r$) of a circle is the distance from the centre of the circle to any point on the edge of the circle. Because the diameter is the distance across the circle, the radius is equal to half the diameter. That is, $r = d \div 2$ which implies that $d = 2 \times r$. If we substitute $2 \times r$ for $d$ in the formula for finding the circumference of a circle, we obtain $C = \pi \times (2 \times r)$. Because we can multiply numbers in any order we like, let us write the formula in this way $C = 2 \times \pi \times r$.

Have a look at the circle at right. The radius is 8 m.




To find the circumference of the circle, we use the formula

$$C = 2 \times \pi \times r$$

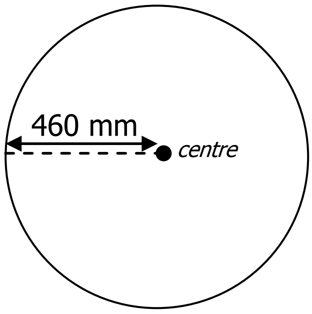
$$C = 2 \times \pi \times 8 \text{ m}$$

$$C = \underline{50.3 \text{ m}}$$

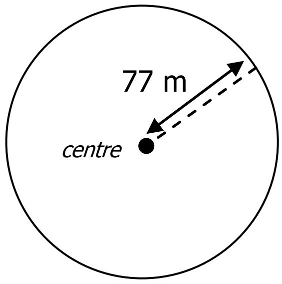
So, the circumference of the circle is 50.3 m.

Find the circumference of each of the following circles:

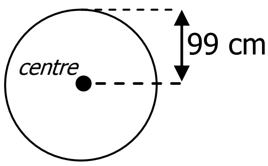
1.



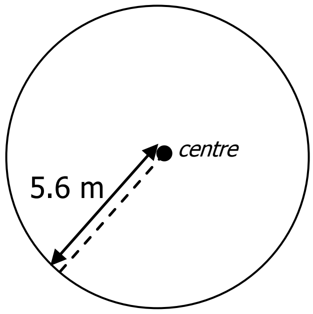

2.




3.




4.

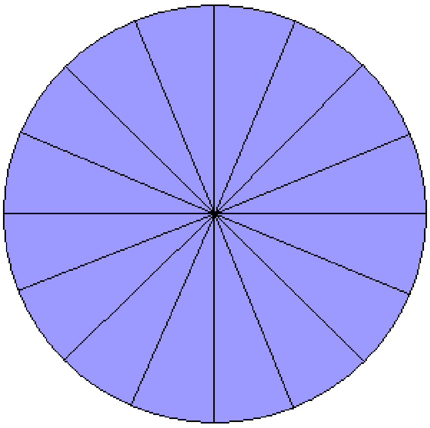



Investigation 2.1

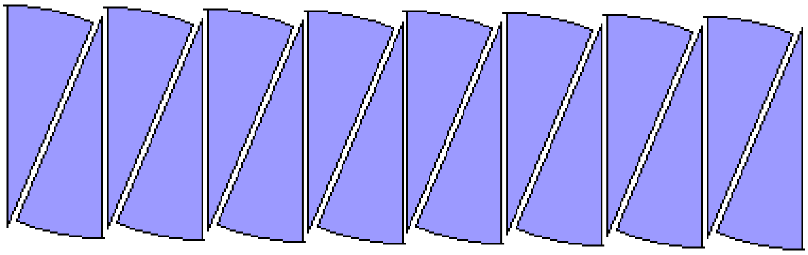
1. Look up $\pi$ or pi on the Internet. Find different places where it is used.

## 2.2 Area

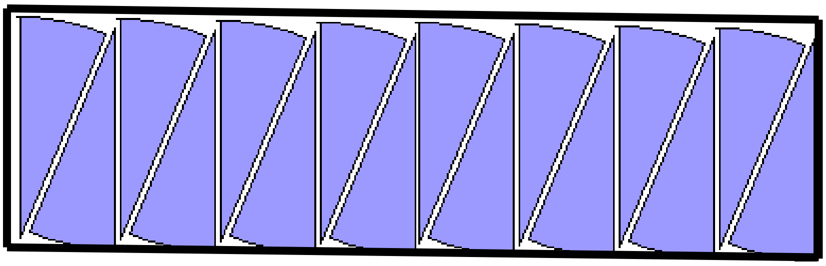
Activity 2.2a



1. Find a piece of paper, a large lid (at least 12 cm in diameter), a ruler, and scissors.
2. Measure the diameter of the lid, and calculate the radius (we shall call this $r$).
3. Place the lid firmly on the piece of the paper, and draw a circle around it. Calculate the circumference ($C = 2 \times \pi \times r$) and the area ($A = r^2 \times \pi$).
4. Draw lines across the circle to form 16 sectors that are approximately equal in area, as shown in the figure at right. (*Hint:* keep halving.)
5. Cut out the 16 sectors, as shown at right.
6. Arrange the sectors in the configuration shown below.



