Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called “Deadly Maths”.

“YuMi” is a Torres Strait Islander word meaning “you and me” but is used here with permission from the Torres Strait Islanders’ Regional Education Council to mean working together as a community for the betterment of education for all. “Deadly” is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre’s motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre’s vision: Growing community through education.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Basic Mathematics

MATHEMATICS BEHIND WHOLE NUMBER PLACE VALUE AND OPERATIONS

BOOKLET VB1
USING BUNDLING STICKS, MAB AND MONEY
VERSION 1: 30/06/08

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THIS BOOKLET

This booklet VB1 was the first booklet produced as material to support Indigenous students completing a variety of vocational certificates at Shalom Christian College and Wadja Wadja High School. It was developed for teachers and students as part of an ASISTM project, *Enhancing mathematics for Indigenous Vocational Education and Training students*. The project studied better ways to teach mathematics to Indigenous VET students at Tagai College (Thursday Island campus), Tropical North Queensland Institute of TAFE (Thursday Island Campus), Northern Peninsula Area College (Bamaga campus), Barrier Reef Institute of TAFE/Kirwan SHS (Palm Island campus), Shalom Christian College (Townsville), and Wadja Wadja High School (Woorabinda).

At the date of this publication, the Deadly Maths VET books produced are:

**VB1:** Mathematics behind whole-number place value and operations
   Booklet 1: Using bundling sticks, MAB and money

**VB2:** Mathematics behind whole-number numeration and operations
   Booklet 2: Using 99 boards, number lines, arrays, and multiplicative structure

**VC1:** Mathematics behind dome constructions using Earthbags
   Booklet 1: Circles, area, volume and domes

**VC2:** Mathematics behind dome constructions using Earthbags
   Booklet 2: Rate, ratio, speed and mixes

**VC3:** Mathematics behind construction in Horticulture
   Booklet 3: Angle, area, shape and optimisation

**VE1:** Mathematics behind small engine repair and maintenance
   Booklet 1: Number systems, metric and Imperial units, and formulae

**VE2:** Mathematics behind small engine repair and maintenance
   Booklet 2: Rate, ratio, time, fuel, gearing and compression

**VE3:** Mathematics behind metal fabrication
   Booklet 3: Division, angle, shape, formulae and optimisation

**VM1:** Mathematics behind handling small boats/ships
   Booklet 1: Angle, distance, direction and navigation

**VM2:** Mathematics behind handling small boats/ships
   Booklet 2: Rate, ratio, speed, fuel and tides

**VM3:** Mathematics behind modelling marine environments
   Booklet 3: Percentage, coverage and box models

**VR1:** Mathematics behind handling money
   Booklet 1: Whole-number and decimal numeration, operations and computation
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OVERVIEW

The ASISTM VET project funded in 2008 by the Australian Schools Innovations in Science, Technology and Mathematics (ASISTM) scheme had 6 sites: Wadja Wadja High School at Woorabinda, Shalom Christian College in Townsville, Palm Island Post Year 10 Campus (run by Kirwan State High School and Barrier Reef Institute of TAFE), Tagai College Secondary Campus at Thursday Island, Thursday Island campus of Tropical North Queensland Institute of TAFE, and Northern Peninsula Area College at Bamaga. All these sites have Indigenous students and the project focused on developing instruments and materials to assist the teaching of mathematics needed for certification for Indigenous VET students with little previous success in school.

Discussions with Wadja Wadja High School and Shalom Christian College VET and Mathematics staff decided that the project should develop basic mathematics material to assist low achieving VET students learn prerequisites for VET courses such as understanding of number, operations, time and simple shape and measurement. This first booklet, VB1, looks at 2 and 3 digit whole numbers in terms of numeration, addition and subtraction, and multiplication and division. It focuses on concepts and strategies associated with viewing number in separated or place-value terms.

This booklet is supported by virtual online materials which provide practice on computers to support basic mathematics. These files use virtual copies of bundling straws and rubber bands, multi-base arithmetic blocks, and money to build understanding of 2 and 3 digit place value, addition and subtraction algorithms using place-value strategies, and multiplication and division algorithms also using place-value strategies. The online materials that are designed for this booklet can be accessed in the folders BUNDLING, MAB and VIRTUAL MONEY, under BASIC MATHS in the virtual resource folder at http://ydc.qut.edu.au/yumi-deadly-resources/past-projects.html. The files are designed to be used by students to practice activities initially developed with physical materials. The computer files use Power point to allow students to manipulate virtual copies of the physical materials which are the bundling sticks, MAB and money. They are simple and basic, do not self correct, and can be easily manipulated and extended by teachers. Both VB1 and VB2 booklets are accompanied by online materials.

This booklet covers:

(1) the nature of virtual materials in mathematics teaching as perceived by this booklet;

(2) the physical material activity with bundling sticks, MAB and money that should precede the virtual material work focussing on place value, notably reading and writing of numbers;

(3) the physical material activity on addition and subtraction algorithms using the place-value or separated strategy of combining activity with separate place-value positions; and

(4) the physical material activity on multiplication and division algorithms using the place-value strategy.

The virtual materials cover using virtual copies of bundling straws, MAB and money to:

(1) represent 2 and 3 digit numbers written as names with virtual materials;
(2) write 2 and 3 digit number-names for virtual-material representations of these numbers;
(3) represent 2 and 3 digit numbers written as symbols with virtual-materials;
(4) write 2 and 3 digit numbers in symbols for virtual-material representations of these numbers;
(5) place-value strategy for the addition algorithm;
(6) place-value strategy for the subtraction algorithm;
(7) additive subtraction strategy for subtraction of 2 and 3 digit numbers;
(8) place-value strategy for the multiplication algorithm; and
(9) place-value sharing strategy for the division algorithm.
1. VIRTUAL MATERIALS AND MATHEMATICS LEARNING

1.1 Role of virtual materials

Current pedagogical beliefs emphasise that the abstraction of mathematical concepts and processes is best served by a combination of work with appropriate manipulatives and reflection with peers and teacher. Manipulatives are most obviously physical but mental manipulation can also be undertaken with pictures and diagrams. Reflection is with language and symbols.

