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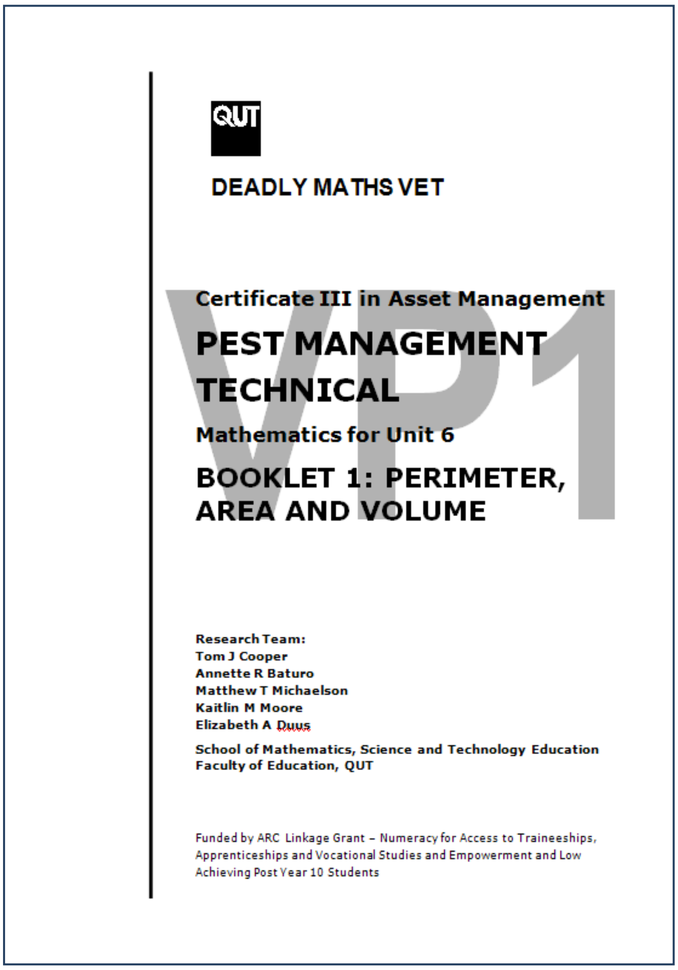
Certificate III in Asset Management
Pest Management Technical
Booklet VP1: Perimeter, Area and Volume



DEADLY MATHS VET

Certificate III in Asset Management

PEST MANAGEMENT TECHNICAL

Mathematics for Unit 6

BOOKLET 1: PERIMETER, AREA AND VOLUME

VP1

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School of Mathematics, Science and Technology Education
Faculty of Education, QUT

Funded by ARC Linkage Grant – Numeracy for Access to Traineeships, Apprenticeships and Vocational Studies and Empowerment and Low Achieving Post Year 10 Students

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Islander's Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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CRICOS No. 00213J

This booklet was developed as part of a project which ran from 2005–2008 and was funded by an Australian Research Council Linkage grant, LP0455667: *Numeracy for access to traineeships and apprenticeships, vocational studies, empowerment and low-achieving post Year 10 students.*

Queensland University of Technology

DEADLY MATHS VET

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PEST MANAGEMENT TECHNICAL

MATHEMATICS FOR UNIT 6

BOOKLET VP1

PERIMETER, AREA AND VOLUME

DRAFT 1: 16/4/08

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Deadly Maths Group

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BACKGROUND

This booklet (Booklet VP1) was the first booklet produced as material to trial with pesticide students completing Unit 6 as part of the Open Learning Institute of TAFE course Certificate III in Asset Management (Pest Management – Technical).

This trial was part of the ARC Linkage project LP0455667: *Numeracy for Access to Traineeships, Apprenticeships and Vocational Studies and Empowerment and Low Achieving Post Year 10 Students*. This project studied better ways to teach mathematics to VET students at Bundamba State Secondary College, Gold Coast Institute of TAFE, Metropolitan South Institute of TAFE, Open Learning Institute of TAFE, and Tropical North Queensland Institute of TAFE (Thursday Island Campus).

This booklet focuses on how to teach formulae for perimeter, area and volume as required for Unit 6. It leads onto Booklet VP2 which looks at ratio and proportion for mixing chemicals for spraying.

The chief investigators of this project were Dr Annette Baturo and Professor Tom Cooper. The research assistants were Matthew T Michaelson, Kaitlin Moore, and Elizabeth Duus. The project manager was Gillian Farrington.

CONTENTS

PREAMBLE

The Certificate III in Asset Maintenance (Pest Management – Technical) is a new course to train pesticide sprayers. It contains two units, 6 and 10, which focus on using proportion to mix the required chemicals for a spraying job.

Mixing is based on calculating correct amount of chemical for mixing with water to cover and/or fill perimeters, areas, and volumes for simple plane shapes (e.g. rectangles and circles), solids (e.g. prisms, pyramids, cylinders, and cones) and for more complex shapes and solids that are combinations of these shapes and solids.

Formulae for perimeter are straightforward, simply being reductions of distance around something, except for the circle which is 3-and-a-bit (pi, which is abbreviated using the symbol π) × diameter. Formulae for area and volume are related to each other and the array meaning of multiplication: the area of a rectangle can be seen to be length × width; the area of parallelogram is the same as this; the area of a triangle is half of this; the area of a circle can be related to a rectangle radius wide and π × radius long. Volume of prisms and cylinders is simply area of base × height, while pyramids and cones are one-third of this.

Unit 6 provides formulae for perimeters, areas, and volumes commonly found where spraying chemicals. This booklet provides information on how formulae for perimeters, areas and volumes may be understood and developed and assists memorisation.

This booklet may, therefore, be of use to you in the course for two reasons. First, it will provide understanding that lies behind the formulae. Second, it will help you remember the formulae because when a rule or process is simply learnt by rote, it can be easily forgotten and difficult to apply. Remembering the method and transferring your knowledge to new labels are easier when you understand how and why things are done as they are.

The focus of this booklet is, therefore, to look at the mathematics behind perimeter, area, and volume with understanding. This is done through focusing on:

1. providing an overview of the mathematics behind perimeter, area, and volume;
2. giving methods for developing and understanding formulae for the perimeter of rectangles, triangles, and circles;
3. giving methods for developing and understanding formulae for the area of rectangles, parallelograms, triangles, and circles;
4. giving methods for developing and understanding formulae for the volume of prisms and cylinders;
5. giving methods for developing and understanding formulae for the volume of pyramids and cones;
6. providing the strategy for finding the area and volume of combinations of shapes and solids.

1. OVERVIEW

Unit 6 of the Certificate III in Asset Maintenance: PEST MANAGEMENT – TECHNICAL spends section 8.1 looking at the calculations the students will need to make with regard to Perimeter, Area and Volume. This booklet provides more detailed information and preparatory activities for understanding the formulae that have to be used to calculate area and volume that will determine the amount of chemical to be sprayed.

The booklet covers: (1) perimeter for rectangles, squares, triangles, regular polygons and circles, (2) area for rectangles, parallelograms, triangles and circles; and (3) volume for rectangular and triangular prisms and pyramids and cones and cylinders. It also covers perimeter, area and volumes for combination shapes.

The Certificate III booklet lists the formulae, so this booklet attempts to build understanding of why the formulae is as it is. It does this two ways:

(1) by discovering formulae – for example, perimeter of polygons, circumference of circle, area of rectangle, and volume of prisms; and

(2) by relating other formulae to already known formulae – for example, parallelogram is same as rectangle, triangle is half rectangle, circle is a rectangle r by $\pi \times r$, volume of prism is area of polygon base × height, and pyramids and cones are 1/3 of prisms and cylinders.

Relating formulae is achieved by activity with materials:

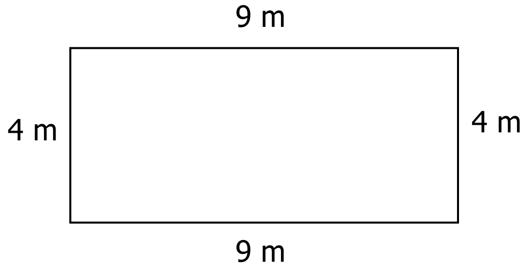
- cutting and pasting paper copies of shapes to show relation to known shapes;
- cutting and rejoining parts of a shape to form something similar to a known shape;
- building up solid shapes from their bases; and
- pouring rice from one solid shape to a known solid shape to see how many of the one shape fills the other.

2. PERIMETER

2.1 Perimeter of the rectangle

The **perimeter** of a shape is the total distance around it. In other words, the perimeter is the distance over which a fence or wall would need to be erected to fully enclose the shape. For most shapes, calculating the perimeter is easy, because the only operation involved is addition.

For example, have a look at the rectangle at right.

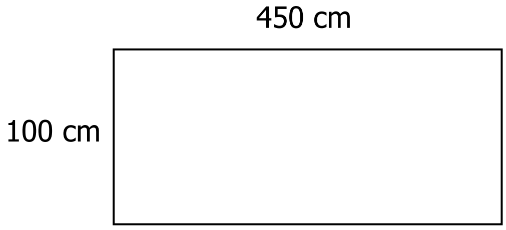


The total distance around the rectangle is simply the result of adding all of the sides together:

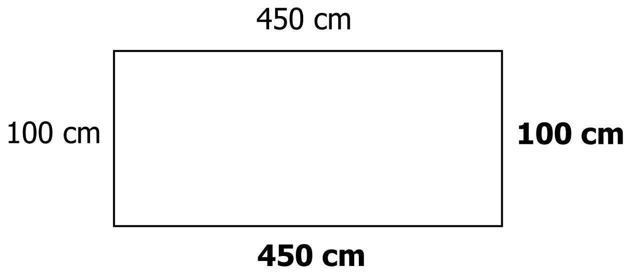
9 m + 4 m + 9 m + 4 m = 26 m.

Remember that a rectangle will always have opposite sides that are of the same length. So, if given a rectangle with the lengths of only two sides marked, we automatically know the lengths of the opposite sides.

For example, have a look at the rectangle at right.



We know what the lengths of the opposite sides are: the side length opposite the 450 cm must be 450 cm, and the side length opposite the 100 cm must be 100 cm. So, we can add these lengths to the drawing so that we do not forget what they are.

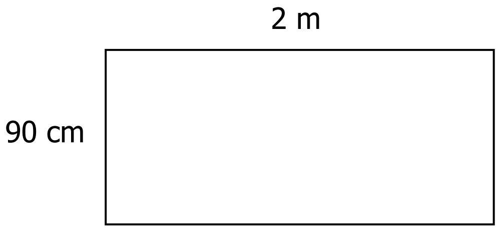


Now, the total distance of this rectangle is simply:

450 cm + 100 cm + 450 cm + 100 cm = 1100 cm.

Note that, because you can add numbers in whatever order you like, you can start adding from any side of the rectangle that you choose.

We have also to take account of different units as on right. What is the perimeter? What are the units, m or cm? Does this make sense?



90 + 2 + 90 + 2 = 184 ??

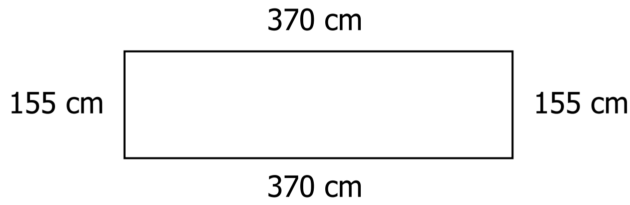
We have to change to the same units. Here, the best way is to change the 2 m to cm (200 cm). This makes the perimeter:

90 cm + 200 cm + 90 cm + 200 cm = 580 cm

Activity 2.1a

Find the perimeter of each of the following rectangles. Make sure that all of your answers include the correct units of length (e.g., m for metres, cm for centimetres).

1.