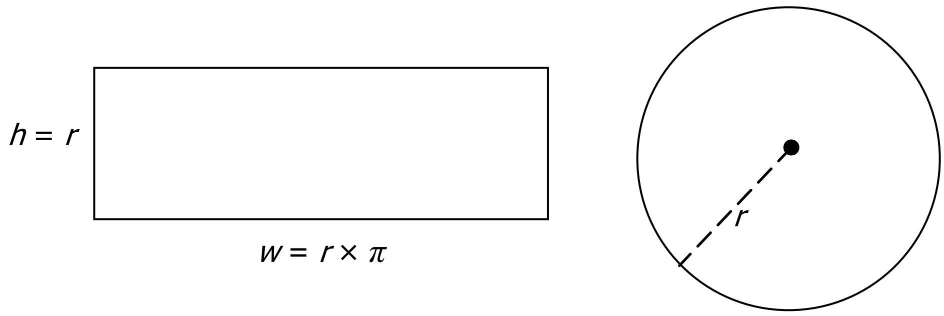
7. Note that this configuration looks very much like a rectangle.



8. What is the height? What is the length? Can you see how these relate to radius and circumference?
9. Measure the height and length of the rectangle. Calculate the area of the rectangle. Divide the length of the rectangle by the radius. Did you get an answer that is close to $\pi$? Remember that $\pi$ is approximately equal to 3.14.
10. Did you recognise that the height of the rectangle is the same as the radius of the lid? What does this mean for length?

Activity 2.2b

Since the height ($h$) of the rectangle is equal to the radius ($r$) of the circle and the width ($w$) of the rectangle is (approximately) equal to the radius ($r$) of the circle times $\pi$ (see figures below), we can develop the formula for finding the area of a circle.




To calculate the area of the rectangle, you would have used $A = h \times w$. Since $h = r$ and $w = r \times \pi$, we can substitute

$$A = h \times w$$

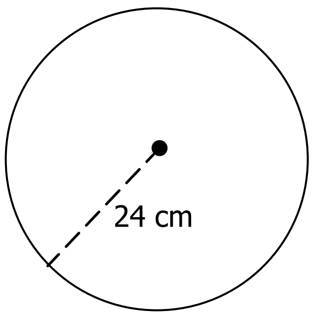
$$A = r \times r \times \pi$$

$$A = r^2 \times \pi.$$

Remember that $r^2 = r \times r$.

So, we have developed the formula for finding the area of circle, $A = r^2 \times \pi$ where $r$ is the radius of the circle. Given the circle's radius, all we have to do is multiply the radius by itself and then multiply the result by $\pi$.

Have a look at the circle at right.




We can see that $r = 24$ cm. So, to calculate the area of the circle, we have

$$A = r^2 \times \pi$$

$$A = (24 \text{ cm})^2 \times \pi$$

$$A = 24 \text{ cm} \times 24 \text{ cm} \times \pi$$

$$A = \underline{1809.6 \text{ cm}^2}$$

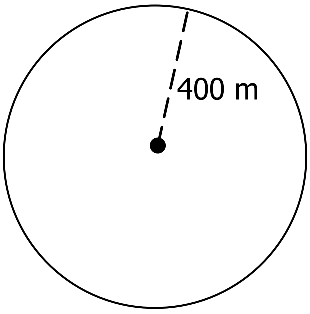
That is, the area of the circle is approximately equal to 1809.6 cm².

Remember that, if given the diameter of a circle instead, you must divide it by 2 to find the radius before using the formula for calculating the area of a circle.

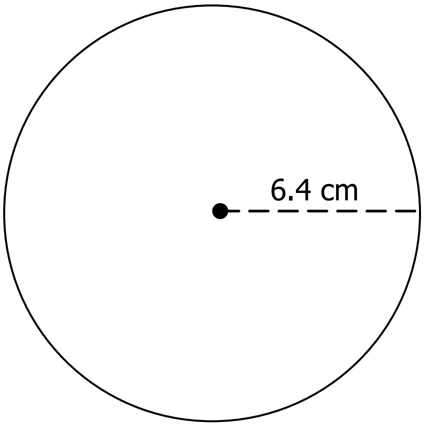
Activity 2.2b continued

Find the area of each of the following circles.

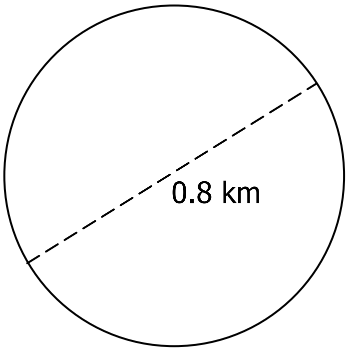
1.



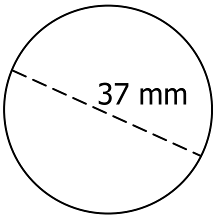

2.




3.



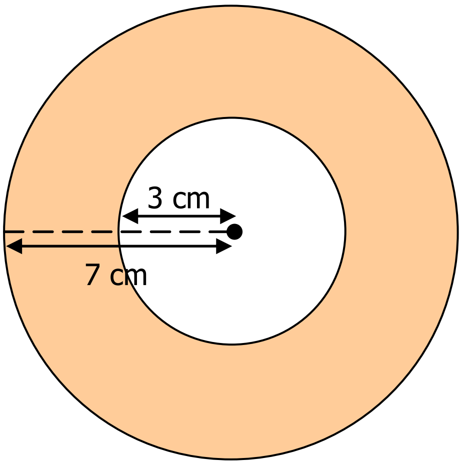

4.