Virtual materials provide another collection of manipulations to add to the physical, and pictorial and diagrammatic. Therefore, teaching mathematics can be seen to involve the use of manipulation (physical, virtual, pictorial and diagrammatic materials, written symbols, spoken words) in order to facilitate student development of mental models (internal representations). The kinaesthetic actions associated with physical and virtual manipulatives (physical and mouse movements) assist abstraction by providing mental images to scaffold the symbolism.

Most activity with physical materials involves sliding, joining, separating, grouping, ungrouping, partitioning, turning and flipping actions. All of these actions are available on computer through mouse movements and images of the concrete materials (virtual materials) using computers with commonly available generic software. Thus, computer activities with virtual materials reflect activities with concrete and pictorial materials and their efficacy is bound up with the effectiveness of physical materials.

Parts of what learners construct from interaction with materials are built into the medium of the materials. Students can mentally replicate (in their schemas) the relations and transformations represented by the concrete material, and abstract this mental replication to symbols and mental models; however, there is a gap between action and expression that is difficult to bridge. Physical materials are often very multi-sensory (e.g., they involve colour or have interesting textures or shapes) which can hinder the abstraction process. Pictorial materials are more abstract than concrete materials as the child is expected to imagine any manipulation that may have been required to transform. Physical and virtual material are also inflexible and can be messy.

Thus, virtual materials and actions can be effective in teaching mathematics because they:

1. are a bridge between physical and pictorial being not as overt as physical representations nor as covert as pictorial representations;
2. can be “debugged, reconstructed, transformed, separated and combined together” and saved for later reuse with the same or other students;
3. provide teachers with unique knowledge of all students’ proficiency with all components of the manipulations as the manipulations can be saved and stored for later assessment;
4. can integrate with physical materials in a way that enhances mathematics learning (because of the way they reinforce physical materials); and
5. have capacities for actions, activities and representations not easily available with concrete materials; for example, shapes can be enlarged by specific amounts, turned by specific degrees.
In this way, virtual materials use the visual, symbolic and operational power of the technological media and provide another pedagogical and didactical tool for the media.

Note: overall, the virtual materials developed for this booklet are a very different use of computers in mathematics education than that commonly seen in schools, at least in Queensland. As computer activities, they are relatively simple – students manipulate computer drawn copies of real materials using PowerPoint. As such, virtual materials have comforting similarities to concrete activities and this aspect seems to make it easier for teachers to translate their mathematics teaching to virtual materials and, thus, to computers.

The best way to use virtual materials is to integrate it with other representations:

1. virtual materials should follow work with the physical materials that they copy;
2. virtual materials also work well if integrated with physical materials (interchanging from physical to virtual);
3. virtual materials require student expertise and familiarity with the PowerPoint actions they use (e.g. copy, paste, click and move)

An effective way to teach mathematics is to use the Payne-Rathwell model as below. This means starting with real world problems, modelling with physical, virtual and pictorial materials, and then introducing language and symbols. Then, this should be followed with activities to connect all these 5 forms in all directions (reversing). The models used should follow the order on right, starting with real world moving to physical, virtual and pictorial and finishing with patterning activities (as below).

<table>
<thead>
<tr>
<th>Payne &amp; Rathmell triangle</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real world situations</td>
<td>Sequence</td>
</tr>
<tr>
<td>Models</td>
<td>Physical</td>
</tr>
<tr>
<td>Language</td>
<td>Virtual</td>
</tr>
<tr>
<td></td>
<td>Pictorial</td>
</tr>
<tr>
<td></td>
<td>Patterns</td>
</tr>
</tbody>
</table>

### 1.2 Examples of virtual materials

1. Place value for two-digit whole numbers can be effectively developed by activities with bundling sticks (singly and in bundles of ten) and a tens/ones place-value chart as follows.

<table>
<thead>
<tr>
<th>Child constructs or sees</th>
<th>Child says</th>
<th>Child writes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tens</td>
<td>forty-three</td>
<td>4 3</td>
</tr>
<tr>
<td>Ones</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The strength of this learning approach is that it is multi-representational (providing visuals, language & symbols) and dynamic (showing transformations and changes as well as
Concrete materials to represent place value and size relationships between places

Place value chart used in conjunction with concrete materials or digit cards to represent position and order of the places

Pictorial representation

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Concrete to pictorial material represents a sequence of abstraction because concrete materials can be physically manipulated whereas pictures cannot – the child is expected to imagine any manipulation.

(2) Virtual base-10 blocks on a computer can be manipulated ("clicked and dragged") similar to the real blocks and provides another way in which numeration can be taught. As follows, such virtual materials should provide a conceptual bridge from concrete to pictorial representations.

(3) Space and shape activities can also be taught by sliding, flipping and turning, as follows. In particular, students can easily undertake tessellations with virtual materials. Assembling a class set of real materials is time-consuming.

(4) Tessellations can be easily completed virtually. Sliding, flipping and turning virtual shapes requires only one template, which can be downloaded for individual student’s use. The students themselves can then quickly copy the shapes required and, with respect to tessellations, have access to a variety of colours to enhance the final product (as follows).

1.3 Examples of virtual material lessons

(1) 2-digit numeration. Two-digit numeration activity uses base-10 blocks (as follows) to represent numbers. The mouse actions involved in picking up and placing the blocks on the
virtual place value chart are very similar to the hand movements that pick up and concrete place blocks on a real place value chart.