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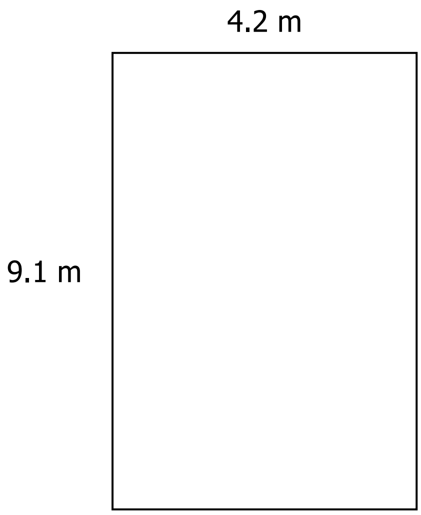

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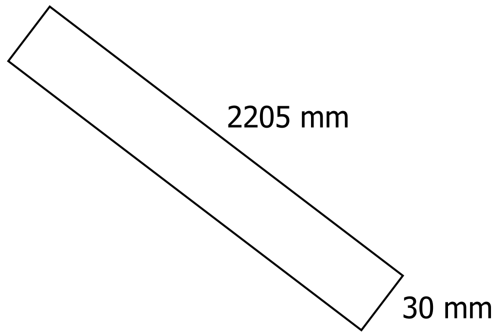

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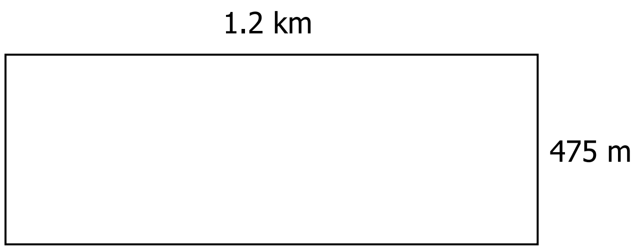
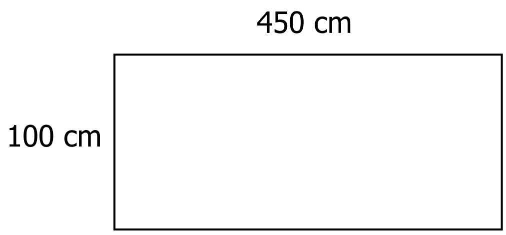


Remember that, in order to be able to add quantities together, they **must** be in the same units.

Because the lengths of opposite sides of a rectangle are always the same, we can find a quicker way to calculate the perimeter of rectangles.



Have another look at the rectangle at right.

Recall that the solution to finding the perimeter of this rectangle was:

450 cm + 100 cm + 450 cm + 100 cm = 1100 cm.

If we rearrange the order in which we add the side lengths of the rectangle, we get:

450 cm + 450 cm + 100 cm + 100 cm = 1100 cm.

Remember that we can add numbers in whatever order we like and we get the same answer.

Now, if we group our addition like this,

(450 cm + 450 cm) + (100 cm + 100 cm)

= 900 cm + 200 cm,

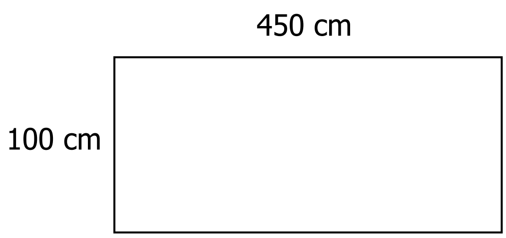
= 1100 cm

then we can see that the result of adding each group is the same as doubling each length and then adding them together.

So, we have now found another way to calculate the perimeter, P, of rectangles:

$$P = 2 \times a + 2 \times b$$

where a and b are the lengths of the rectangle.



Back to this rectangle again. If we use the formula, then we see that a = 450 cm and b = 100 cm (or a = 100 cm and b = 450 cm, because the order of addition does not matter). So,

$$P = 2 \times a + 2 \times b$$

$$P = 2 \times 450 + 2 \times 100$$

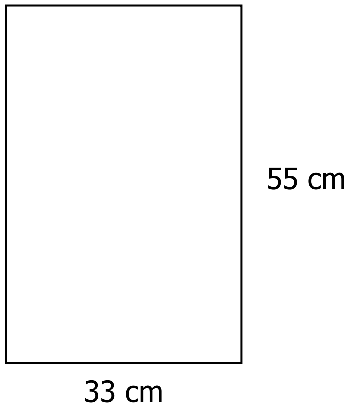
$$P = 900 + 200$$

$$P = 1100 \text{ cm}$$

Activity 2.1b

Use the new perimeter formula to find the perimeter of each of the following rectangles. Remember to show all of your working, as in the example on the previous page.

1.



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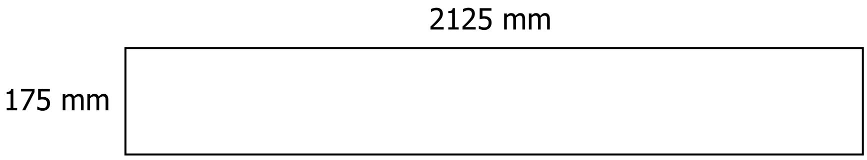

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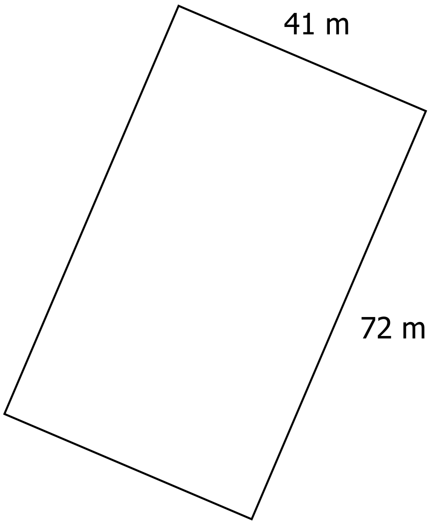

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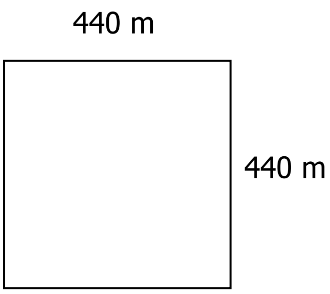
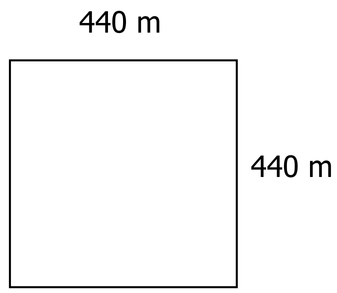


2.2 Perimeter of the square

Have another look at the last rectangle in the previous activity (also shown at right). Your solution should have looked something like this:



$$a = 440 \text{ m},\ b = 440 \text{ m}$$

$$P = 2 \times a + 2 \times b$$

$$P = 2 \times 440 + 2 \times 440$$

$$P = 880 + 880$$

$$P = 1760 \text{ m}$$

You may have noticed that, in the second to last step, you added 880 and 880, which is the same as doubling 880. So, in effect, what you ended up doing to find the perimeter of this rectangle was doubling 440 and then doubling the result. Doubling twice is the same thing as multiplying by 4, because $2 \times 2 = 4$. Therefore, in this specific type of rectangle, in which the two known side lengths are equal, all you need to do is multiply one side length by 4:

$$a = 440 \text{ m}$$

$$P = 4 \times a$$

$$P = 4 \times 440$$

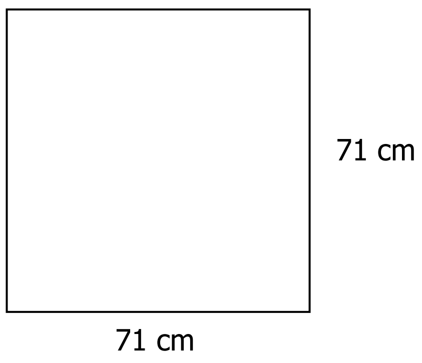
$$P = 1760 \text{ m}$$

When the two known side lengths of a rectangle are equal, the rectangle is also called a **square**. A square is a rectangle with equal side lengths. Since a square has four sides of equal length, it makes sense that the perimeter would equal 4 times the side length.

<u>Activity 2.2</u>

Use $P = 4 \times a$ to find the perimeter of each of the following squares.

1.



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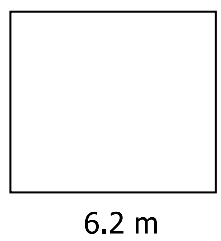
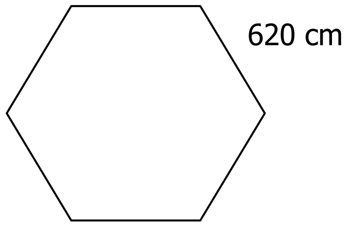


2.3 Perimeter of other regular shapes

Now that we have found the perimeter of rectangles and squares, it is time to have a look at some other regular shapes. A **regular shape** is a shape for which all the sides have the same length. Remember that the perimeter of a shape is just the distance around it. All you need to do to find the perimeter of a regular shape is to multiply the length of any side by the total number of sides.

Have a look at the regular shape at right. Because it has six sides of equal length, the shape is called a **regular hexagon**. To find the perimeter of the regular hexagon, we need to multiply the number of sides by the length of one side:



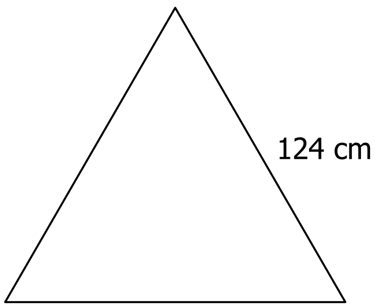
6 sides × 620 cm = <u>3720 cm</u>.

So, the perimeter of the regular hexagon is 3720 cm.

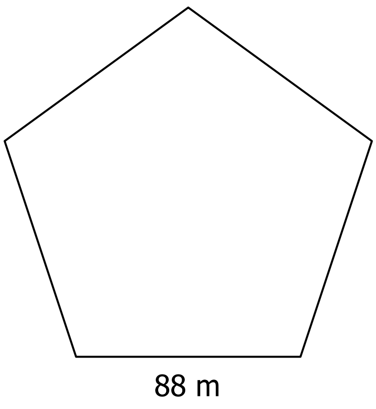
<u>Activity 2.3</u>

Find the perimeter of each of the following regular shapes.

1.



2.



2.4 Perimeter of the circle

The perimeter of a circle has a special name, the **circumference**. The circumference of a circle is still the distance around the circle or the distance over which a fence would need to be erected to fully enclose the circle. The reason why a special name is used for a circle is because calculating the circumference of a circle requires a special formula. Let's find this formula.

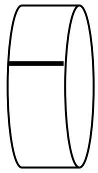
Activity 2.4a

To find the formula for calculating the circumference of a circle, follow the procedure below:

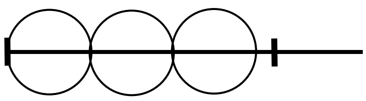
1. Find a sheet of paper, three different-sized lids, a ruler, and a pen. Label each lid **small**, **medium**, and **large**, according to their relative sizes.

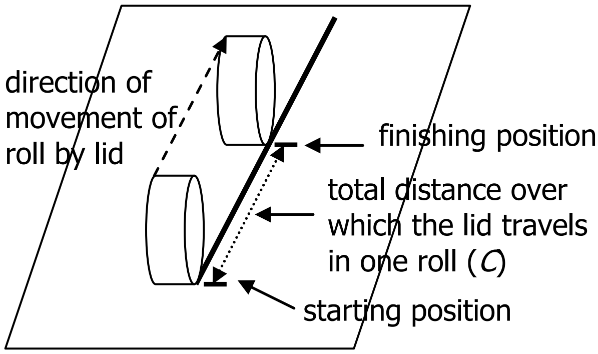
2. Use a ruler to draw three straight lines across a sheet of paper.

3. For each of the three lids in turn, mark a line on it in one place on its side (as shown in the figure at right). Turn the lid on its side, and position the lid at one end of one of the lines you drew on the paper so that the line you drew on the lid touches the paper. Roll the lid one time along the paper (as shown in Figure on right below). Mark the position where it stopped on the paper.



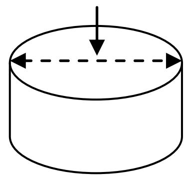
4. For each of the three lids, lay the lid flat and see how many fit on the line (as in figure on left below). Note there are 3 and a bit circles in the line.





5. Is it the same 3 and a bit for all the lids? What does this mean?

6. For each lid in turn, measure this distance called *C* on the line (see figure above right) and measure the distance called *d* directly across the centre of the lid from edge to edge (as shown in the figure at right). Make sure you label the lid for which *C* and *d* are calculated.



7. Write the data you collected in Step 6 on the table below (remember to include the units of length that you are using) and then divide *C* by *d* for the third column:

Lid	*C*	*d*	$C \div d$
Small			
Medium			
Large			

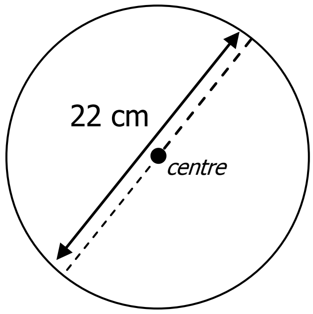
8. What did you get in the third column for each of the lids? Was it close to 3 and a bit? What does this mean?