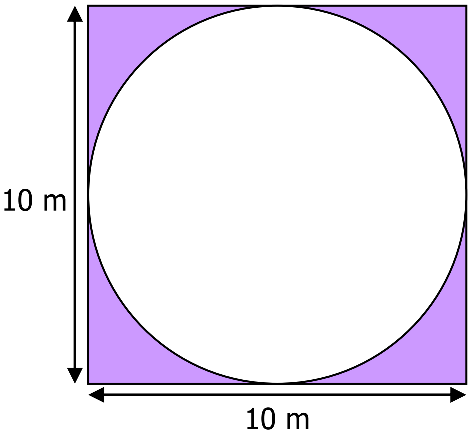

Investigation 2.2

Find the area of the shaded region in each of the following figures.

1.




2.




3. Make up your own complicated circular area and give it to a friend to solve.

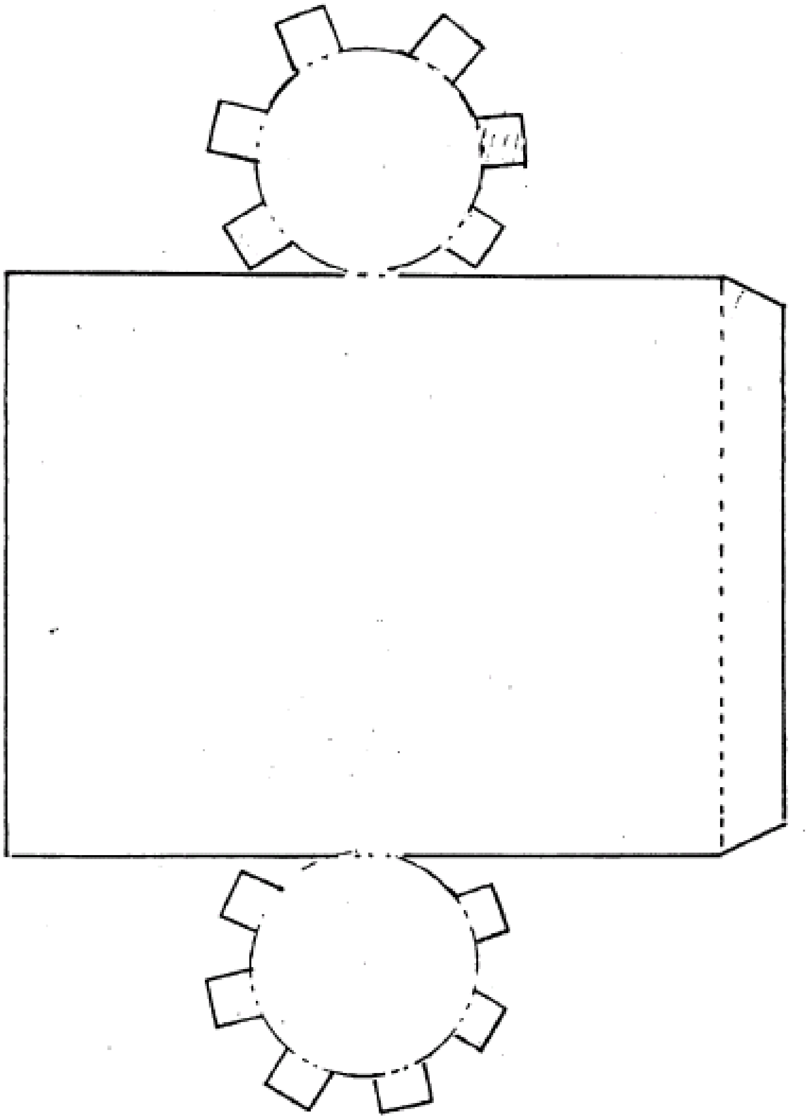
 

# 3. CIRCLE CONSTRUCTION

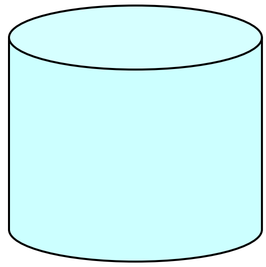
## 3.1 Cylinders

Activity 3.1a

1. Cut out the figure below along the solid lines.



2. Fold the shape along the dashed lines. Form a cylinder like the one shown at right.



3. Describe any challenges you experienced in constructing the cylinder. What could you have done to make it easier to construct the cylinder?

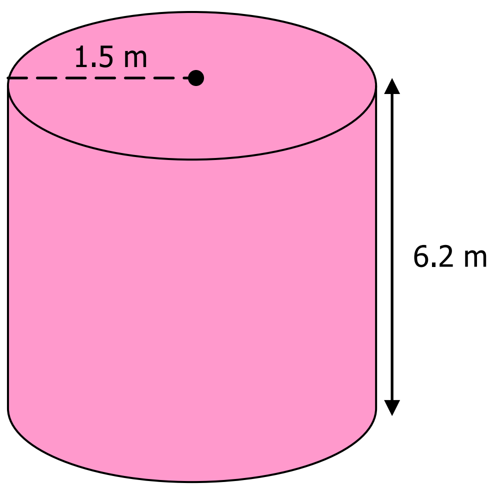
4. Name three things that are in the shape of a cylinder.

5. List some of the characteristics that make them similar. Are they symmetrical? Do they roll on the floor?

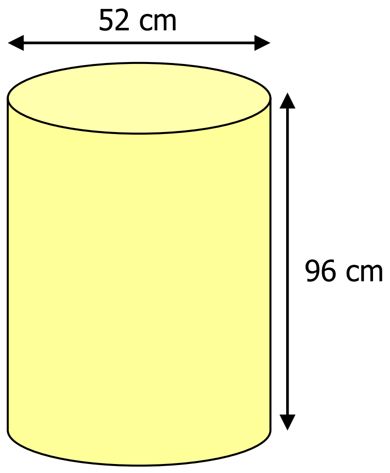
Activity 3.1b

Identify the radius (*r*) and height (*h*) in each of the following cylinders.