(2) Polygons. Students “click and drag” shapes to assess the extent to which the students understood the concept of a polygon.

(3) Telling the time. Students "told time" on a virtual clock (as follows). This was after they had to stand up and physically rotate their bodies using their extended arms to point to a given position (e.g., 4 o’clock) on a large circle (clock face) drawn on the floor.

(4) Flips, slides, and turns. Flips, slides and turns can be introduced through tangrams (as follows).
(5) **Patterning.** Assisting recall of basic multiplication facts by exploring patterns by colouring squares on virtual hundred boards to show multiples of 5, 3 or 9 and so on. The action of the mouse to colour the squares is very different to the action of a coloured pen on a paper 100s board. However, the ability of the repeat button to quickly colour squares and the *PowerPoint* program to edit errors made the virtual boards very attractive to the students.

(6) **Scales.** Placing number on number lines to reinforce teaching of scales (as below).

Identify the number at A. Show 330 on the scale.

![Number line diagram]

The virtual scales developed to practice this identification and finding of numbers were based on authentic scales, for example, tachometers, measuring cylinders and altimeters. The students could move arrows or empty and fill cylinders to identify the numbers. The students found this motivating and the virtual activities were well received.
2. WHOLE NUMBER NUMERATION

2.1 Meanings and materials

Whole number numeration has 4 components/meanings.

(1) **Place Value**  Students should know that values of numbers are determined by the position of their digits in relation to the 1’s position (e.g. 257 is 2 hundreds, 5 tens and 7 ones). When students see a number, they should realize that what the number means is determined by position and the value of that position (where the value is based on multiples of 10).

(2) **Counting**  Students should know that numbers follow the same counting pattern in each place-value position, e.g. counting forward – 472, 482, 492, 502, 512 (go upto 9, then back to 0, number on left increases by 1); counting backward – 5324, 5224, 5124, 5024, 4924 (go down to 0, then back to 9, number on left decreases by 1).

(3) **Rank**  Students should know that, regardless of how many different digits are in it, the number represents one position on a number line and that its overall value is determined by how far down the line it is.

(4) **Multiplicative structure**  Students should know that adjacent place value positions are related by multiplication and division by 10 (i.e., move one place to the left in x10, move one place to the right is ÷10, move two places to the left is x 100, ad so on).

This booklet focuses on the first meaning/component, place value. This meaning considers numbers in a **separated** way – as a combination of digits relating to separate place-value positions. Because the value of place-value positions increases to the left, a digit to the left is 10 times the value of the digit to its right. It is the best understanding from which to develop the traditional separated strategies for algorithms.

To do this the booklet focuses on 3 materials. These materials show value (the material to the left shows the 10 times change) and position (they can be placed on place-value charts (PVCS)):

(1) **Bundling straws (and 1-10 Place Value Charts (PVC’s))**  Straws can be used to represent ones and tens bundled into bundles of 10 to represent tens.

(2) **Multi Base Arithmetic blocks (MAB) (and 1-10-100 PVCs)**

    MAB units, longs, flats and blocks can be used to represent 1s, 10s, 100s and 1000s on PVCs (This booklet focuses on 3-digits) The blocks need to be traded to change position unlike bundling straws which can be bundled and un-bundled.

(3) **Money P1-10-100 PVC**

    Although not representing the size relationship as accurately as bundling straws and MAB, there is some effectiveness in using money on PVC ($1, $2 counts & $5 roles for ones, $10, $20 and $50 notes for tens, and $100 notes for hundreds)

The crucial thing is to allow the students time to familiarize themselves with the materials and to learn how the materials represent numbers before using the virtual materials. Therefore, we now give some activities with handling straws and MAB before looking briefly at money.
2.2 Two digit numbers and bundling straws

The idea here is to develop activities that show that 64 is 6 bundles of 10 and 4 ones and is stated as “sixty-four”. The problem is difficulties with the English language: 17 should be “onety seven”, 35 should be “threety five”, and 70 should be “seventy zero”. The special names for the teens and the leaving out of the zero in language are particular problems. Therefore these numbers are done last and given special attention.

A sequence of activities for introducing 2 digit numbers with bundling straws are given below. They should be completed before the virtual materials on bundling straws are used.

Materials: 1-10 PVC (see end of this section), bundling straws rubber bands, writing paper, pencil, dice.

Language: Ones, tens, groups, bundle

Activity 1: Getting to know the Place Value Chart & Bundling Straws

(1) Show the PVC. Say: Read the place names as I point to them (pointing from right to left). Which place is largest in value? [If students don’t know, say: Would you rather have 100 dollars, 10 dollars or 1 dollar? Why?] Which place is smallest in value? [If students don’t know, say: What would be the smallest amount of money – 100 dollars, 10 dollars or 1 dollar?]

(2) Show some loose bundling straws. Say: These are called ones. Where would you put them on the Place Value Chart? [Ones place]

(3) Show a bundle of 10. Say: These are called tens. Where would you put them on the Place Value Chart? [Tens place]

(4) Make sure the student knows where to put the tens and ones by doing the following. Hand the student some ones, ask them what they’re called and to put them on the PVC in the right place. Hand the student another bundle of 10 and ask them what they’re called and to put it on the PVC in the right place.

   REPEAT UNTIL THE STUDENT KNOWS THAT THE ONES GO IN THE ONES PLACE AND THE TENS IN THE TENS PLACE.

Activity 2: Making groups of 10

(1) Making groups of 10. Say: Pick up a handful of ones. We’re going to see how many tens we can make out of these ones. Do you have enough ones to make a group of 10? Show me. Put a rubber band around them to hold them together.

   Where will you put the 10 on the Place Value Chart – the tens place [pointing to the place] or the ones place [pointing]? Do you have enough ones to make another ten? Repeat until no more tens can be made. Then, say: What number have you made? Count by tens first [ten, twenty, thirty, forty, etc]].

