YuMi Deadly Maths

Year 7 Teacher Resource:

NA – What's the overlap?

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

YuMi Deadly Maths Year 7 Teacher Resource: **NA – What's the overlap?**

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ACKNOWLEDGEMENT

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in which the mathematics ideas for this resource were developed,
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Year 7 Number and Algebra

What's the overlap?

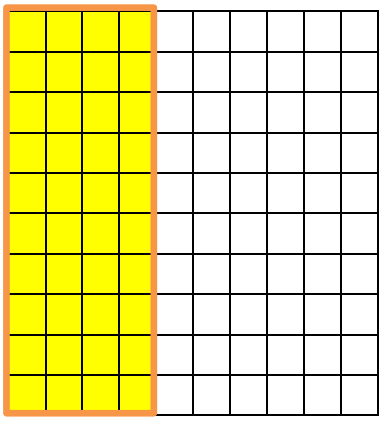
Learning goal	Students will multiply a decimal by a whole number and by a decimal. They will solve problems involving decimal multiplication using efficient strategies.
Content description	Number and Algebra – Real numbers • Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
Big idea	Number – Factor-factor-product
Resources	Maths Mat, elastics of different colours, coloured squares and strips, grid paper, coloured pencils

Reality

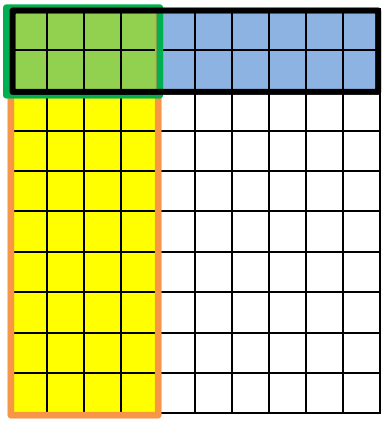
Local knowledge Discuss what a decimal number is [e.g. Is saying: *A number with fractional place values (tenths, hundredths, etc.)* good enough? Does it have to be less than one? Can it be a negative?] *How can we tell that a number is a decimal?* [It contains place-value positions that include tenths.] *What distinguishing mark or signpost does it have as a consequence of having a tenths position?* [It has a decimal point.] *Where is the place of the decimal point?* or *Where does the decimal point always sit?* [The decimal point never changes; it is always placed with the ones and on the right of the ones, separating the whole number place values from the decimal place values.] *Where do we find numbers that can have decimals?* [money; metric measurements].

Prior experience Check that students understand the multiplicative structure of the decimal place-value system and are fluent in moving the digits to the left of the decimal point when multiplying by powers of 10 (the number gets bigger) and to the right when dividing by powers of 10 (the number gets smaller).

Kinaesthetic Maths Mat: Identify the whole, the large square, as the unit. Have four students take an orange elastic and show $^{4}/_{10}$, 0.4 **of the whole**, vertically on the Maths Mat (4 rows of tenths or 40 hundredths – diagram A). Another two students take a black elastic and show $^{2}/_{10}$, 0.2 **of the whole** horizontally on the Maths Mat. Have other students place green coloured squares where the orange and black elastics overlap. Students place a green elastic around the part where the blue and yellow parts overlap (diagram B). *What fraction does this green part show?* $^{2}/_{10}$ **of the** $^{4}/_{10}$, 0.2 of 0.4 (or $^{4}/_{10}$ **of** $^{2}/_{10}$, 0.4 of 0.2) [8 hundredths, 0.08]. Note the commutative principle of multiplication: $a \times b = b \times a$ (Students at corners hold elastics of all/both colours.)



A. $^{4}/_{10}$ or $^{40}/_{100}$, 0.4 or 0.40



B. $^{2}/_{10} \times {}^{4}/_{10} = {}^{8}/_{100}$, $0.2 \times 0.4 = 0.08$

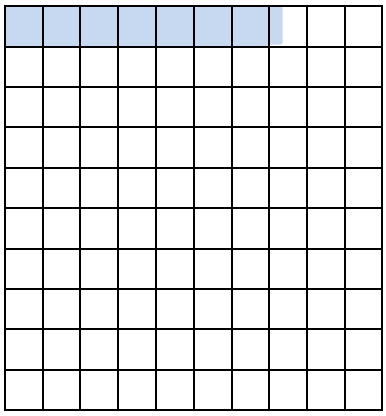
What fraction does the yellow part bounded by the orange elastic show? [four tenths vertically or 40 hundredths **of the whole**]. *What fraction does the blue part bounded by the black elastic show?* [$^{2}/_{10}$ or 20 hundredths **of the whole**]. *Is the green section part of the yellow section? Is it also part of the blue section? What do we call the part where two roads meet?* [intersection]. *The intersection or overlap of the two decimal factors gives the result (product) when we multiply decimal factors together. What do you notice about the result when two decimals less*

than one are multiplied? [The answer is smaller than both the decimals (factors).] *Compare the number of decimal place-value positions in the factors with the number of decimal place-value positions in the answer (product). What do you see? Use the process above to find if this happens in other similar examples, e.g. 0.6 × 0.8.*