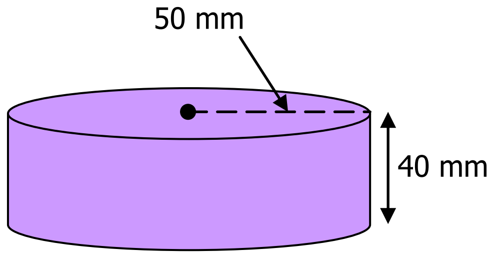
1.



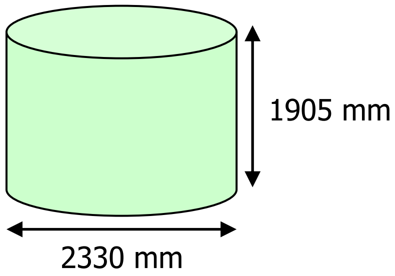

2.



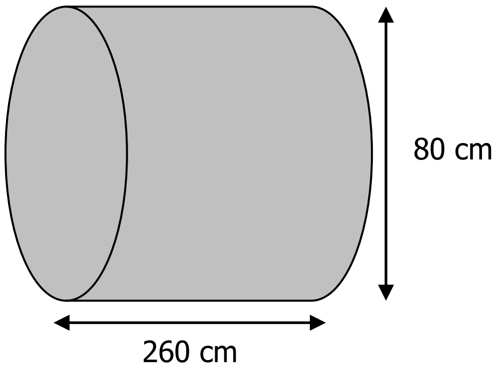

3.



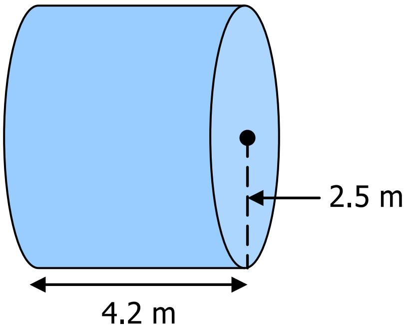

4.




5.




6.




Investigation 3.1

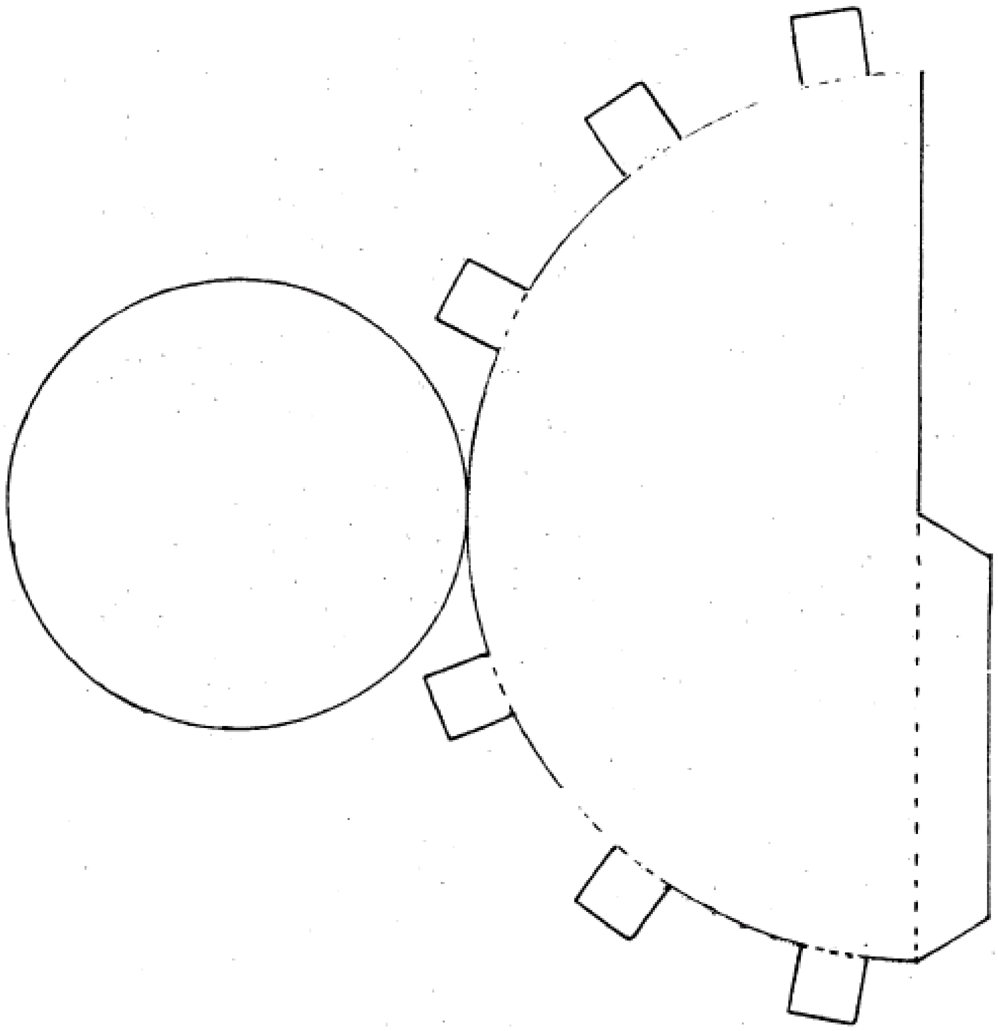
1. Why are most tanks cylinders? Why are non cylindrical tanks mostly more expensive?