(2) Repeat the activity above a couple of times with a different number of ones.

Activity 3: Making Numbers

(1) Say: Show me the number that has 2 tens 7 ones on the Place Value Chart. Show me the 2 tens part. Show me the 7 ones part.

   Repeat with these numbers: 4 tens 2 ones 3 tens 8 ones 1 ten 5 ones 5 tens 0 ones
(2) Say: Show me the number, twenty-five, on the PVC. Show me the twenty part. How many tens make twenty? Show me the 7 part.
Which is worth more – the twenty part or the 7 part?
Repeat for: thirty-one; forty-three; sixty

**Activity 4: Writing Numbers**

(1) Tutor: Use the bundling straws and Place Value Chart to make numbers like the one shown in the diagram below. Ask the student to write the number and then read it.

**Activity 5: Counting and trading ones for tens**

(1) Tutor: Show 36 with bundling straws on the PVC. Ask the student to write the number.
Put out another one straw, saying: I’m adding 1 more one. Write the number now.
Repeat until you get to 39. Put out 1 more and ask: Do I have enough ones to trade for a ten? [Trade] What number do I have now? Write it.
Keep going for a few more numbers. Ask the students to read the numbers he or she has written.

(2) Swap roles. Let the student make the numbers while you write the numbers.

(3) Repeat the above but have the student use the calculator to count as you build the numbers with bundling straws.

**Activity 6: Ungrouping tens to make ones − bundling straws**

(1) Put 2 tens 7 ones on the PVC. Say: How many tens do you have? How many ones?

(2) Roll the dice & take that number of ones away. [Suppose 4] Say: Do you have enough ones to take away 4 ones? [Yes] Show me.

(3) Roll the dice again. [Suppose 5] Say: Do you have enough ones to take away 5 ones? [If No, then say:] How can you get more ones from this number? [Ungroup a 10 and put the 10 ones in the Ones place.] How many tens do you have now? [1] How many ones?

(4) Repeat until the student has no more straws left.

(5) Play "Lose 5 tens 6 ones" if you have 2 or more players. Each player starts with 5 tens 6 ones of bundling straws on the Place Value Chart.
Take turns in rolling the dice and taking off that many ones, trading tens for ones when necessary. First player to lose all the straws is the winner.
2.3 Three digit numbers and MAB

The idea here is to show with physical materials that 375 is three hundreds, seven tens and five ones and is stated “Three hundred and seventy five”. Once again, there are English language problems with teens and zeros – 513 should be (five hundred and onety three”, 450 should be “four hundred and fivety zero”, and 702 should be “seven hundred and zeroty two” for perfect language patterns.

A sequence of activities for introducing 3 digit numbers with MAB is given below. They should be completed before the virtual materials on MAB are used.

Materials: 1-10-10 PVC (see end of this section), MAB, pencil, paper, dice or playing cards, digit cards as below.

Language: units, longs & flats, hundreds, trading

Activity 1: Getting to know the MAB blocks; Trading

(1) Check whether your student knows that 10 MAB ones = 1 MAB ten by having him/her line up the small blocks beside the long block (see the picture below).

(2) Then check that s/he knows that 10 MAB tens = 1 MAB hundred.

(3) Play “Lose 6 tens 6 ones” with MABs on the Place Value Chart. Put out 66 on chart with MAB. Throw two dice, add and remove that number of ones from chart, regrouping as go. Record number of MAB with calculator as play the game. First to 0 wins. Don’t forget to ask the questions: Do you have enough ones to take away XXX ones? Make sure the student is familiar with the word, trade, and its meaning.

Activity 2: Introducing hundreds

(1) Ask the student to show 99 with MABs on the PVC.

(2) Say: Show me the ninety part. How many tens? Show me the nine part? How many ones? Add 1 more one. How many ones do you have now? Do you have enough ones to trade for a ten? [Student should trade] How many tens do you have now? Do you have enough tens to trade for a hundred? [Student should trade] What number do you have now?

(3) Keep adding ones [make sure the student puts them in the ones place] until you get to 112. Ask the student to read the number now. Ask how many hundreds? How many tens? How many ones?

(4) Ask the student to show each of these numbers with MABs on the Place Value Chart.

- five hundred and seventy-four
- three hundred and thirty-three
- two hundred
- eight hundred and thirty-two
- seven hundred and forty
- four hundred and nine
Activity 3: Trading Games

(1) Play “Win 3 hundred.” Start with 6 tens. Roll the dice and put out that number of tens (make sure the student puts the tens in the tens place and trades when possible). Ask the student to say the number each time (e.g., one hundred and twenty).

(2) Play “Lose 2 hundred.” Start with 2 hundred, roll the dice and take away that number of tens, trading when necessary.

Activity 4: Calculator Counting

Student: Using the calculator.

(1) Counting by hundreds. Start with 364 on the PVC. Ask the student to read the number [three hundred and sixty-four]. Tutor to write the number on the paper.

(2) Say: Add 1 hundred. [T. puts out 1 more H and writes the new number under the starting number; the student adds 100 on the calculator.]

(3) Keep doing this until you have 964. Ask the student to check whether it’s the same as your blocks number.

(4) Swap calculator and blocks & paper and play new game. When the game is finished, ask the student what place changed first when s/he was counting by 100s. Check the written numbers to see if only the 100s changed.

Activity 5: Reading numbers with zeros and tens

(1) Tutor gives a real-world story: Malcolm counted this many cars in Rockhampton.

(2) Tutor builds the number.

(3) Student says the number: Two hundred and sixteen

(4) Tutor says: Show me the 2 hundred part; show me the ten part; show me the six part. Discuss what the number should/could have been called: 2 hundred and “onety-six”

(5) Compare the number with two hundred and sixty and two hundred and sixty-one. Put out MAB for both these numbers. Tutor says: How many ones in two hundred and sixteen, how many in two hundred and sixty, and how many in two hundred and sixty-one? Repeat for tens and hundreds.