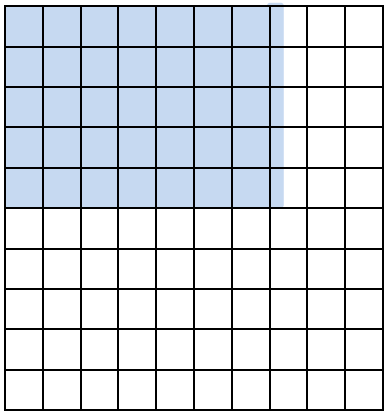
Abstraction

Body

Maths Mat, coloured squares and coloured strips: Identify each small square as the unit. Have students place coloured squares and strips (that are equal to tenths of the squares) on the mat to show 7.4 given that one square is the unit – diagram A (7 full squares showing ones and 4 strips showing tenths).



A. 1 × 7.4 = 7.4



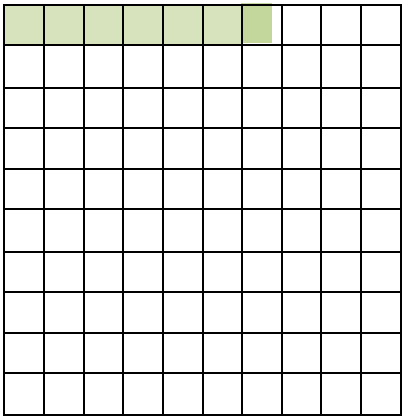
B. 5 × 7.4 = 5 × 7 + 5 × 0.4

= 35 + 20 tenths or 2 ones

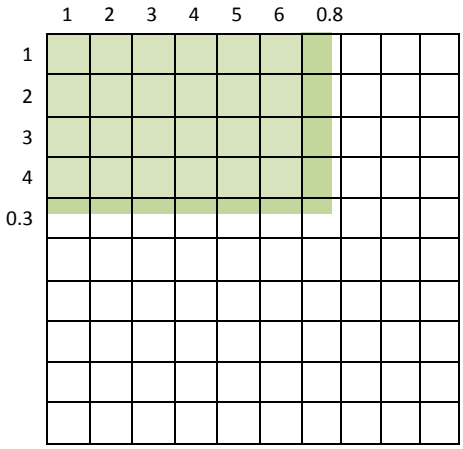
= 37 (check with calculator)

Estimation: 5 × 7 [35]. Have students cover the mat with squares and strips so that the first row is repeated 4 times (diagram B). *How many times are 7.4 squares shown on the mat?* [5 times]. *How many 7s are there?* [5 lots of 7]. *How many 0.4s are there?* [5 lots of 0.4]. *Together, what do these represent?* [5 × 7.4]. *What process did we use to find the result of 5 × 7.4?* [We multiplied the 7 ones by 5 and multiplied the 4 tenths by 5, then we added the results together.] Repeat the process with other examples of a decimal number multiplied by a whole number, e.g. 3.6 × 8. *What happened when a decimal was multiplied by a whole number?* [The result was a bigger decimal.] **Compare** the number of decimal place-value positions in the factors with the number of decimal place-value positions in the answer. (Note the distributive principle: $a \times (b + c) = a \times b + a \times c$)

Use a similar process for multiplying a decimal by a decimal, e.g. 4.3 × 6.8. Estimate. [4 × 7]



C. 1 × 6. 8



D. 4.3 × 6.8 = 4 × 6.8 + 0.3 × 6.8

= 4 × 6 + 4 × 0.8 + 0.3 × 6 + 0.3 × 0.8

= 24 + 3.2 + 1.8 + 0.24

= 29.24 (check with calculator)