2. Why are most spaceships cylinders? Why are most submarines cylinders?

## 3.2 Cones

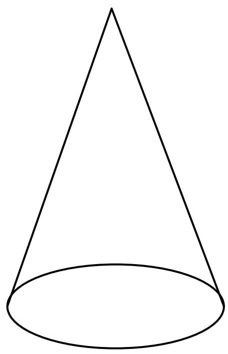
Activity 3.2a

1. Cut out the figure below along the solid lines.



2. Fold the shape along the dashed lines.

3. Form a cone like the one shown at right.



4. Describe any challenges you experienced in constructing the cone.

5. What could you have done to make it easier to construct the cone?

6. Name three things that are in the shape of a cone.

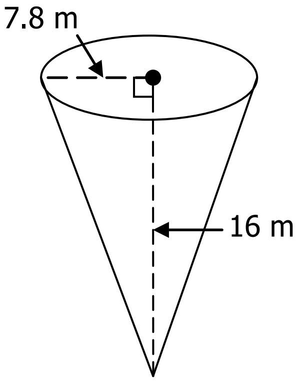
7. List some of the characteristics that make them similar.

8. Are they symmetrical? Do they roll on the floor?

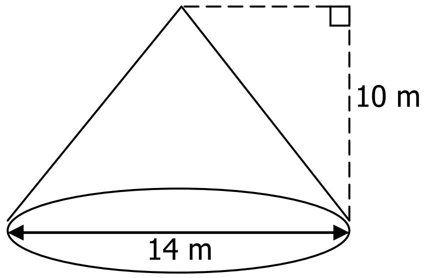
Activity 3.2b

Identify the radius (r) and height (h) of each of the following cones.

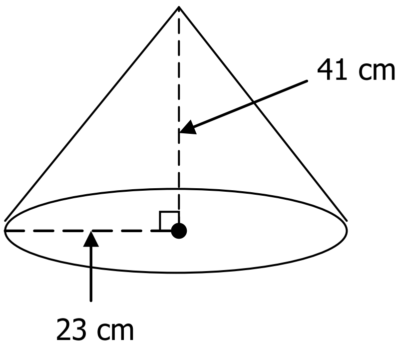
1.



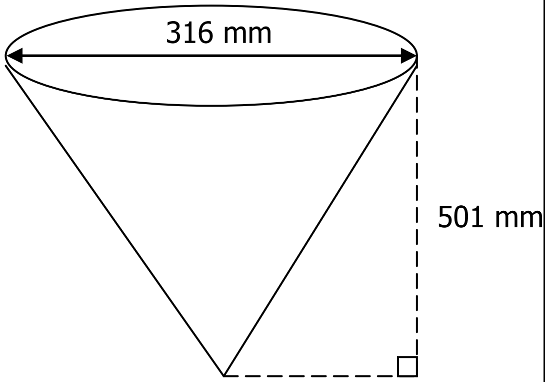

2.



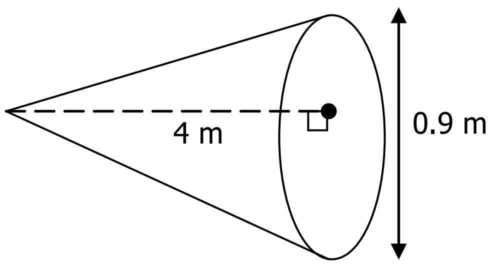

3.



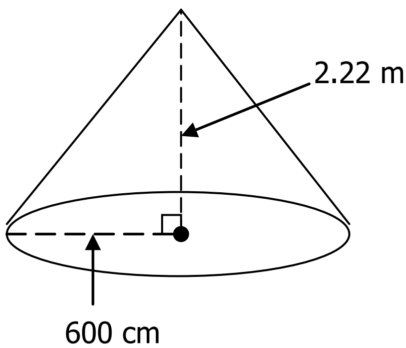

4.




5.




6.




Investigation 3.2

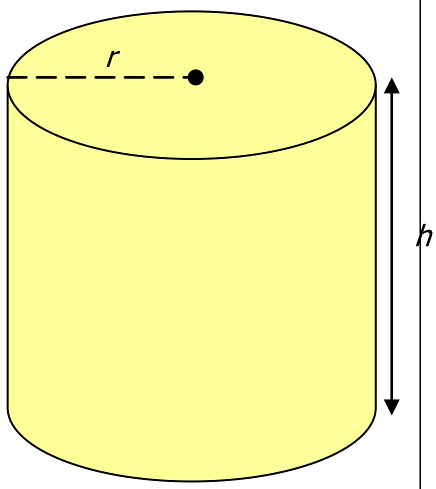
1. Find or construct a cone and cylinder of the same base and height.

2. Using rice, see how many cones can fill one cylinder (of the same base and height). What does this mean for formula of a cone?