You should have observed that the value for $C \div d$ is equal to 3-and-a-bit. This value represents the ratio of the **circumference** (C) of a circle to the **diameter** (d) of the circle. A better approximation of this ratio is known to be 3.14 and is more formally referred to as **pi** and is symbolised using the Greek letter π.

So, we have developed the formula $C \div d = \pi$. Rearranging the formula, we obtain $C = \pi \times d$. Hence, the formula for calculating the circumference of a circle is $C = \pi \times d$. That is, to find the circumference of a circle, we multiply the diameter of the circle by π.

By the way, your calculator may have a π button. If so, then you can use this button to calculate the circumference of a circle. If not, then you can just use 3.14 in place of π.

Have a look at the circle at right.



The diameter (d) is 22 cm. To find the circumference of the circle, we use the formula:

$$C = \pi \times d$$

$$C = \pi \times 22 \text{ cm}$$

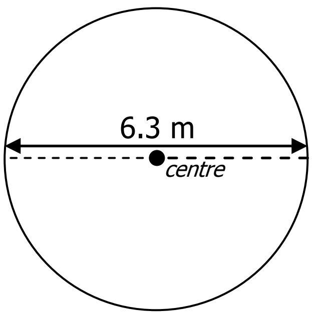
$$C = \underline{69.1 \text{ cm}}$$

So, the circumference of the circle is 69.1 cm.

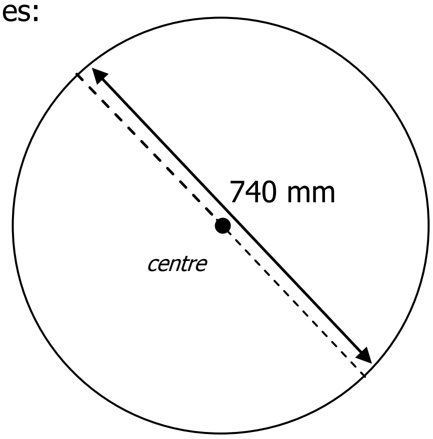
Activity 2.4b

Find the circumference of each of the following circles:

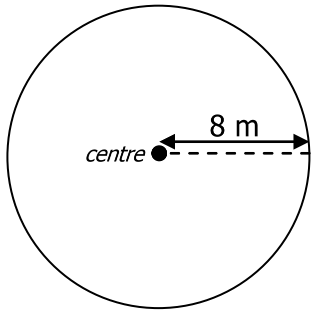
1.



2.



The **radius** (r) of a circle is the distance from the centre of the circle to any point on the edge of the circle. Because the diameter is the distance across the circle, the radius is equal to half the diameter. That is, $r = d \div 2$ which implies that $d = 2 \times r$. If we substitute $2 \times r$ for d in the formula for finding the circumference of a circle, we obtain $C = \pi \times (2 \times r)$. Because we can multiply numbers in any order we like, let us write the formula is this way $C = 2 \times \pi \times r$.



Have a look at the circle at right. The radius is 8 m.
To find the circumference of the circle, we use the formula

$$C = 2 \times \pi \times r$$

$$C = 2 \times \pi \times 8 \text{ m}$$

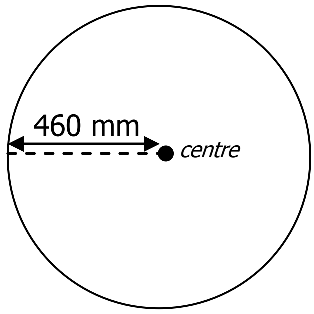
$$C = \underline{50.3 \text{ m}}$$

So, the circumference of the circle is 50.3 m.

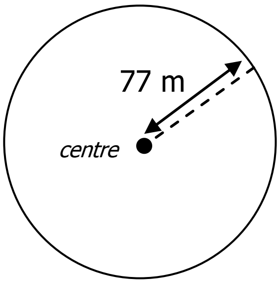
Activity 2.4c

Find the circumference of each of the following circles:

1.



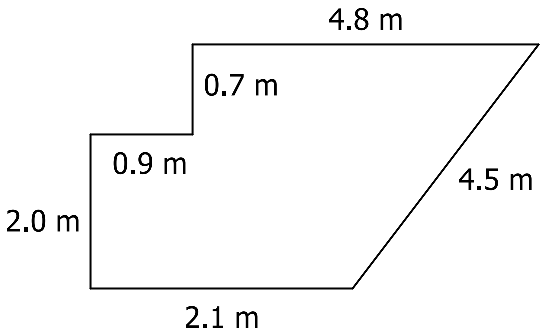
2.



2.5 Perimeter of combination shapes

Because perimeter is simply the distance around a shape, finding the perimeter of combination shapes is easy. All that we have to do is add up the length of each side.

Have a look at the combination shape at right.



The perimeter of this shape is the sum of the length of each side:

$$P = 2.0 \text{ m} + 2.1 \text{ m} + 4.5 \text{ m} + 4.8 \text{ m} + 0.7 \text{ m} + 0.9 \text{ m}$$

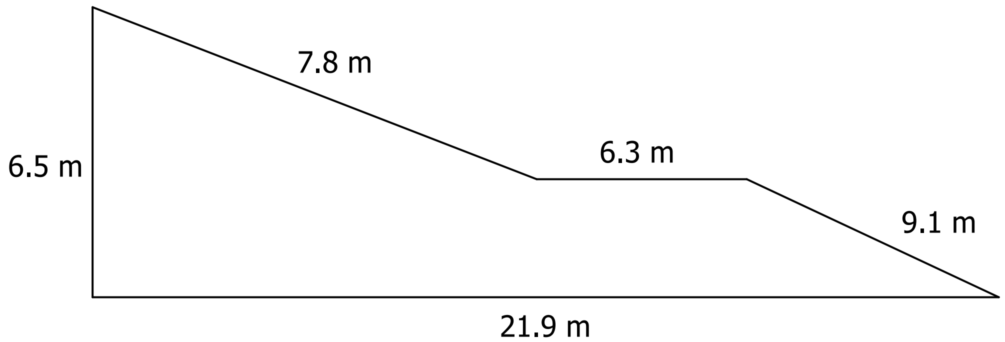
$$P = \underline{15.0 \text{ m}}$$

So, the perimeter of the combination shape is 15.0 m.

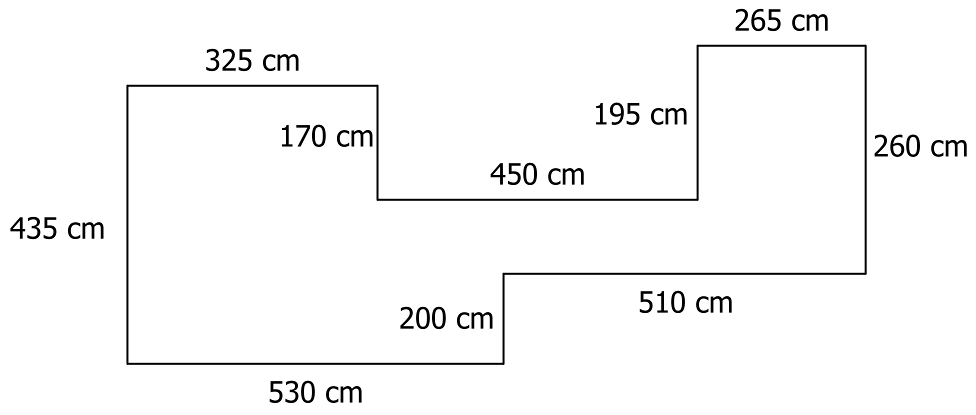
Activity 2.5

Find the perimeter of each of the following combination shapes.

1.

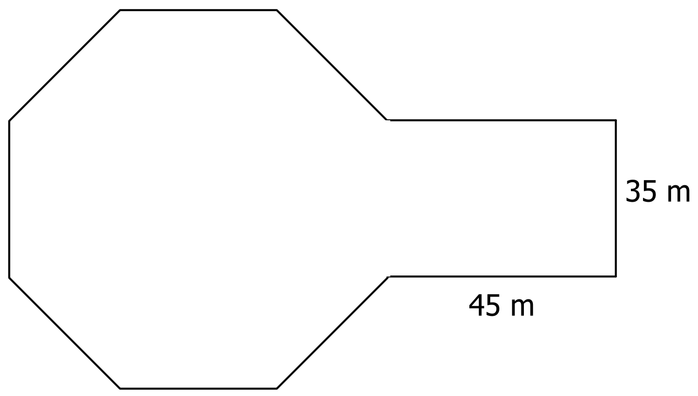


2.



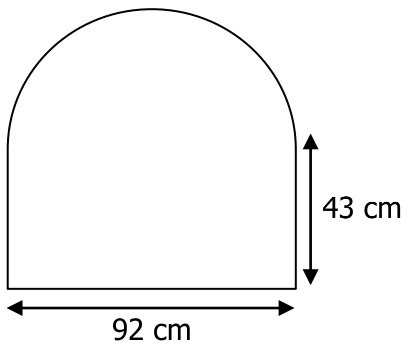
265 cm
325 cm
195 cm
170 cm
260 cm
450 cm
435 cm
510 cm
200 cm
530 cm

3.



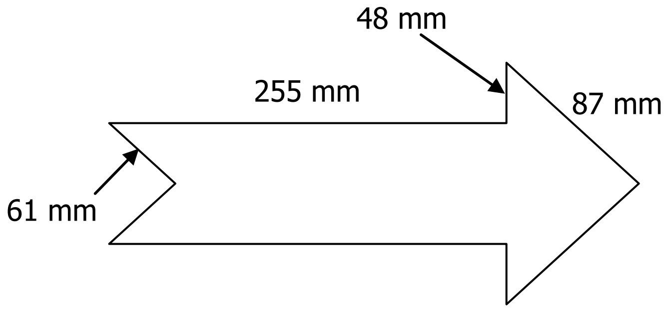
35 m
45 m

4.



43 cm
92 cm

5.

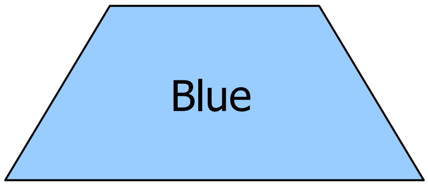


48 mm
255 mm
87 mm
61 mm

3. AREA OF SIMPLE SHAPES

3.1 Notion of area

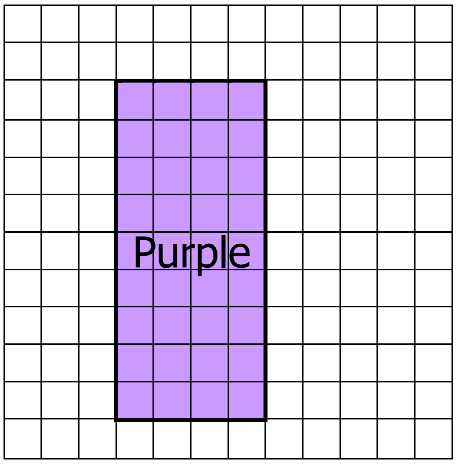
The **area** of a two-dimensional shape is a measure of the size of the surface contained inside the shape. For the shape at right, the area is the blue shaded part.



Finding the area of a shape depends on the shape itself. Different shapes have different formulae for finding the area.

3.2 Area of the rectangle

Have a look at the rectangle on the grid at right.

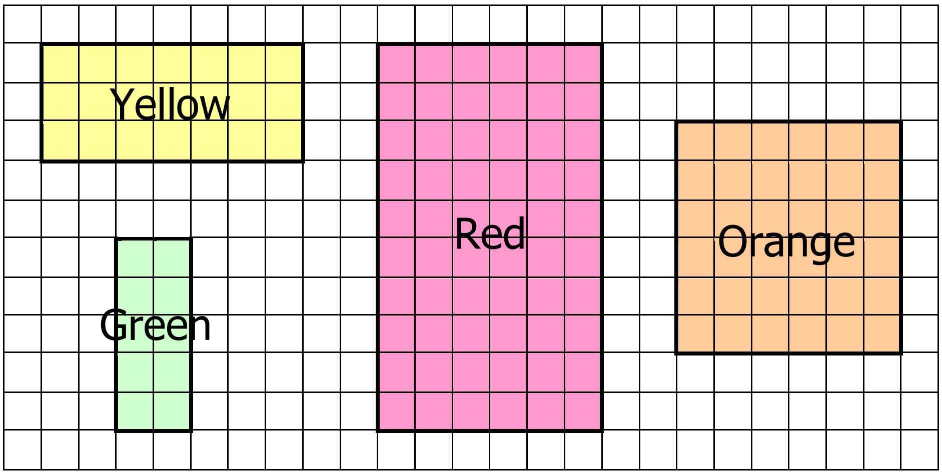


The area is the shaded purple region. The shaded purple region contains a total of 36 squares. Therefore, the area of the shaded purple region is equal to 36 squares.