(6) Repeat for other numbers. For example: 311; 518; 470; 407; 200.

(7) Swap roles – the tutor says the number and the student shows it with MAB. Don’t forget to ask the student to show the hundreds part of the number, the tens and so on.
**Activity 6: Writing numbers with tens and zeros**

1. Tutor gives a real-world story: Malcolm counted this many cars in Rockhampton.
2. Tutor builds the number.

```
<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>
```

3. Student says the number Two hundred and seventeen
4. Tutor says: Show me the 2 hundred part; put a digit card underneath to show this part. Show me the tens left over (the ten part); put a digit card underneath to show this part. Show me the left over ones (the five part); put a digit card underneath to show this part.
5. Tutor asks the student to make a small PVC on paper and then write the number shown on the large PVC with MAB.
6. Compare with 270 and 271. Make with MAB and add digit cards. Discuss the differences in the hundreds, tens & ones and the similarities in the language.
7. Repeat for other numbers. For example: 412; 813; 630; 702; 300
8. Swap roles – the tutor writes a number on the small PVC & says the number; the students show it with MAB.
9. Don’t forget to ask the student to show the hundreds part of the number, the tens and so on.

**Activity 7: Relationship between place-value positions**

1. Tutor says: Place a 4 on the ones of the PVC. Tutor asks: How many ones, how many tens, how many hundreds? Ask students to write number on a small PVC (as on right) and to put on a calculator.