## 3.3 Volume Formulae

Activity 3.3a

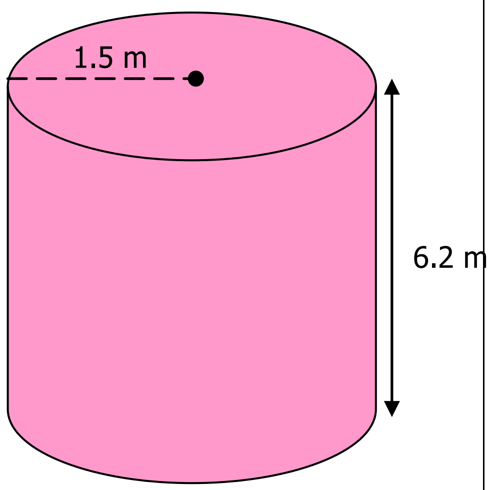
A **cylinder** is a solid with ends in the shape of a circle. The figure at right is an example of a cylinder. Note that the two dimensions of the cylinder are the radius ($r$) and height ($h$).




The formula for calculating the volume of a cylinder is easy to discover. We take the area of the base, which is a circle, and multiply it by the height of the cylinder. Since we have already learned that the area of a circle is $A = r^2 \times \pi$ where $r$ is the radius of the circle, we can derive the formula for finding the volume of a cylinder:

$$V = r^2 \times \pi \times h.$$

Have a look at the cylinder at right.




We can see that the radius ($r$) is 1.5 m and the height ($h$) is 6.2 m. The volume of the cylinder is

$$V = r^2 \times \pi \times h$$

$$V = (1.5 \text{ m})^2 \times \pi \times 6.2 \text{ m}$$

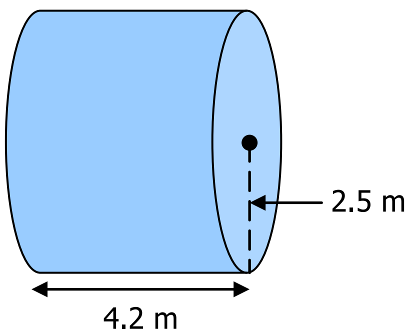
$$V = 1.5 \text{ m} \times 1.5 \text{ m} \times \pi \times 6.2 \text{ m}$$

$$V = \underline{43.8 \text{ m}^3}.$$

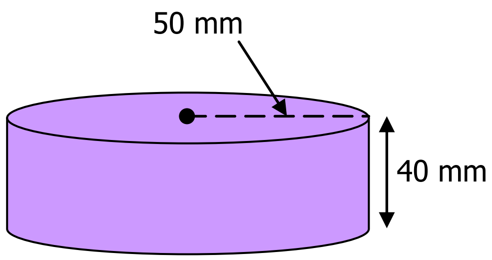
So, the volume of the cylinder is 43.8 m³.

Find the volume of each of the following cylinders.

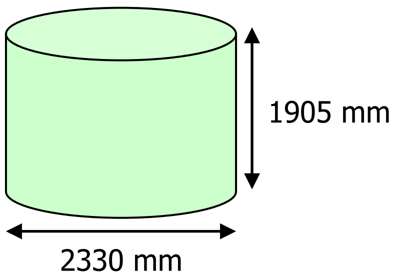
1.



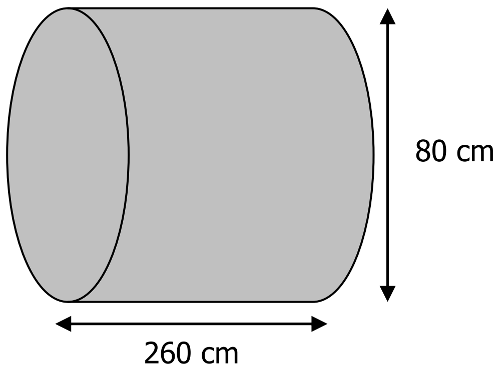

2.




3.

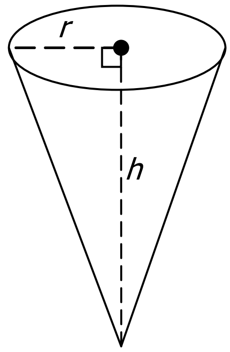



4.



Activity 3.3b

A **cone** is a solid with a circular base whose height converges at a single point, called a vertex. The figure at right is an example of a cone. Note that the two dimensions of the cone are the radius ($r$) and height ($h$).




The formula for calculating the volume of a cone is similar to the formula for calculating the volume of a cylinder, except that the volume of a cone is 1/3 the volume of a cylinder. So, the formula for calculating the volume of a cone is $V = r^2 \times \pi \times h \div 3$ where $r$ is the radius of the circular base and $h$ is the height.

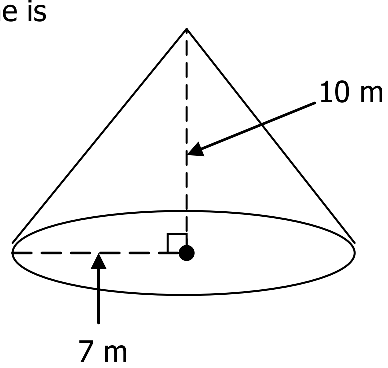
Have a look at the cone at right. We see that the radius ($r$) of the cone's base is 7 m and the height is 10 m. Therefore, the volume ($V$) of the cone is