Note that the rectangle is 4 squares wide and 9 squares long. Is there an operation that can be used to get 36 from 4 and 9?

Activity 3.2a

1. Find the area of each of the following rectangles by counting the number of squares inside it.



2. Complete the table below with the length (*a*), width (*b*), and area (*A*) of each of the rectangles in Question 1.

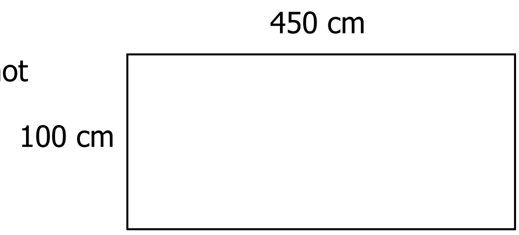
Rectangle	**Length (*a*)**	**Width (*b*)**	**Area (*A*)**
Yellow			
Green			
Red			
Orange			

3. What is the relationship among length (a), width (b), and Area (A) for each rectangle?

From the previous activity, you should have realised that, instead of counting the squares inside a rectangle, we can find the rectangle's area (A) by multiplying its length (a) by its width (b). Hence,

$$A = a \times b.$$

Have a look at the rectangle at right. We can see that a = 100 cm and b = 450 cm (or a = 450 cm and b = 100 cm, because the order of multiplication does not matter). So,



$$A = a \times b$$

$$A = 100\text{ cm} \times 450\text{ cm}$$

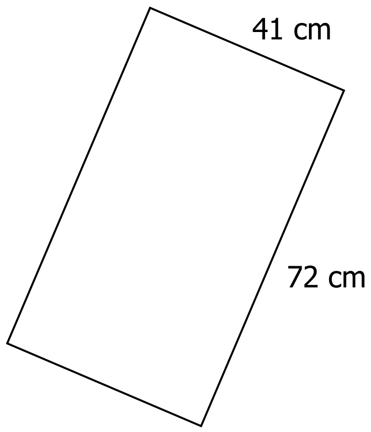
$$A = \underline{45\,000\text{ cm}^2}.$$

That means that the area of the rectangle is 45 000 cm². But, what does the ² mean in cm²? It means "squared." That is, cm² stands for "centimetres squared" or "square centimetres". Because area is a measure of the dimensions of two sides, its units must be squared. So, when finding the area of a shape, remember to put in the squared symbol ² (e.g. mm², cm², m², km²).

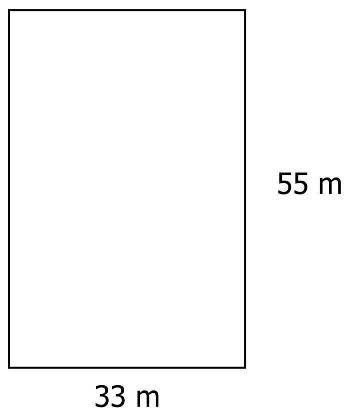
<u>Activity 3.2b</u>

Find the area of each of the following rectangles by using the formula $A = a \times b$. Remember that your answer will be in square units (e.g. mm², cm², m², km²).

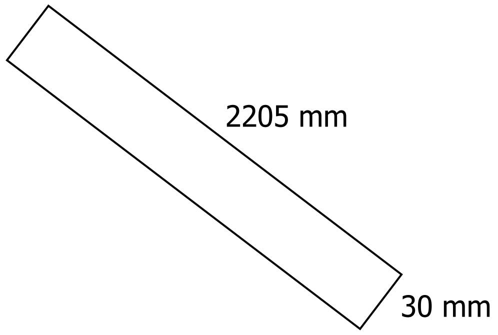
1.



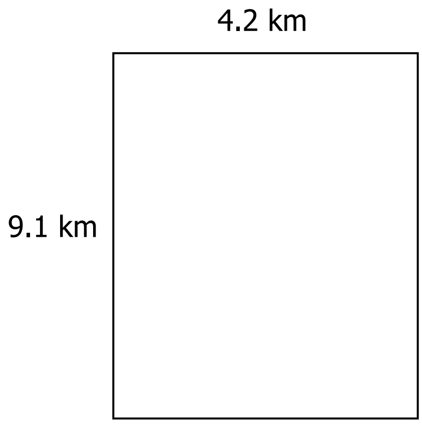
2.



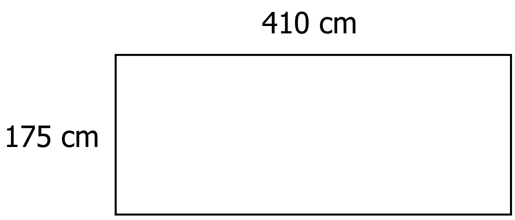
3.



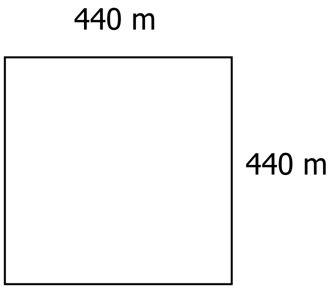
4.



5.



6.



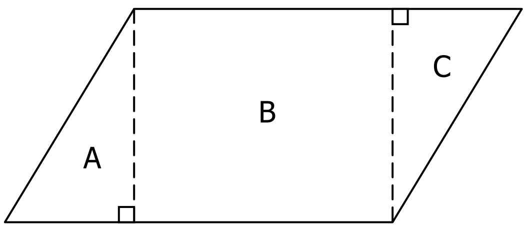
3.3 Area of the parallelogram

Have a look at the shape at right.

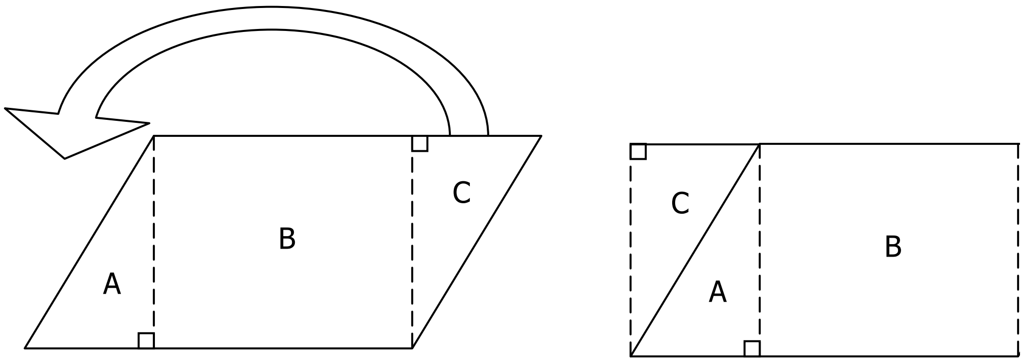
This shape is called a **parallelogram**, because opposite sides are parallel lines. We can think of a parallelogram as a rectangle that has been blown over by the wind.

Activity 3.3a

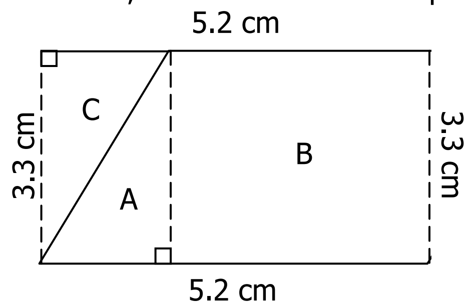
1. Find a piece of paper, a ruler, and scissors.
2. On the piece of paper, draw a parallelogram like the one shown above. Make sure that the opposite sides are parallel.
3. Cut out the parallelogram, keep the parallelogram, and discard the rest of the paper.
4. As shown in the figure below, cut the parallelogram along the dashed lines and label each region A, B, and C.



5. Move Region C to the left of Region A, as shown.



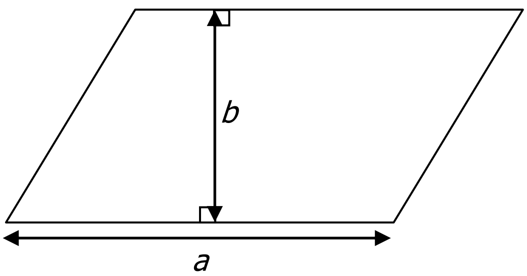
6. Measure the length of each side, as shown in the example below.



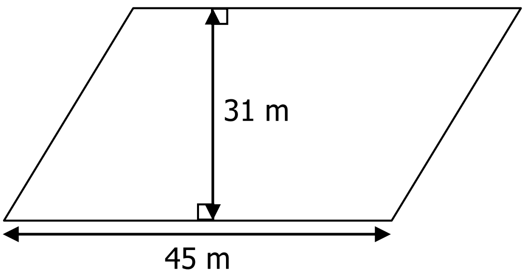
7. Find the area of the rectangle.

From the previous activity, you should have recognised that the area of a parallelogram is the same as the area of the rectangle formed by the parallelogram when the one end is moved to the other end. Therefore, we have discovered the formula for the area (A) of a parallelogram to be $A = a \times b$ where a is the length and b is the height.

Note, however, that the height of a parallelogram is the distance across the parallelogram and is at right (90°) angles with the length, as shown in the parallelogram at right. The length of the slanted side does not need to be known to calculate the area of a parallelogram.



Have a look at the parallelogram at right.



We can see that a = 45 m and b = 31 m. So,

$$A = a \times b$$

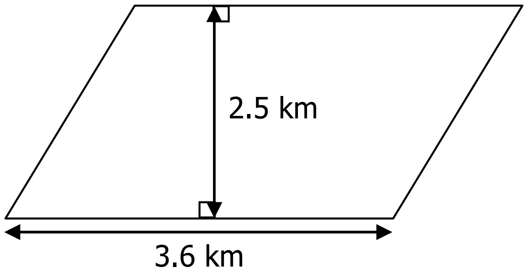
$$A = 45 \text{ m} \times 31 \text{ m}$$

$$A = \underline{1395 \text{ m}^2}.$$

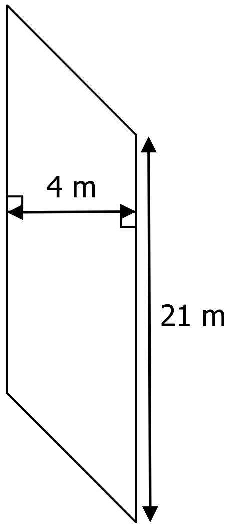
Activity 3.3b

Find the area of each of the following parallelograms.

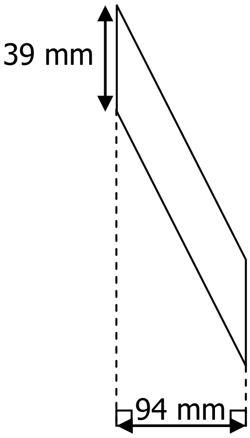
1.



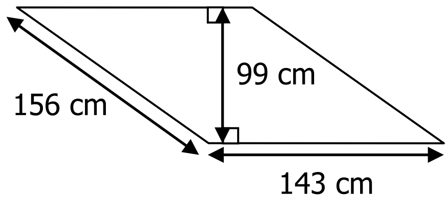
2.



3.



4.

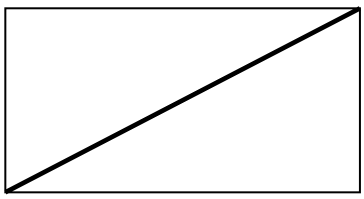


3.4 Area of the triangle

Now, we will investigate how to find the area of a triangle.

Activity 3.4a

1. Find a rectangular piece of paper, a ruler, and scissors.
2. Measure the lengths of the sides of the rectangle.
3. Calculate the area of the rectangle.
4. Use the ruler to draw a diagonal line across the paper, as shown on right.



5. Cut the paper along the diagonal line to form two right-angled triangles.
6. Calculate the area of each right-angled triangle.
7. Analyse the relationship between the area of the rectangle and the area of each of the right-angled triangles.

In the previous activity, you should have recognised that the area of a right-angled triangle is equal to half the area of the rectangle from which the triangle is formed. Therefore, we have the formula for finding the area (A) of a right-angled triangle: $A = a \times b \div 2$, where a and b are the lengths of the perpendicular sides of the triangle. The perpendicular sides form the right angle of the triangle.

Have a look at the right-angled triangle at right.