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<table>
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<tr>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>
```

2. Tutor says: Move the 4 to the tens of the PVC. Tutor repeats questions and asks students to write number on a small PVC. Tutor asks: How do you change 4 ones on calculator to make 4 tens? Show x10.
3. Repeat moving 4 from tens to hundreds. Tutor asks: How do you change 4 tens on calculator to make 4 hundreds? Show x 10. Tutor asks: Can you see a pattern? What happens if a number moves from ones to tens to hundreds?
4. Repeat the above moving the 4 from hundreds to tens to ones.
5. Repeat 1 to 4 above using the following starting points: 6, 23
6. Digit cards on PVC. Place digit or digits on PVC and write number on calculator. Move digit(s) left or right. Ask students to work out the multiplication or division to change the calculator number in the same way.
Reverse the procedure. Type a number on the calculator (one digit, a ten or a hundred) and place digit card on PVC. Multiply or divide by 10 or 100. Ask students to move the digit to match. Ask if the students can see a pattern (move to the left, x10; move to the right /10)

**Activity 8: Game – "Close, closer, closest"**

1. Put 200 on PVC with MAB for each player. Each player enters 200 on calculator.
2. Remove 10, J, Q, Ks from a card deck. Shuffle cards. Deal two cards to each player.
3. Players, in turn, form a two-digit number from their cards, add this MAB to PVC (completing all trading) and add number to calculator. They read the number and say how many tens and ones at each play.
4. The game ends when the first player passes 600. The winner is the player closest to 600 at that point.
5. **Option:** Start at 600 and remove the two-digit number at each turn. Closest player to 200 wins.

### 2.4 Three digit numbers and money

The idea here is to use money to reinforce 3 digit number understanding. Money does not relate perfectly in size but it is real and an application, so can motivate. It also allows real world situations to be explored (e.g. shopping, getting change).

Before doing the virtual activities on money:

1. make students familiar with money;
2. do the virtual activities with real money;
3. play shop – adding prices and giving change.

Note the usefulness of the additive subtraction method. This is sometimes called ‘The Shopkeepers’ Algorithm’ as it was used in shops before computer/calculator.

Prepare students for their being 3 options. ($1 & $2 coins, & $5 note) in the ones position and 3 options ($10, $20 and $50 notes) in the tens position. Prepare students also for having too many tens in the tens position (e.g. two $10s, 3 $20s and a $50) & having to trade to get the correct number/amount of money.
### 10/1 PLACE VALUE CHART

<table>
<thead>
<tr>
<th>TENS</th>
<th>ONES</th>
</tr>
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<tbody>
<tr>
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<td></td>
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</table>
### 100/10/1 PLACE VALUE CHART

<table>
<thead>
<tr>
<th></th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
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</table>
3. WHOLE NUMBER ADDITION AND SUBTRACTION

3.1 Methods for adding and subtracting.

There are three strategies for adding and subtracting:

1) **Separation (Place Value).** This method is based on considering number as separate place-value positions. The computations are undertaken by separating the number into place-value positions, operating on each position and then combining. For example:

\[\begin{array}{c}
247 \\
+ 386 \\
\hline
633
\end{array}\]

\[\begin{array}{c}
10 \\
6015 \\
- 268 \\
\hline
447
\end{array}\]

The method acts on the hundreds, tens and ones separately – it uses place-value oriented materials.

2) **Sequencing.** This method is based on a rank or number line orientation to number. The computations keep one number as is and add/subtract bits of the other number in sequence. For example:

\[\begin{array}{c}
247 \\
547 \\
627 \\
633 \\
\hline
715 \\
515 \\
455 \\
447 \\
\hline
- 200 \\
- 60 \\
- 8
\end{array}\]

The 300, the 60 and the 8 are added separately and the 200, 60 and 6 are subtracted separately. It is not necessary to have any order – the numbers could be added or subtracted in reverse order, or in a mixed up order.

3) **Compensation.** This method is again based on rank. Numbers near the ones in the computation are chosen to make the operation easy and then the changes compensated for. For example:

\[\begin{array}{c}
247 \\
+ 386 \\
\hline
647 \\
- 14 \\
\hline
633
\end{array}\]

\[\begin{array}{c}
247 \\
+ 400 \\
\hline
647 \\
- 14 \\
\hline
633
\end{array}\]

\[\begin{array}{c}
715 \\
- 268 \\
\hline
415
\end{array}\]

\[\begin{array}{c}
715 \\
- 300 \\
\hline
415
\end{array}\]

\[\begin{array}{c}
715 \\
+ 32 \\
\hline
447
\end{array}\]

For 247+386, we added 14 too much and so we had to remove it. For 715-268, we took off 32 too much, so we had to add it.

These three methods are all available for students to use. However, for this booklet and the virtual materials, we will be focusing mainly on the separation methods.
3.2 Teaching separation methods for addition

Separation methods are best taught with place value charts (PVCs) and size materials such as bundling sticks, MAB and money placed on top of these PVCs. Below are plans of activities on how to use bundling sticks to teach this separation method for addition. MAB and money can be used similarly to the bundling sticks.

It is crucial that students experience addition with real bundling sticks, MAB and money before moving to do the virtual bundling stick, MAB and money activities.

Materials: Bundling sticks, calculators, 10/1 place value chart (PVC), materials for games and activities (card decks, dice, newspaper advertisements for things to sell in a shop.).

Activity:


2. Ask: What do we add first? [Ones] Say: Join the ones. Ask: Do you have enough ones to make a 10? [yes] Say: Bundle ten sticks to make a 10. Ask: How many ones are left? Say: Record this on your place value chart.

Ask: What do we add next? [Tens] Then say: Join the tens. Say Calculate the final tens and ones. Write this on your place value chart.
Note: It is possible to do tens first:

3.3 Teaching separation methods for subtraction

Separation methods are best taught with place value charts (PVCs) and size materials such as bundling sticks, MAB and money placed on top of these PVCs. Below are plans of activities on how to use bundling sticks to teach this separation method for addition. MAB and money can be used similarly to the bundling sticks.

It is crucial that students experience addition with real bundling sticks, MAB and money before moving to do the virtual bundling stick, MAB and money activities.

Materials: Bundling sticks, calculators, 10s-1s place value chart, materials for games (card deck, dice), worksheets A and B.
**Activity:**

(1) Pose a problem: You pay a $37 bill with $50; how much change? Ask: How many tens and ones in 50? [5 tens, 0 ones] Say: Show this on place value chart with bundling sticks. Say: Record 50–37 on paper.

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
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<tr>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>− 3</td>
<td>7</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