$$V = r^2 \times \pi \times h \div 3$$

$$V = (7\text{ m})^2 \times \pi \times 10\text{ m} \div 3$$

$$V = 7\text{ m} \times 7\text{ m} \times \pi \times 10\text{ m} \div 3$$

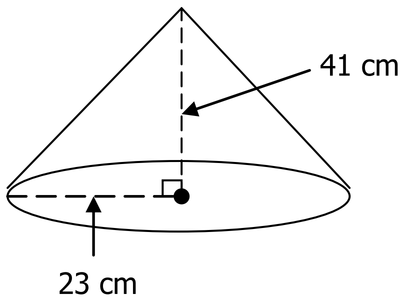
$$V = \underline{513\text{ m}^3}.$$



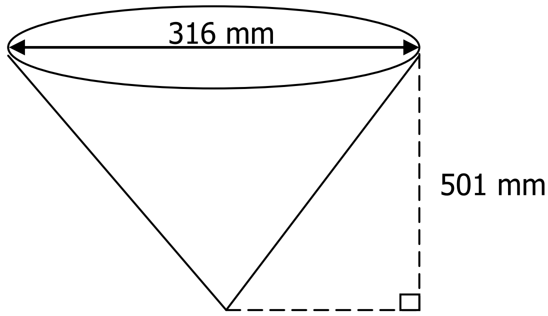

Hence, the volume of the cone is 513 m³.

Find the volume of each of the following cones.

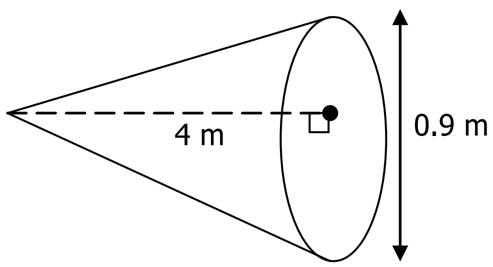
1.



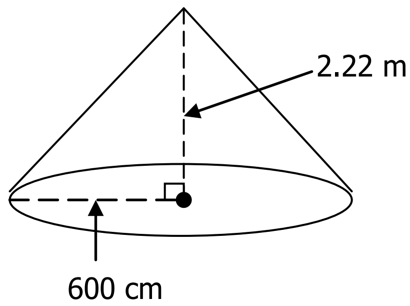

2.




3.




4.




Investigation 3.3

1. Where do we use cones in our every day life? E.g., for icecream? Why is a cone a useful shapes for these purposes?

2. Which culture builds cone shaped buildings? Why?

# 4. CIRCLE RELATIONSHIPS

## 4.1 Area vs Perimeter

Activity 4.1a

1. Obtain some cm graph paper (see attached at back of this booklet).

3. Cut string and join it so that it makes a perimeter of 24 cm.

4. Place the string on the cm graph paper in the shape of a L, a rectangle, a square, a triangle and a circle and calculate the area. Count squares more than ½ in and ignore squares less that ½ in.

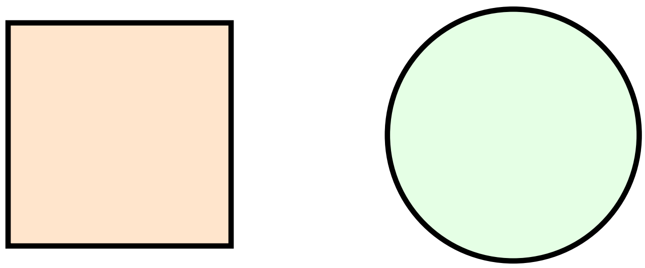
5. Writer down the results on a table

| L | RECTANGLE | SQUARE | TRIANGLE | CIRCLE |
|---|---|---|---|---|
| | | | | |

6. Which has the highest area. What does this mean?

Activity 4.1b

1. Let's be more exact than 5.1a. The square and circle below have the same perimeter. Find the perimeter, using each grid cell as 1 unit of measure.



2. Count the number of grid cells enclosed by the square. This is the area of the square.

3. Count the number of whole grid cells enclosed by the circle. Estimate the percentage of area that each part grid cell covers and add all the percentages together. Add this to the number of whole cells. This is the area of the circle.

4. Compare your answers for the area of the square and the area of the circle from Questions 2. and 3. What does this mean?

5. Let's be more accurate. Use a ruler to calculate the length of each side of the square. Calculate the area of the square. Use a ruler to calculate the diameter of the circle. Calculate the area of the circle, using the formula.

6. Compare the calculated area of the square and the calculated area of the circle from Questions 7. and 8. Is the result similar to Question 4. Again, what does this mean.

7. What do you understand to be the difference between the area of a rectangle and circle with the same perimeter? Will this always be true? Check it with some other squares and circles.

8. Check it by starting with a circle and square of the same area and calculate the perimeter and circumference?

<u>Investigation 4.1</u>

1. Why are square houses the cheapest houses? Check this by looking at some advertisement for houses?

2. Would the findings of Activities 5.1a and b mean that a round house would be cheaper than a square house?

3. What would be the advantages and disadvantages of such a house?

4. Design a house with round walls and cone like roofs – be creative?

## 4.2 Surface Area vs Volume

Activity 4.2a

1. Construct a rectangular prism, a cube ,a cylinder and a sphere of the same volume out of plasticine. Do this by weighing out the same amount of plasticine for each solid and moulding it into the shapes as best you can. Don't make your cylinder long.

2. Now cover the four shapes with paper as best you can (the sphere is very difficult) so there is very little scrunching or overlap of the pieces of paper. Remove the paper and use cm graph paper to calculate the area of coverage of the solid shapes. This is called Surface area.