Note that the labelled side lengths form the right angle of the triangle. Therefore, they are a and b. Now, to calculate the area of the right-angled triangle, we have

$$A = a \times b \div 2$$

$$A = 52 \text{ cm} \times 125 \text{ cm} \div 2$$

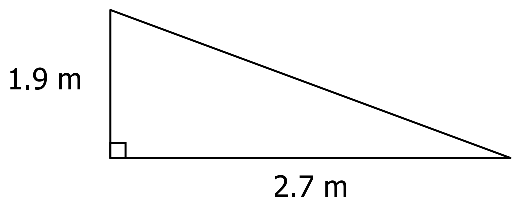
$$A = \underline{3250 \text{ cm}^2}.$$

So, the area of the right-angled triangle is 3250 cm².

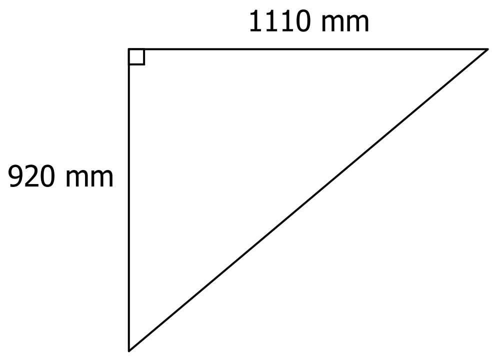
Activity 3.4b

Find the area of each of the following right-angled triangles.

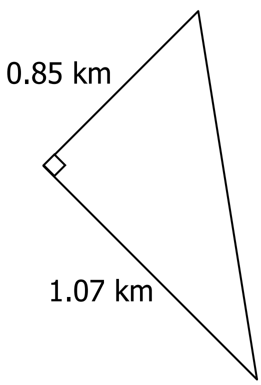
1.



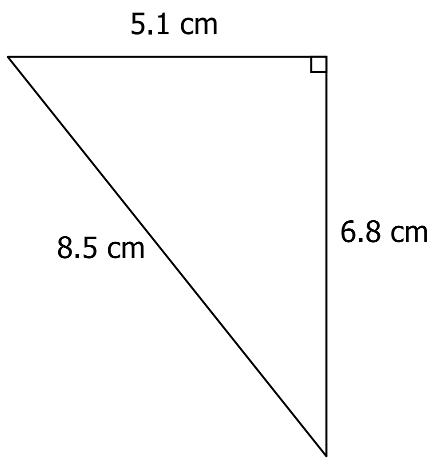
2.



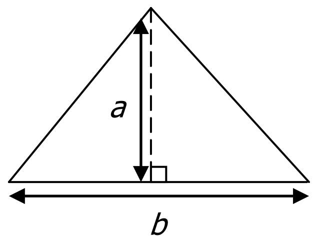
3.



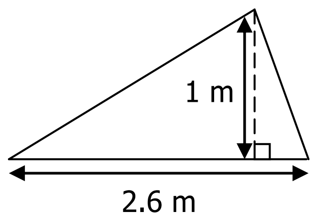
4.



If a triangle is not right-angled, as in the triangle at right, the formula is still $A = a \times b \div 2$. However, a is the length of the height of the triangle, and b is the length of the base of the triangle. Note that the lines a and b form a right angle. Also, recognise that the lengths of the unmarked sides are not required for the area formula.



Have a look at the triangle at right. Note that it is not right-angled. The formula for finding the area of any triangle is $A = a \times b \div 2$. In this triangle, a = 1 m and b = 2.6 m. Therefore, the area of this triangle is



$$A = a \times b \div 2$$

$$A = 1\text{ m} \times 2.6\text{ m} \div 2$$

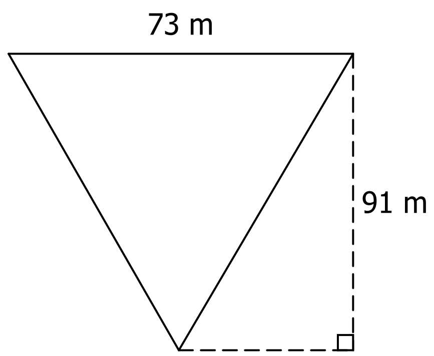
$$A = \underline{1.3\text{ m}^2}.$$

So, the area of the triangle is 1.3 m².

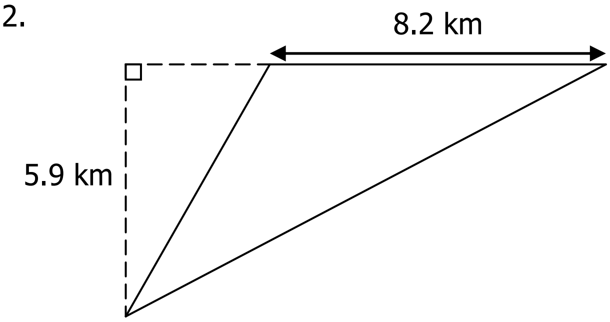
Activity 3.4c

Find the area of each of the following triangles.

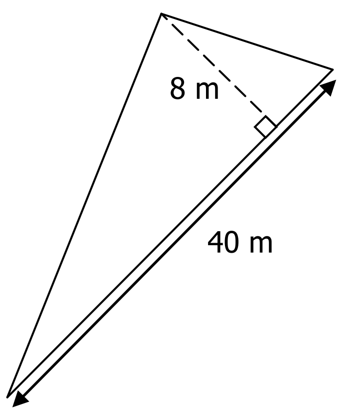
1.



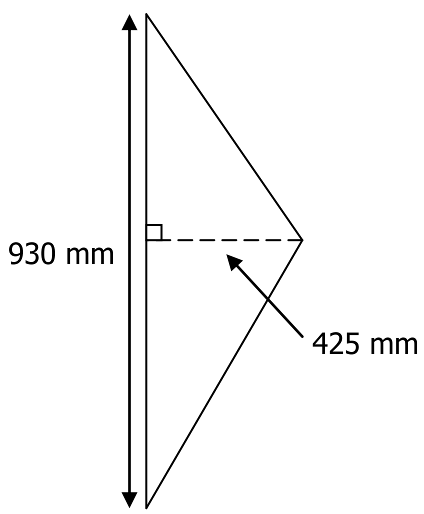
2.



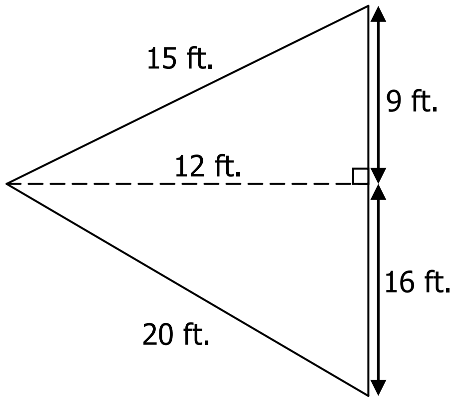
3.



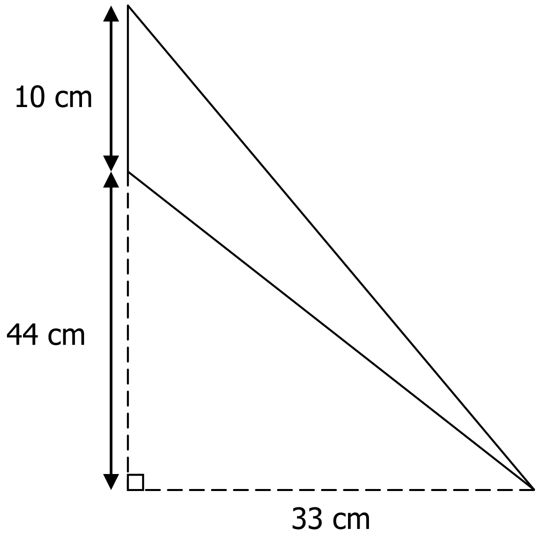
4.



5.



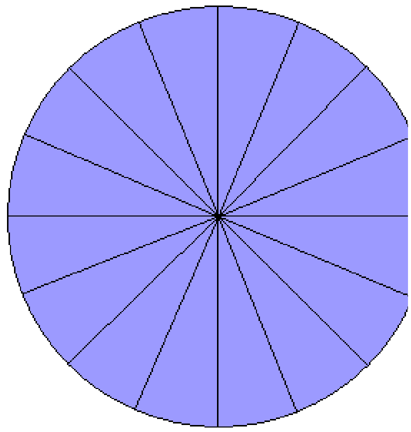
6.



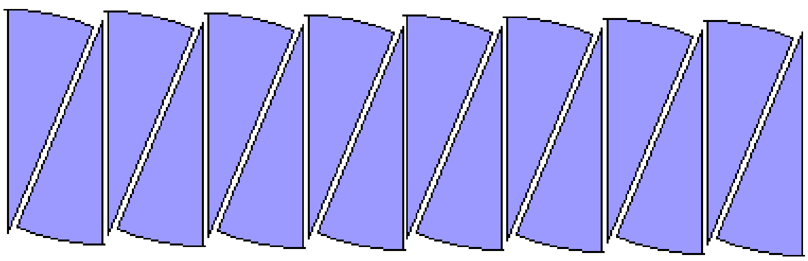
3.5 Area of the circle

Now, we will investigate how to find the area of a circle.

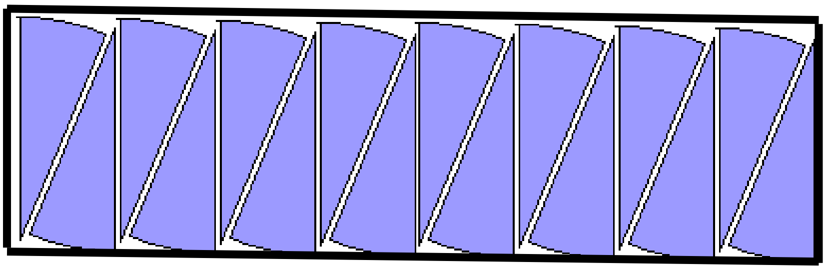
<u>Activity 3.5a</u>



1. Find a piece of paper, a large lid (at least 12 cm in diameter), a ruler, and scissors.
2. Measure the diameter of the lid, and calculate the radius (we shall call this r).
3. Place the lid firmly on the piece of the paper, and draw a circle around it. Calculate the circumference ($= 2 \times \pi \times r$) and the area ($= r^2 \times \pi$).
4. Draw lines across the circle to form 16 sectors that are approximately equal in area, as shown in the figure at right. (Hint – keep halving.)
5. Cut out the 16 sectors, as shown at right.
6. Arrange the sectors in the configuration shown below.



7. Note that this configuration looks very much like a rectangle.

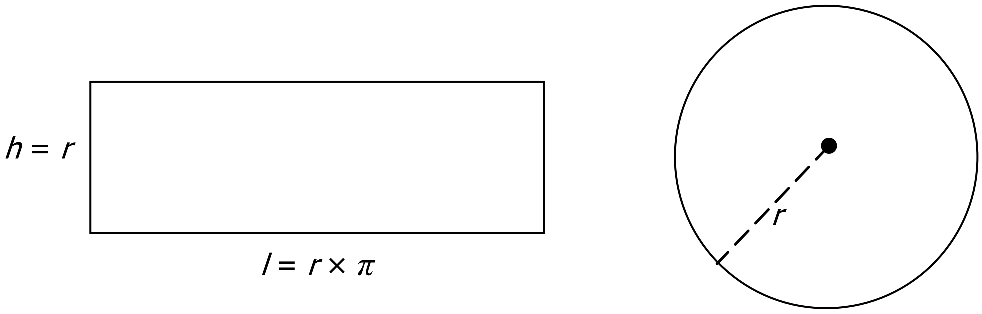


8. What is the height? What is the length? Can you see how these relate to radius and circumference?

9. Measure the height and length of the rectangle. Calculate the area of the rectangle. Divide the length of the rectangle by the radius. Did you get an answer that is close to π? Remember that π is approximately equal to 3.14.

10. Did you recognise that the height of the rectangle is the same as the radius of the lid? What does this mean for length?

Since the height (h) of the rectangle is equal to the radius (r) of the circle and the length (l) of the rectangle is (approximately) equal to the radius (r) of the circle times π (see figures below), we can develop the formula for finding the area of a circle.