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How does the answer compare with the estimate? [28 is very close to 29.24]. **Compare** the number of decimal place-value positions in the factors with the number of decimal place-value positions in the answer (discuss). *What answer do you get when you multiply 43 by 68?* [Use calculators, 2 924]. *How does this answer compare with the multiplication with the decimal numbers that had the same digits?* [The digits are in the same order but in different places; the answer is 100 times larger in the whole number multiplication.] *Why is this so?* [In the whole number multiplication, ones are being multiplied by ones to give **2 924 ones**. In the decimal multiplication, tenths are multiplied by tenths to give **2 924 hundredths** ($\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$).] *Would it be possible to write a decimal multiplied by a decimal in the same way as a whole number multiplied by a whole number is written?* [Yes]. *What would the result look like?* [The digits in the answer would be the same but the decimal place-value positions in the answer would be the same number of decimal place-value positions that were in the factors.]

Hand Distribute grid paper to students. Students use the processes above to colour squares on the grid paper to represent the various types of multiplication with decimals listed above.

Mind *Visualise your desk at home. If it measures 1.2 m by 0.7 m, how much floor space does it occupy? In your mind, compare the size of your bed. If it measures 2 m by 1.2 m, how much space does it take? What do you notice about the two answers?* [Multiplying by a decimal less than one means the answer gets smaller; multiplying by a whole number or a decimal more than one means the answer gets bigger.]

Creativity Students create their own images to show multiplication with decimals.

Mathematics

Language/ symbols decimal, decimal point, decimal place-value position, fraction, whole number, tenths, hundredths, multiply, bigger, smaller

Practice Ensure that students estimate the answer before the calculation Use positive numbers only.

1. Provide examples of multiplication with two decimals less than one. Students follow the above process. Discuss the number of decimal place-value positions in the factors and compare with the number of decimal place-value positions in the answer. *What conclusion may be made?* Check answers with calculators.
2. Provide examples of multiplication of a decimal by a whole number using the above process. Discuss the answers and draw conclusions. Check answers with calculators.
3. Provide examples of multiplication of decimal by decimal (larger than one) using the above process. Discuss the answers and draw conclusions. Check answers with calculators.
4. Provide examples of decimal multiplication using the algorithm as for whole number multiplication. Check with calculators. *Do you have the same number of decimal place-value positions in the answer as you have in the factors?*
5. Students create a thinkboard that demonstrates multiplication with decimals.

Connections Relate to whole numbers and fractions, percentage, area, volume, commutative and distributive principles.

Reflection

Validation Students discuss situations where multiplication with decimals occurs, e.g. metric measurement, money, baking. In pairs, students validate their partner's thinkboard.

Application/ problems Provide applications and problems for students to apply to different real-world contexts independently, e.g. *The new carpet is 3.6 m long and 2.8 m wide. How much floor space does the carpet cover?*

Tom's car can travel 13.6 km per litre on the highway. If his fuel tank holds 55.4 litres, how far can he travel on one full tank of petrol?

A baker earns $42.50 per hour. How much will she earn if she works for 23.8 hours?

Extension

Flexibility. Use different models to represent multiplication of decimals, e.g. array, area, volume and algorithm. Familiarise students with different types: decimal × decimal (smaller and larger than one), decimal × whole, and decimal × decimal × decimal.

Reversing. Students are able to move between telling a decimal multiplication story ↔ writing and representing multiplication with decimals ↔ interpreting decimal diagrams and recognising the intersection or overlap as the result, starting from and moving between any given point.

Generalising. *The decimal point NEVER changes; it always sits with the ones. Place-value positions never change; numbers move across the place-value positions. Multiplying by a decimal less than 1 always gives a smaller product. The product of two decimals less than 1 is always smaller than the two decimals that were multiplied. The number of decimal place-value positions in the factors is the same as the number of decimal place-value positions in the product.*
Note: 9.6 × 4.5 = 43.2 – two decimal place-value positions in the factors but one decimal place-value position in the product because 43.2 is the same as 43.20; 0.2 (2 tenths) is equivalent to 0.20 (20 hundredths). The norm is to drop final decimal zeros.

Changing parameters. If 5 × 7.4 = 5 × (7 + 0.4) = 5 × 7 + 5 × 0.4, how do you expand:

(a) number of hats × (number of boys + number of girls); or

(b) a × (b + c)?

Teacher's notes

- Ensure that students have a sound understanding of the overlap of the decimals in the area model before proceeding to the array model. Reinforce the understanding of changes in place value and that in equivalent decimals, zeros on the end can be dropped. Reinforce the habit of estimation to check for reasonableness of the answer.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of the decimal 3.5, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of 6.25.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that **aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

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