3. Compare the shapes and the areas with a table

| | RECT PRISM | CUBE | CYLINDER | SPHERE |
|---|---|---|---|---|
| VOLUME | Same | Same | Same | Same |
| AREA | | | | |

4. Does the rectangular prism have a greater surface area than the cube? Does the cube have a greater surface area than the cylinder? What about the sphere?

5. Could you make a cylinder with less surface area?

6. What does all this mean?

Activity 4.2b

1. Blow up a balloon. Cover it with paper mache (newspaper and glue). Let it dry.

2. Burst the balloon. Does it hold a lot of rice (make a whole in the top)? Why?

Investigation 4.2

1. A circular house has more space for less wall than any other shape – so it should be the cheapest form of house? So what about a sphere in terms of enclosing volume?

2. For example, is a sphere the best form of space ship if we want to fit a lot of people in?

## 4.3 Shape vs strength

Activity 4.3

1. Take the paper mache sphere from Activity 5.2b. Is it strong?

2. Why are fruits generally in the shape of spheres? Why are eggs in the shape of spheres? Why are emu eggs so strong? Why is the submarine that really goes deep sphere like?

Investigation 4.3

1. Use the internet to investigate the strength of circles, cylinders, cones and spheres.

2. Investigate the materials that are made with circular shapes. Do they cover most things which are pressurised? For example, gas bottles?

3. Look at pressure pack cans (of, say, flyspray) – how do they incorporate circles and spheres?

4. Why are the earth, moon, and sun spheres? Why does spinning clay give a circular result?

## 4.4 Circle metaphors

Investigation 4.4

1. Investigate the role of circles as metaphors in different cultures. Use the Internet. Talk to people.

2. Look at these metaphors – "circle of life", "circle or wheel of time", "circle of friends", "coming full circle", and so on. What does circle men for them? How/why is the circle related to ecology, balance, recycling?

3. Look at circle in other cultures, e.g., Ying and Yang in Chinese culture.

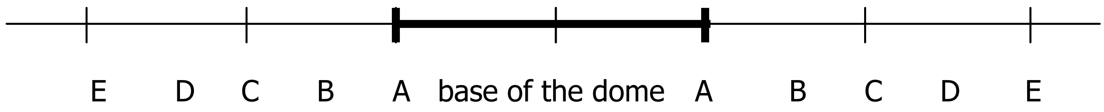
4. Look at circle in your local Indigenous culture – investigate use of circle and semicircle, either with dots or as firm lines. Are there local meanings of the circle?

# 5. DOMES

## 5.1 Constructing domes

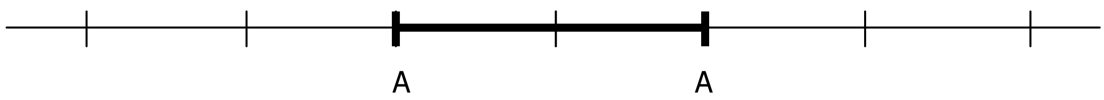
Activity5.1

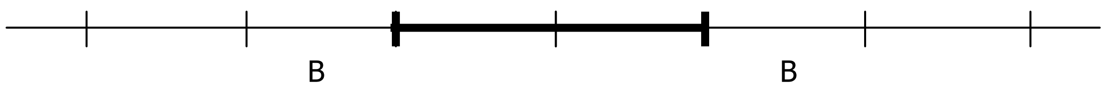
1. Domes can be explored with a line and a compass. Draw the following line:

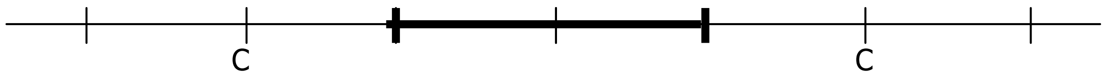


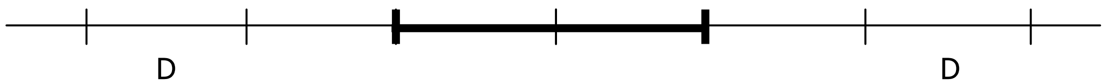

2. Consider that this goes through the middle of a circular house with a dome top. The dome changes as the point from which the circle is drawn moves away from the centre.

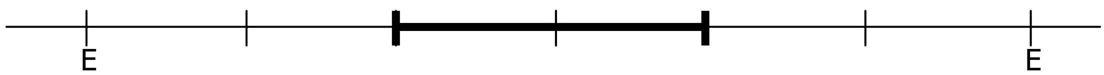
3. Draw a dome with the point from which the circle is drawn at A, B, C and D






4. Discuss the differences with respect to the domes made at each point – which is steeper which is more shallow?

## 5.2 Dome strength

Activity 5.2

1. Take a large copy of Dome C from Activity 6.1 and turn it upside down. Obtain a chain and hang it upside down from two points with the same width as the base of the dome – superimpose the chain over the upside down chain. Lengthen the chain so that its end starts to approach the bottom (what was the top) of the upside down dome

2. What do you see? Compare the shape of the C dome against the shape taken by the chain.

3. What does this mean?

Investigation 5.1 and 5.2

1. Look in the Internet for domes. What types are there? Where are they used? Why were they used? Investigate their advantages and disadvantages.

2. Look in the Internet for the relationship between how chains & chain nets fall and the strength of roofs (when these chain shapes are turned upside down as roofs).