To calculate the area of the rectangle, you would have used $A = h \times l$. Since $h = r$ and $l = r \times \pi$, we can substitute (remembering that $r^2 = r \times r$).

$$A = h \times l$$

$$A = r \times r \times \pi$$

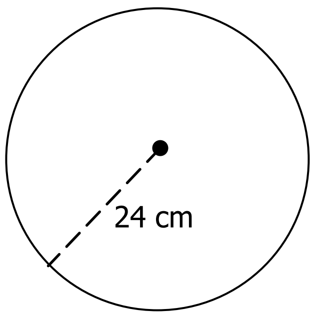
$$A = r^2 \times \pi.$$

So, we have developed the formula for finding the area of circle, $A = r^2 \times \pi$ where r is the radius of the circle. Given the circle's radius, all we have to do is multiply the radius by itself and then multiply the result by π.

Have a look at the circle at right.

We can see that $r = 24$ cm. So, to calculate the area of the circle, we have

$$A = r^2 \times \pi$$



$$A = (24 \text{ cm})^2 \times \pi$$

$$A = 24 \text{ cm} \times 24 \text{ cm} \times \pi$$

$$A = \underline{1809.6 \text{ cm}^2}$$

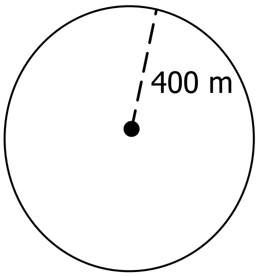
That is, the area of the circle is approximately equal to 1809.6 cm².

Remember that, if given the diameter of a circle instead, you must divide it by 2 to find the radius before using the formula for calculating the area of a circle.

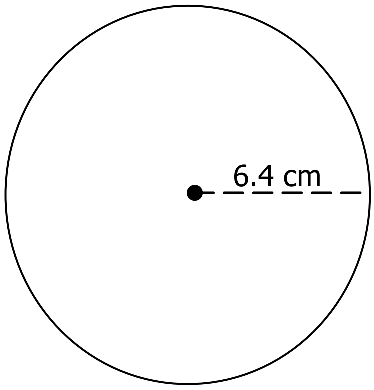
<u>Activity 3.5b</u>

Find the area of each of the following circles.

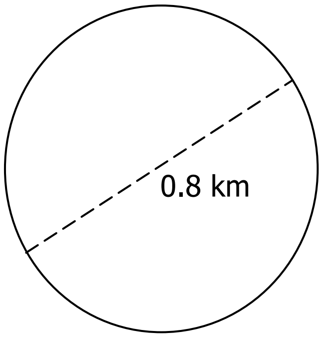
1.



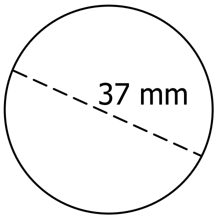
2.



3.



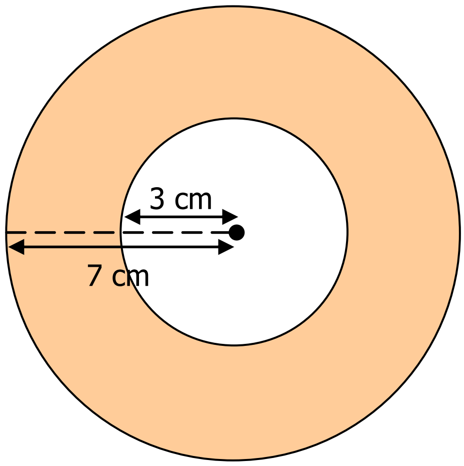
4.



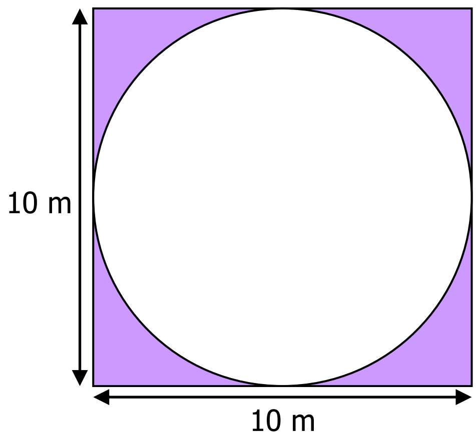
<u>Activity 3.5c</u>

Find the area of the shaded region in each of the following figures.

1.



2.



4. VOLUME OF PRISMS AND CYLINDERS

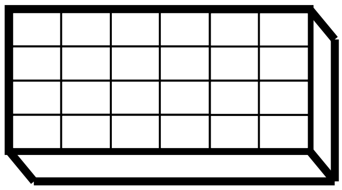
4.1 Notion of volume

Volume is a measure of how much space is enclosed inside a three-dimensional object.

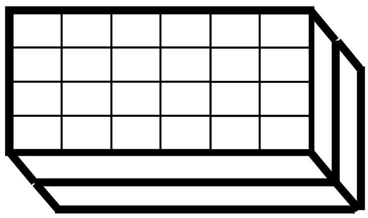
Remember that the area of two-dimensional shapes is measured in square units (e.g. mm^2, cm^2, m^2, km^2). It follows, that the volume of three-dimensional shapes is measured in cubic units (e.g. mm^3, cm^3, m^3, km^3).

4.2 Volume of rectangular prisms

A **rectangular prism** is the formal name for a rectangular box. A rectangular prism looks like the figure at right when shown in two dimensions. The volume of the rectangular prism is the amount of space enclosed inside it.

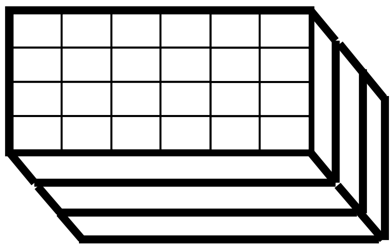


Have a look at the block at right. Note that the face of the block is 6 units long and 4 units wide. Therefore, the area of the face of the block is 6 units × 4 units = 24 square units. Note that the block is 1 unit height. Therefore, the volume of the block is 24 cubic units. That is, a measure of the amount of space inside the block is 24 cubic units.



If we put another block of the same dimensions against the first one, as shown at right, then you will see that the prism now has a height of 2 units. The volume, therefore, is twice that of the first block. So, the volume of this prism is 48 cubic units.

If we add a third block to the prism from before, then the prism will now be 3 units high (see the figure below right). Therefore, the volume of the prism will be 24 × 3 = 72 cubic units.



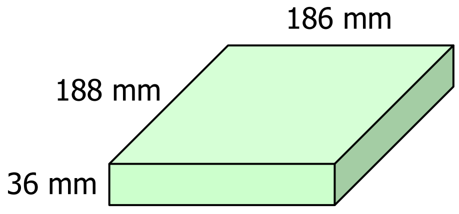
Hence, the volume (V) of a rectangular prism is equal to the area (A) of the face of the prism times the height (c) of the prism. Since the area (A) of the face is simply length (a) times width (b), we have the formula for calculating the volume of a rectangular prism: $V = a \times b \times c$. Because we are multiplying three quantities together, it makes sense that units of volume must be cubic (e.g. mm^3, cm^3, m^3, km^3).

<u>Activity 4.2a</u>

1. Construct the base of a rectangular prism with small cubes. How many cubes?

2. Build a 2nd, 3rd and 4th tier. How many cubes? What does this mean for relating Volume to Area of base?

Have a look at the rectangular prism at right.



We see that the a = 186 mm, b = 188 mm, and c = 36 mm. (Remember that, because we can multiply numbers in any order and get the same result, we can assign a, b, and c to any of the quantities and obtain the same result.) The volume of the rectangular prism is

$$V = a \times b \times c$$

$$V = 186 \text{ mm} \times 188 \text{ mm} \times 36 \text{ mm}$$

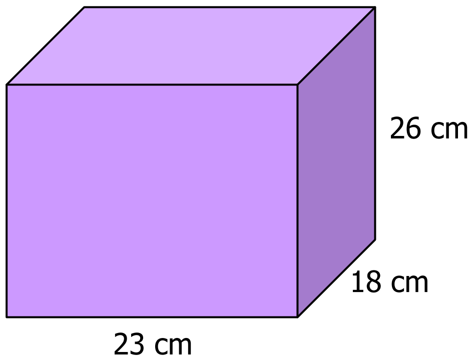
$$V = \underline{1\,258\,848 \text{ mm}^3}.$$

So, the volume of the rectangular prism is 1 258 848 mm^3, which is the measure of how much space is enclosed inside the rectangular prism.

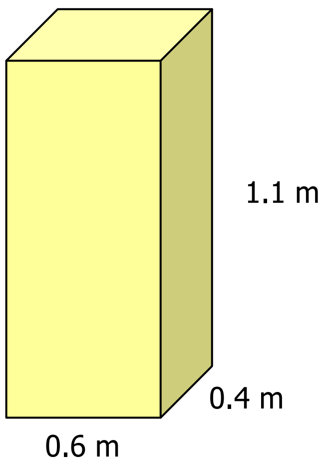
<u>Activity 4.2b</u>

Find the volume of each of the following rectangular prisms.

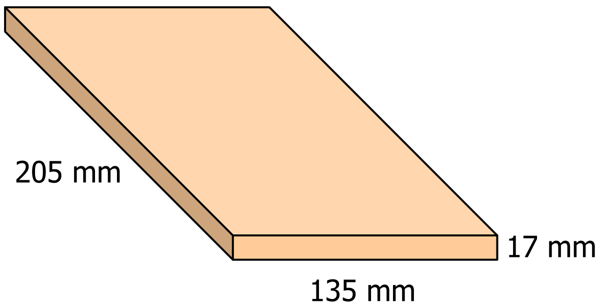
1.



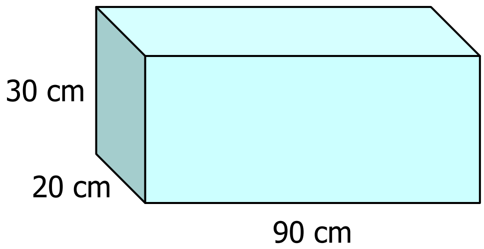
2.



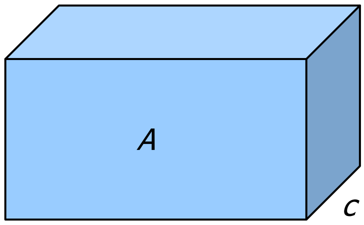
3.



4.



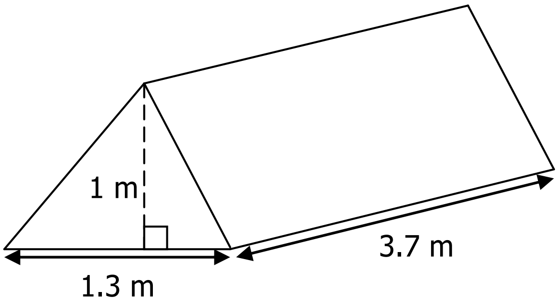
4.3 Volume of other prisms



Remember that the volume (V) of a rectangular prism is equal to the area (A) of the face times the length of the prism (c), as shown in the figure at right. That is, $V = A \times c$.

This formula also applies for prisms that are not rectangular.

Have a look at the triangular prism at right.



To find the volume of the triangular prism, we must first find the area of its face. Note that the triangle has a height (a) of 1 m and a base (b) of 1.3 m. To find the area of the triangle, we use the formula from before:

$$A = a \times b \div 2$$

$$A = 1\text{ m} \times 1.3\text{ m} \div 2$$

$$A = 0.65\text{ m}^2.$$

So, the area of the face is 0.65 m².

Now, we use the volume formula.

$$V = A \times c$$

$$V = 0.65\text{ m}^2 \times 3.7\text{ m}$$

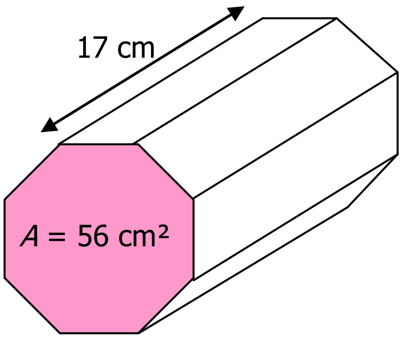
$$V = \underline{2.405\text{ m}^3}.$$

Hence, the volume of the triangular prism is 2.405 m³.

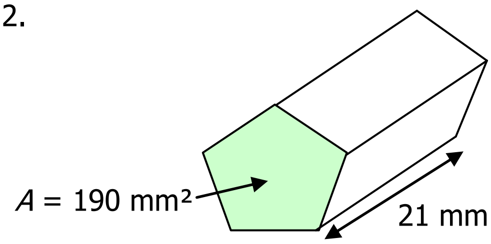
Activity 4.3

Find the volume of each of the following prisms.

1.



2.