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<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>− 3</td>
<td>7</td>
</tr>
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</table>

(3) Say: Subtract 7 ones by sliding 7 sticks down the chart. Subtract 3 tens by sliding 3 bundles down the chart. Ask: How many is left? [1 ten and 3 ones or 13] Say: Record this on paper.

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<tr>
<th>Tens</th>
<th>Ones</th>
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</thead>
<tbody>
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<td></td>
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</table>

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>− 3</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Note there are other ways it could be done, for example:

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>− 3</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Down 7</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
(4) Repeat steps 1 to 3 for another problem. Always ask: Do you have enough ones to subtract the ones? If not, what do you do? How many ones does this give you? How many tens?

(5) Discuss what the students have done. Ask: How would you do this without sticks? Can you think in the mind what to do? Repeat steps 1 to 3 for 54-27. Try to encourage the students not to use bundling sticks and place value charts.

### 3.4 Teaching the additive subtraction method

The additive subtraction method is based on rank/number line understanding of number. However, it can be taught via place-value materials and its usefulness makes it something which is worth considering in this booklet. Each set of virtual materials (bundling sticks, MAB and money) has a file of activities on the additive subtraction method.

The additive subtraction method is based on “thinking addition” in subtraction. That is, changing, for example, “8 subtract 3” to “3 plus what equals 8”. Thus we think of 8 – 3 not as removing 3 from 8 but as how many more are needed to build from 3 to 8.

This method is very useful for time and money (where it is called “the shopkeepers algorithm”) and other forms of measurement.

To moderate the method for 2 and 3 digit numbers, change the order of the numbers to the small numbers first as follows:

<table>
<thead>
<tr>
<th>Thinking</th>
<th>Action</th>
<th>Algorithm</th>
</tr>
</thead>
</table>
| 64 – 27 is thought of as 27 + what = 64 | We build slowly from 27 to 30 then to 60 and finally to 64, adding the increments. | 27 \> 3  
|               |                                             | 30 \> 30 
|               |                                             | 60 \> 60 
|               |                                             | 64 \> 4  
|               |                                             | 47 \> 47 |

| 628 – 349 is thought of As 349 + what = 628 | We build slowly from 349 to 350, 350 to 400, 400 to 600, and finally 600 to 629, adding all the Increments. | 249 \> 1  
|                                              |                                                            | 350 \> 50  
|                                              |                                                            | 400 \> 200 |
|                                              |                                                            | 600 \> 28  
|                                              |                                                            | 628 \> 279 |

This can be done with bundling sticks, MAB and money by making 2 spaces. In the first space put the small number (e.g., for 87-38, this is the 38). Then add sticks, MAB or money until build from small to large number. These sticks are placed in the 2\textsuperscript{nd} space. Adding all the representations in the 2\textsuperscript{nd} space gives the answer.

So to introduce additive subtraction:

1. Relate “8 – 3 = what” to “3 + what = 8” (e.g., join and separate sets of numbers),
2. use sticks, MAB and money to build from smaller to larger numbers; and
3. practice the algorithm.
4. WHOLE NUMBER MULTIPLICATION AND DIVISION

4.1 Methods for multiplying and dividing

Similar to addition and subtraction, there are three methods for multiplying and dividing 2 and 3 digit numbers:

(1) **Separation** – this is the traditional method where things are done by separating into place values, for example:

\[
\begin{array}{c}
37 \\
\times 8
\end{array}
\]

\[
\begin{array}{l}
56 \\
240
\end{array}
\]

\[
\begin{array}{l}
296
\end{array}
\]

\[
\begin{array}{c}
83 \\
\times 7
\end{array}
\]

\[
\begin{array}{l}
72 \\
27
\end{array}
\]

\[
\begin{array}{l}
0
\end{array}
\]

Sharing tens

Sharing ones

(2) **Sequencing** – here one number (the larger for 2 x 1 digit) is held unseparated and parts of the other number are done in sequence, for example:

\[
\begin{array}{c}
37 \\
\times 8
\end{array}
\]

\[
\begin{array}{c}
74 \\
148
\end{array}
\]

\[
\begin{array}{c}
296
\end{array}
\]

\[
\begin{array}{c}
9 \\
\times 3
\end{array}
\]

\[
\begin{array}{c}
27 \\
90
\end{array}
\]

\[
\begin{array}{c}
27
\end{array}
\]

\[
\begin{array}{c}
0
\end{array}
\]

50 groups of 9

20 groups of 9

10 groups of 9

3 groups of 9

83

(3) **Compensations** – a simple multiplication or division is found and then compensated for, for example:

\[
\begin{array}{c}
37 \\
\times 8
\end{array}
\]

\[
\begin{array}{c}
37 \\
\times 10
\end{array}
\]

\[
\begin{array}{c}
37 \\
\times 10
\end{array}
\]

\[
\begin{array}{c}
37 \\
\times 10
\end{array}
\]

\[
\begin{array}{c}
370 \\
370
\end{array}
\]

\[
\begin{array}{c}
333 \\
296
\end{array}
\]

\[
\begin{array}{c}
296
\end{array}
\]

For this booklet, we focus on method (1) separation. This is based on place-value separated materials such as sticks, MAB and money.

4.2 Teaching separation multiplication

Separation methods are best taught by PVCs (which show position) and value/size–based materials (sticks, MAB and money). Below are plans of activities to teach multiplication using MAB. Sticks and money can be used similarly to MAB.

It is crucial that students experience multiplication with real/physical bundling sticks, MAB and money before moving to virtual copies of the sticks, MAB and money.

Separated multiplication is based on lots of, e.g, 24 x 3 is 3 lots of 24. That is 24 + 24 + 24.
Materials: MAB blocks, place value chart (PVC), pen, paper, calculator, attached resources.

Activity:

1. **Meaning.** Ask students: What does 4×37 mean? Focus on set model ["4 lots of 37"]. Ask: Can you think up a story?

2. **Separation.** Ask students: How can we show 37 with MAB. Ask: How many tens? [3]. How many ones left over? [7]. Direct, put out 3 tens in tens place. Put out 7 ones in ones place.

3. **Distributive Law.** Ask, what does 4×37 mean? [4 lots of 37]. State, we have one 37 on PVC with MAB. How can we show 4×37? [4 lots of 37]. Put this on PVC with MAB. State, look at tens. How many 30s? Look at ones. How many one’s? Get students to finish 4 lots of 37 = ________ of 30s and ________ of ones.

4. **Computation.** Put out 4 lots of 37 on PVC and record as you go. Ask, how many in each lot? [37]. How many lots? [4]. How do we write this? [37 on top, × 4 under and a line to show equals].

   **Step 1**

   ![Step 1 Diagram]