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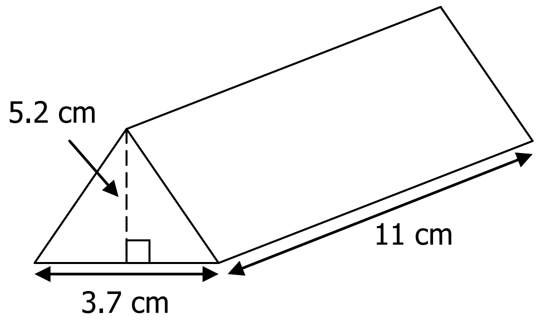

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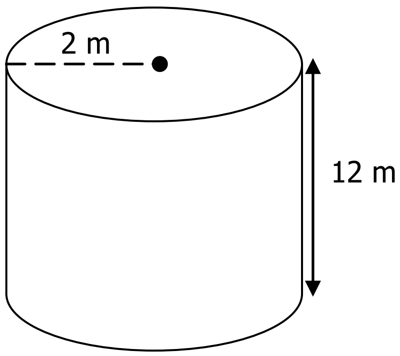
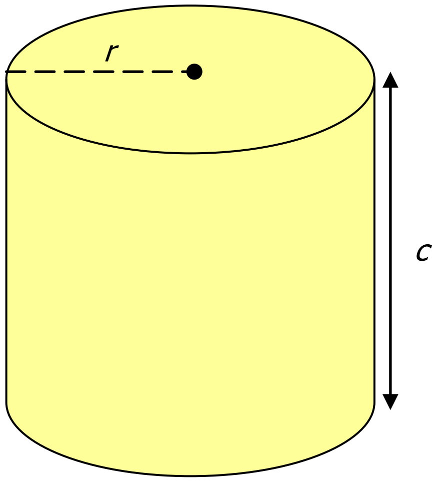


4.4 Volume of the cylinder

In Question 4. from the previous activity, you would have found the volume of a cylinder. A **cylinder** is just like a prism, except that the shape on the face is a circle, as shown in the figure at right.



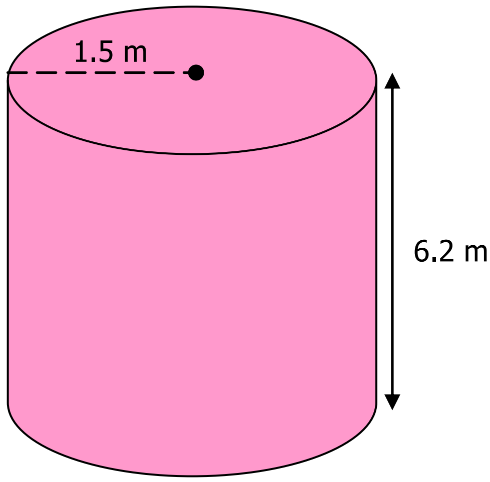
You should have recognised that finding the volume of a cylinder is the same as finding the volume of a prism. First, you find the area of the face. Then, you multiply the area by the length of the cylinder.

Since we have already learned that the area of a circle is $A = r^2 \times \pi$ where r is the radius of the circle, we can derive the formula for finding the volume of a cylinder:

$$V = A \times c$$

$$V = r^2 \times \pi \times c.$$

Have a look at the cylinder at right.



We can see that the radius (r) is 1.5 m and the length (c) is 6.2 m. The volume of the cylinder is

$$V = r^2 \times \pi \times c$$

$$V = (1.5 \text{ m})^2 \times \pi \times 6.2 \text{ m}$$

$$V = 1.5 \text{ m} \times 1.5 \text{ m} \times \pi \times 6.2 \text{ m}$$

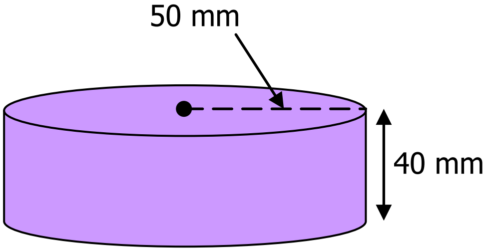
$$V = \underline{43.8 \text{ m}^3}.$$

So, the volume of the cylinder is 43.8 m^3.

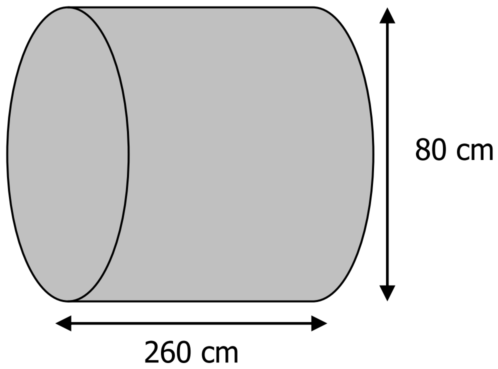
Activity 4.4

Find the volume of each of the following cylinders.

1.



2.

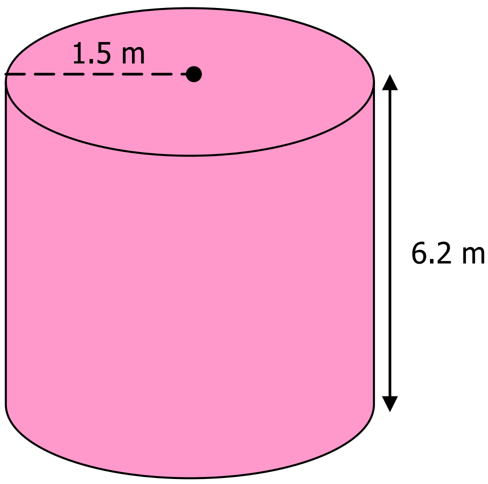


4.5 Capacity

As we learned before, volume is the amount of space inside a prism or cylinder. Volume can also be viewed as the maximum amount of solid material that can be used to fill up the prism or cylinder. On the other hand, when we are interested in finding the amount of liquid that can fill up a prism or cylinder, we need to find the capacity of the prism or cylinder. **Capacity** is the volume of liquid.

Finding the capacity of a prism or cylinder is just like finding the volume. However, once we have the volume, we must convert it to capacity, which is usually measured in litres (L). The conversion is 1 m^3 = 1000 L.

Have another look at the cylinder at right from before.



We already know that the volume of this cylinder is 43.8 m^3. If we need to calculate the capacity (C) of this cylinder, then all we have to do is multiply the volume (V) by 1000 L:

$$C = V \times 1000$$

$$C = 43.8 \text{ m}^3 \times 1000$$

$$C = \underline{43\,800 \text{ L}}.$$

So, the capacity of the cylinder is 43 800 L. That is, a maximum of 43 800 L of liquid can fill up the cylinder.

The conversion factor of 1000 works only when you have the volume in m^3. If you have the volume in units other than m^3, then the conversion factor to capacity is different. The table below shows the conversion factors for different units of volume:

1 km^3 = 1 000 000 000 000 L

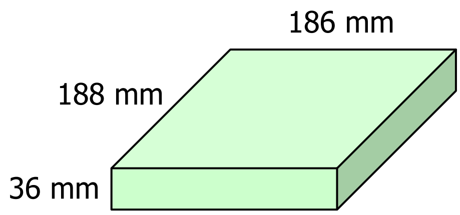
1 m^3 = 1000 L

1 cm^3 = 0.001 L = 1 mL

1 mm^3 = 0.000 001 L = 0.001 mL

We always want to make sure that we use the correct conversion factor given the units of volume that we have.

Have another look at the rectangular prism at right from before.



We already know that its volume is 1 258 848 mm³. If we need to calculate the capacity (*C*) of the rectangular prism, then all we need to do is multiply the volume (*V*) by 0.000 001:

$$C = V \times 0.000\,001$$

$$C = 1\,258\,848 \text{ mm}^3 \times 0.000\,001$$

$$C = \underline{1.26 \text{ L}}.$$

Alternatively, we could have multiplied the volume (*V*) by 0.001 in order to obtain the capacity in mL. In this case,

$$C = V \times 0.001$$

$$C = 1\,258\,848 \text{ mm}^3 \times 0.001$$

$$C = \underline{1258.8 \text{ mL}}.$$

So, the capacity of the rectangular prism is 1.26 L or 1258.8 mL.

<u>Activity 4.5</u>

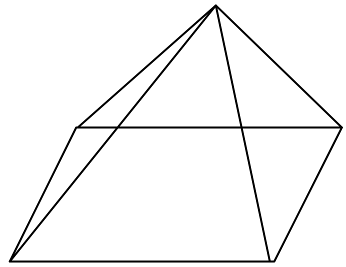
Find the capacity of each of the following prisms or cylinders (in either mL or L) from previous activities:

1. Activity 4.2, Question 2. on page 23.
2. Activity 4.2, Question 3. on page 23.
3. Activity 4.3, Question 1. on page 24.
4. Activity 4.4, Question 1. on page 26.

5. VOLUME OF PYRAMIDS AND CONES

5.1 Volume of the rectangular pyramid

A **rectangular pyramid** is a pyramid with a rectangular base. See the two-dimensional representation of a rectangular pyramid at right.



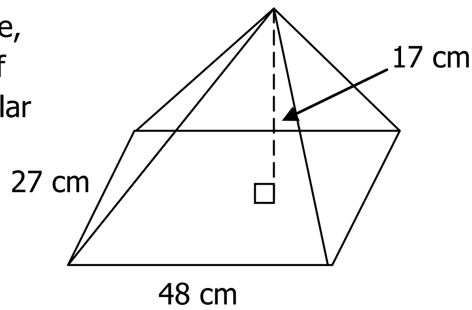
Activity 5.1a

1. Find or construct a rectangular pyramid and prism of the same base and height.
2. Using rice, see how many pyramids can fill one prism (of the same base and height). What does this mean for formula of a rectangular pyramid?

The formula for finding the volume (V) of a rectangular pyramid is similar to the formulae for finding the volume of prisms and cylinders. First, we find the area (A) of the face. In this case, the face we need is the base of the pyramid, which we already know is just a rectangle. Then, once we find the area of the base, we multiply it by the height (h) of the pyramid. Finally, we divide by 3.

As a formula, we have $V = A \times h \div 3$.

Have a look at the rectangular pyramid at right.



Note that the base is made up of lengths 27 cm and 48 cm. Hence, the area of the base is 27 cm × 48 cm = 1296 cm². The height of the rectangular pyramid is 17 cm. So, the volume of the rectangular pyramid is

$$V = A \times h \div 3$$

$$V = 1296\text{ cm}^2 \times 17\text{ cm} \div 3$$

$$V = \underline{7344\text{ cm}^3}.$$

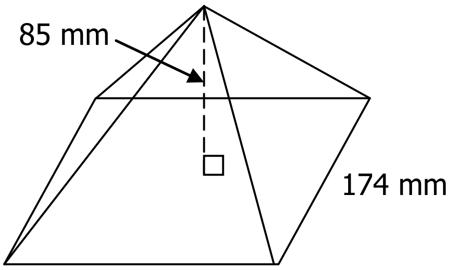
Therefore, the volume of the rectangular pyramid is 7344 cm³.

If we wanted to find the capacity of the rectangular pyramid, we replace cm³ with mL. This gives a capacity of 7344 mL. To change to litres, divide by 1000 or multiply by 0.001 to obtain 7.3 L. So, the capacity of the rectangular pyramid is 7.3 L.

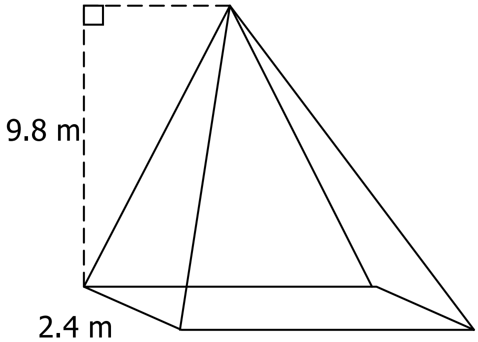
Activity 5.1b

Find the volume and capacity of each of the following rectangular prisms.

1.

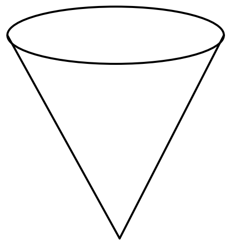


2.



5.2 Volume of the cone

A **cone** is a pyramid with a circle as a base, as shown at right.



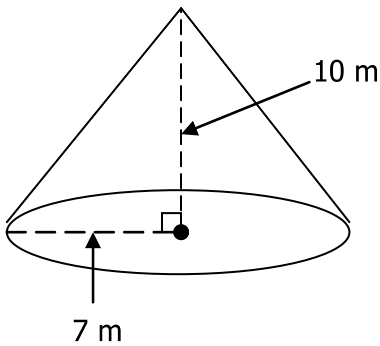
Activity 5.2a

1. Find or construct a cone and cylinder of the same base and height.
2. Using rice, see how many cones can fill one cylinder (of the same base and height). What does this mean for formula of a cone?

The volume (V) of the cone is the same as the volume of the rectangular pyramid: we calculate the area (A) of the base, multiply it by the height (h), and divide by 3. Hence, the formula for finding the volume of a cone is $V = A \times h \div 3$.

Remember that the formula for calculating the area (A) of a circle is $A = r^2 \times \pi$ where r is the radius of the circle.

Have a look at the cone at right.



We see that the radius (r) of the cone's base is 7 m. Therefore, the area of the base is

$$A = r^2 \times \pi$$

$$A = (7\text{ m})^2 \times \pi$$

$$A = 7 \text{ m} \times 7 \text{ m} \times \pi$$

$$A = 153.9 \text{ m}^2.$$

So, the volume (V) of the cone is

$$V = A \times h \div 3$$

$$V = 153.9 \text{ m}^2 \times 10 \text{ cm} \div 3$$

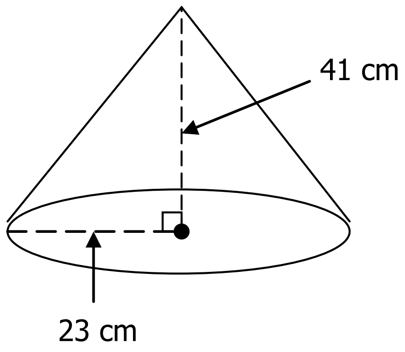
$$V = \underline{513 \text{ m}^3}.$$

Hence, the volume of the cone is 513 m³.

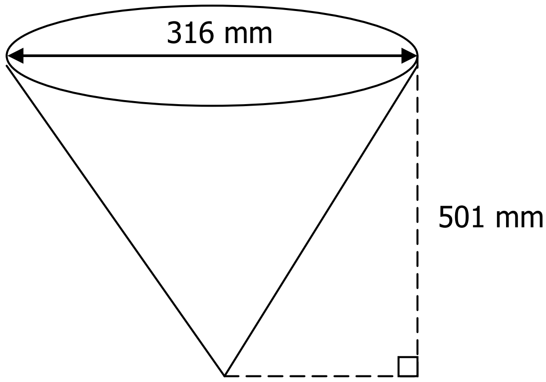
Activity 5.2b

Find the volume and capacity of each of the following cones.

1.



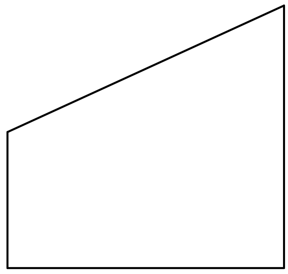
2.



6. AREA AND VOLUME OF COMBINATION SHAPES AND SOLIDS

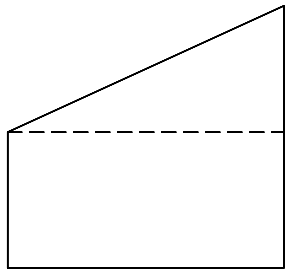
6.1 Breaking-into-parts strategy

Finding the area and volume of combination shapes and solids is really easy. All that we have to do is to break the shape or solid into smaller shapes or solids. We can then find the area or volume of the smaller shapes or solids and then add them together to obtain the area or volume of the combination shape or solid.

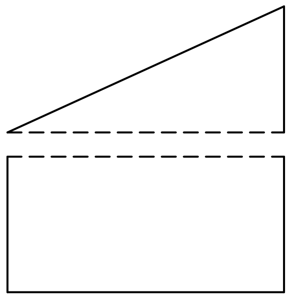


Have a look at the combination shape at right.

We can see that this shape is the combination of a rectangle and a triangle. By drawing a dashed line between the two shapes, we can more easily identify the rectangle and the triangle.

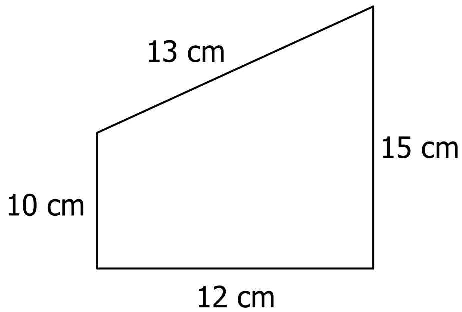


We can even pull the two shapes apart to help better identify them.

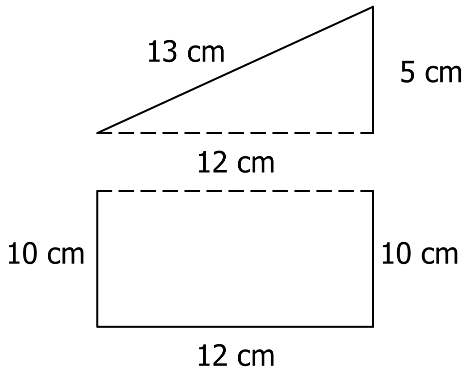


If we are meant to find the area of the combination shape, then we just need to find the area of the rectangle and the area of the triangle separately, and add the two areas together.

Have a look at the same combination shape with measurements at right.



We see that the combination shape can be separated into a rectangle and triangle, as shown below right. Note that the 15-cm length of the right side of the combination shape is made of the 10-cm length, which corresponds with the opposite side length of the rectangle, and the 5-cm length, which is what is left (i.e. 15 cm – 10 cm = 5 cm). To calculate the area of the combination shape, we must first calculate the area of the rectangle (A_R) and the area of the triangle (A_T) separately.



For the rectangle, we recognise that a = 10 cm and b = 12 cm. Therefore, the area of the rectangle is

$$A_R = a \times b$$

$$A_R = 10\text{ cm} \times 12\text{ cm}$$

$$A_R = 120\text{ cm}^2.$$

For the triangle, we recognise that a = 5 cm and b = 12 cm. Please note that the length of 13 cm is not needed in the formula. Therefore, the area of the triangle is

$$A_T = a \times b \div 2$$

$$A_T = 5 \text{ cm} \times 12 \text{ cm} \div 2$$

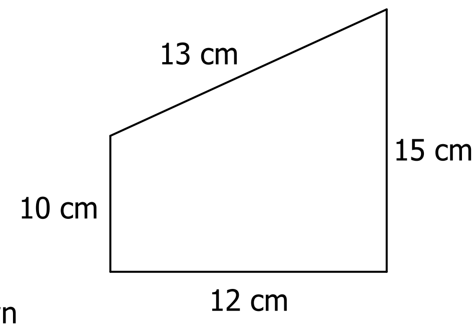
$$A_T = 30 \text{ cm}^2.$$

Now that we have that the area of the rectangle is 120 cm² and the area of the triangle is 30 cm², we can find the area of the combination shape (A):

$$A = A_R + A_T$$

$$A = 120 \text{ cm}^2 + 30 \text{ cm}^2$$

$$A = 150 \text{ cm}^2.$$



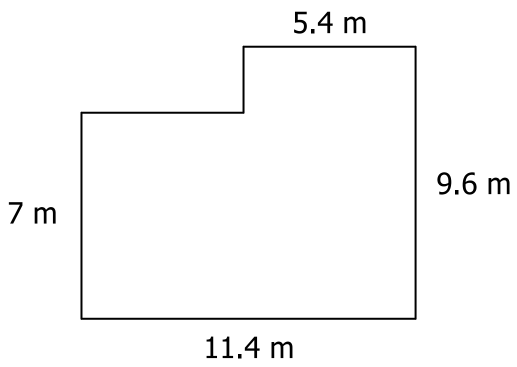
Finally, we have that the area of the combination shape, shown at right again, is 150 cm².

6.2 Area of combination shapes

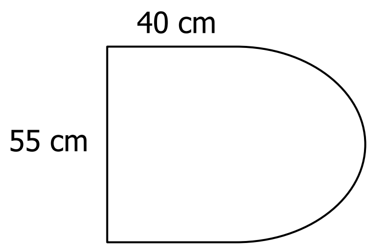
Activity 6.2

Find the area of each of the following combination shapes. Show your working clearly.

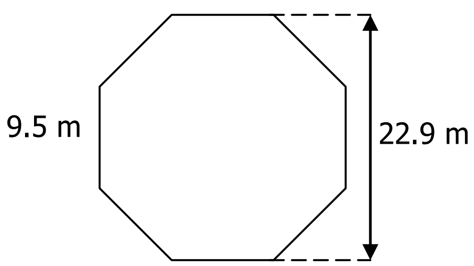
1.



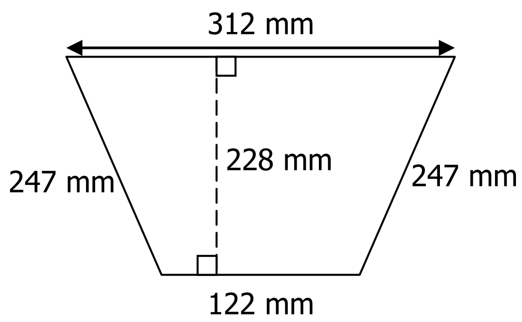
2.



3.



4.



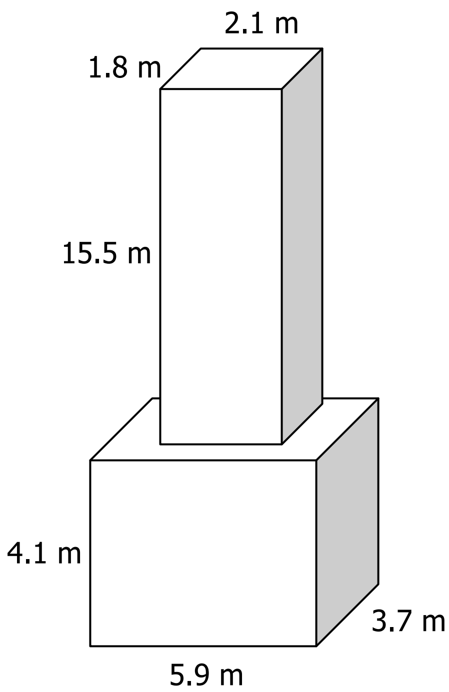
6.3 Volume of combination solids

Finding the volume of combination solids follows the same procedure as finding the area of combination shapes. We identify the individual solids that make up the combination solid. Then, we calculate the volume of each individual solid. Finally, we add up all of the volumes to obtain the volume for the combination solid.

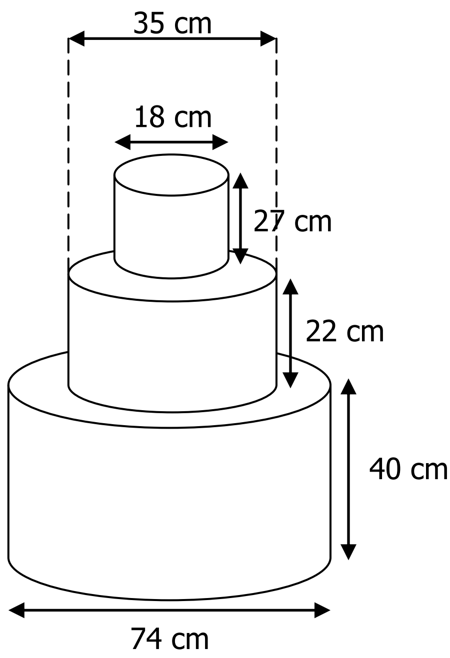
Activity 6.3

Find the volume of each of the following combination solids. Show your working clearly.

1.



2.



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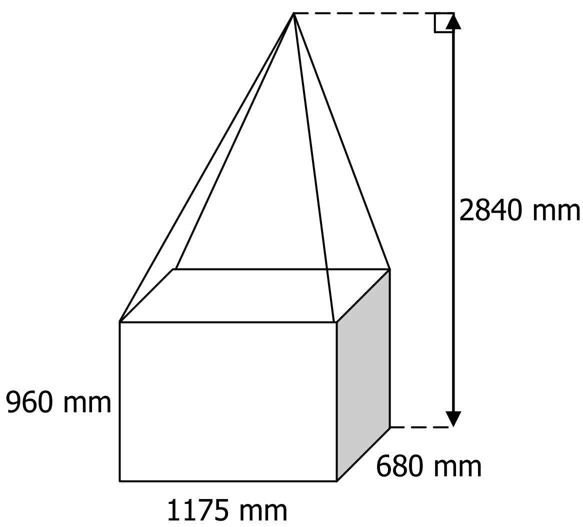

4.