   **Step 2**

   Discuss that we have to find 4 lots of 7 and regroup. Separate 37 into tens and ones and look at the ones. Ask, what is 4×7 (use calculator)? [28]. Ask, have we enough ones to make a ten? [Yes]. Have we enough ones to make 2 tens? [Yes]. Have we enough ones to make 3 tens? [No]. Direct, make the tens. Move to tens position. How many ones are left over? [8]. How many extra tens? [2]. Write 28 under the 4.

   ![Step 2 Diagram]

   **Step 3**

   Say, now look at tens. How many tens in each lot? How many lots? Discuss that we have to find 4 lots of 30 and regroup. Ask, what is 4×30 (use calculator)? [120]. How
many tens? Remind that $4 \times 3 = 12$ means $4 \times 30 = 120$ or 12 tens. Ask, do we have enough tens to form a hundred, two hundred, ...? [one 100]. Make the one hundred and move to hundreds position. How many tens left over? [2]. How many extra hundreds? [1]. Write 120 under the 28.

\[
\begin{array}{|c|c|c|c|}
\hline
100 & 10 & 1 \\
\hline
37 \\ 
\times 4 \\
\hline
28 & 120 \\
\hline
37 \\ 
\times 4 \\
\hline
120 & (4 \times 30) \\
\hline
28 & (4 \times 7) \\
\hline
\end{array}
\]

Step 4
Add the 28 and 120 to give 148, the result of $4 \times 37$. Note: can do the tens first:

\[
\begin{array}{|c|c|c|}
\hline
100 & 10 & 1 \\
\hline
37 \\ 
\times 4 \\
\hline
120 & (4 \times 30) \\
\hline
28 & (4 \times 7) \\
\hline
\end{array}
\]

(5) Repeat Step 4 for example $3 \times 265$:

Step 1
\[
\begin{array}{|c|c|c|}
\hline
100 & 10 & 1 \\
\hline
265 \\
\times 3 \\
\hline
3 lots of 265 \\
\hline
\end{array}
\]

Step 2
\[
\begin{array}{|c|c|c|}
\hline
100 & 10 & 1 \\
\hline
265 \\
\times 3 \\
\hline
15 & (3 \times 5) \\
\hline
\end{array}
\]

Step 3
\[
\begin{array}{|c|c|c|}
\hline
100 & 10 & 1 \\
\hline
265 \\
\times 3 \\
\hline
15 & (3 \times 5) \\
\hline
180 & (3 \times 60) \\
\hline
\end{array}
\]
Step 4

<table>
<thead>
<tr>
<th>100</th>
<th>10</th>
<th>1</th>
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\[
\begin{align*}
265 & \times 3 \\
15 & \quad (3 \times 5) \\
180 & \quad (3 \times 60) \\
600 & \quad (3 \times 200) \\
\hline
795 & \\
\end{align*}
\]

Step 5

Add all the parts. Note: could have done larger place values first as on right:

\[
\begin{align*}
265 & \times 3 \\
15 & \quad (3 \times 5) \\
180 & \quad (3 \times 60) \\
600 & \quad (3 \times 200) \\
\hline
795 & \\
\end{align*}
\]

(6) Repeat Steps 4 and 5 but without materials

(a) \(7 \times 48\): separate 48 into 40 and 8 and multiply
(note: \(7 \times 8 = 56\), \(7 \times 4 = 28\) so \(7 \times 40 = 280\))

(b) \(6 \times 149\): separate 149 into 100, 40 and 9 and multiply
(note: \(6 \times 9 = 54\), \(6 \times 4 = 24\), \(6 \times 1 = 6\) so \(6 \times 40 = 240\) and \(6 \times 100 = 600\))

4.3 Teaching separation division

Separation methods are best taught by PVCs (which show position) and value/size–based materials (sticks, MAB and money). Below are plans of activities to teach multiplication using MAB. Sticks and money can be used similarly to MAB.

It is crucial that students experience multiplication with real/physical bundling sticks, MAB and money before moving to virtual copies of the sticks, MAB and money.

Separated division is based on sharing, e.g., \(24 \div 3\) is sharing 24 lollies amongst 3 people.

Materials: MAB ( for 100, for 10, for 1), plates, place value chart (100, 10, 1), pen, paper, attachments

Activity:

(1) Consider the division example \(92 \div 4\). From booklet 5.2, this can be considered as “92 shared amongst 4” or “how many groups of 4 in 92”. Discuss this with students. Ask: How can we think about \(92 \div 4\)? What does \(92 \div 4\) mean? Also ask: Is there another way to write it? Lead students to remember that \(92 \div 4\) can also be written as \(4 \div 92\). State: For computation we use this second way! [point to \(4 \div 92\)]

(2) State: The first strategy is based on separating the number being divided [quotient] into place values and sharing! Ask: How many tens in 92? [9] How many ones left
over? [2] State: Show this with MAB. [put out 9 tens and 2 ones on place value chart]
Show the 4 to share amongst. [put out 4 plates]

Ask: What shall we share first? [Let students discuss but encourage them to see that in
real life it is easier to share the larger notes first – the tens]

State: Share the tens. [“a ten to you, a ten to you” and so on] Ask: How many tens did

Ask: What can we do with the left over 10 to keep sharing? [discuss but encourage
students to see need to break/trade it for ones] Ask: How many ones does this give
us? [12] State: Share these ones.

many ones are left over? [0] What is the answer? [2 tens and 3 ones – 23]

(3) Repeat Step 2 but record with a pen and paper as go

Step 1  Setting up the sharing.
Step 2  Sharing the larger place value – the tens.

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Step 3  Regrouping.

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Step 4  Sharing the ones.

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</table>

Step 5  State the answer. [23]

(4) Repeat for 771 ÷ 3. State: Act this out with MAB and 3 plates. State: Point at numbers and material as you ask the questions.
Step 1  
Set up. 

How many hundreds?  
How many tens left over?  
How many ones left over?  
How many to share amongst? 

Step 2  
Sharing the hundreds. 

How many hundreds to each plate?  
How many hundreds used?  
How many hundreds left over? 

Step 3  
Regrouping 100s $\rightarrow$ 10s. 

How many tens now?
Step 4  Sharing the tens.

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<th>T</th>
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</tbody>
</table>

\[ \begin{array}{c}
25 \\
6 \\
17 \\
15 \\
2
\end{array} \]

How many tens to each plate?
How many tens used?
How many tens left over?

Step 5  Regrouping 10s → 1s.

<table>
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<tr>
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</tbody>
</table>

\[ \begin{array}{c}
25 \\
6 \\
17 \\
15 \\
21
\end{array} \]

How many ones now?

Step 6  Sharing the ones.

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<th>O</th>
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<tr>
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<td></td>
<td></td>
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</tbody>
</table>

\[ \begin{array}{c}
257 \\
6 \\
17 \\
15 \\
21 \\
21 \\
0
\end{array} \]

How many to each plate?
How many used?
How many ones left over?

Step 7  State the answer. [257]
5. Go through separation strategy without material for example 936 ÷ 4. Point to numbers as ask questions.

   **Step 1**  Set up.

   \[
   4 \overline{)936}
   \]

   *How many to share?*
   *Write this down.*
   *How many to share amongst?*
   *Write this down.*
   *Draw \( \underline{\hspace{1cm}} \) to show division.*

   **Step 2**  Divide the 100s.

   \[
   2 \overline{)936}
   \]

   *How many 100s to divide? [9]*
   *How many 100s to each group? [9 ÷ 4 gives 2 to each group]*
   *How many 100s left? [1]*

   **Step 3**  Regroup and divide the 10s.

   \[
   4 \overline{)936}
   \]

   *Change the 100s to 10s.*
   *How many 10s to divide? [13]*
   *How many 10s to each group? [13 ÷ 4 gives 3 to each group and 1 left over]*
   *How many 10s left? [1]*

   **Step 4**  Regroup and divide the 1s and state answer.

   \[
   4 \overline{)936}
   \]

   *Change the 10s to 1s.*
   *How many 1s to divide? [16]*
   *How many 1s to each group? [4]*
   *How many 1s left? [0]*
   *What is the answer? [234]*

6. Repeat direction 5 for \( \overline{4)828} \). Discuss the zero for tens.