YuMi Deadly Maths

Year 7 Teacher Resource:

MG – Allied shapes

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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YuMi Deadly Maths Year 7 Teacher Resource: **MG – Allied shapes**

ACKNOWLEDGEMENT

We acknowledge the traditional owners and custodians of the lands
in which the mathematics ideas for this resource were developed,
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Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

ydc.qut.edu.au

Year 7 Measurement and Geometry

Allied shapes

Learning goal	Students will calculate the area of triangles and parallelograms using formulas.
Content description	Measurement and Geometry – Using units of measurement • Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem-solving (ACMMG159)
Big idea	Measurement – part-whole, interpretation vs construction
Resources	Maths Mat, elastics, 1 cm grid paper, scissors, glue, geoboards, thinkboard templates

Reality

Local knowledge

Discuss where parallelograms and triangles are found in real life, e.g. tiling, tangram, pattern design, quilting, architecture, art.

Prior experience

Revise properties of quadrilaterals, in particular the distinctive features of parallelograms (opposite sides equal and parallel, opposite angles equal). Clarify that squares, rectangles and rhombuses meet these criteria but are special cases of parallelograms. Not all parallelograms are squares, rectangles or rhombuses. Show some pictures of parallelograms in real life (use Google Images). Repeat the process for triangles. Revise concept of area (the space contained within a boundary or fence, the amount of surface space that has been covered). Revise the transformations of translation and reflection.

Kinaesthetic

Ask four students to make a parallelogram on the Maths Mat with an elastic. *Does this meet the criteria for a parallelogram? What shapes can you see within the parallelogram?* [two triangles and a rectangle]. Have students mark these with two differently coloured elastics.

Only two new students are required on the base lines as the original students at the four corners will hold both the red and the green on one end and the red and the blue on the other end (Diagram 1). *What would happen if we translated the green triangle (a slide) so that it sat next to the blue triangle on the other end?* (Diagram 2) [A rectangle has been made.] *Is the space inside the parallelogram the same as the space inside the rectangle?* [Yes]. *How do you know?* [The triangle that was cut from one end has been slid across and joined on to the other end.]

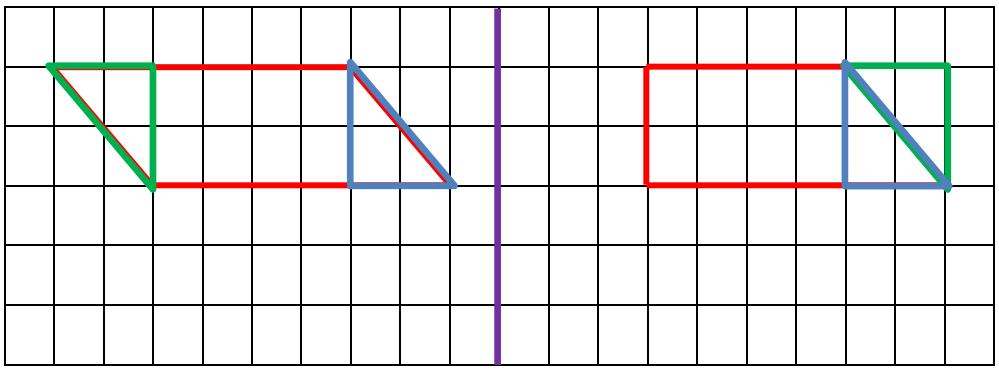


Diagram 1 Diagram 2

In our example, how many small squares are there in the rectangle (that includes the two triangles) [12 squares]. *Now go back to the parallelogram – so how many squares are there in the parallelogram?* [12 squares]. *How do we find the area of the rectangle?* [Base × Height]. *Is this the same as the area of the parallelogram?* [Yes]. *Why? Show me the base of the rectangle. Show me the base of the parallelogram. Show me the height of the rectangle. Show me the height of the parallelogram. What type of angle does the height of the parallelogram form with its base?* [right angle]. *What kind of lines are the green and the blue lines inside the parallelogram?* [Perpendicular to the base]. *Make a parallelogram on the mat that has a base*

of 5 and a perpendicular height of 3. How many squares does this cover? (What is its area?). Give other examples.

Reverse: *With the elastics, make a parallelogram that is not a square, rectangle or rhombus that has an area of 16 squares.* Give other examples. *How can you describe a parallelogram?* [A parallelogram is a rectangle that has been knocked out of shape.]

Abstraction

Body

Repeat the above process to show that a triangle is half the area of a rectangle. The parts of the whole triangle are transformed through reflection to make the rectangle that is two times the triangle. Start with a right triangle then a scalene. Drop the perpendicular height first and discuss the two triangles so formed in Diagram 2. Ask two students to make a reflection of these triangles using elastics of the same colour. Elicit that the two triangles are equal to the original triangle. Establish the formula: *Area of triangle is half the area of a rectangle with a base and perpendicular height equal to the base and height of the triangle*.

So: $\mathrm{A} = \frac{\text{base} \times \text{height}}{2}$

or $\mathrm{A} = \frac{1}{2} \times \text{base} \times \text{height}$

or $\mathrm{A} = \text{base} \times \text{height} \div 2$

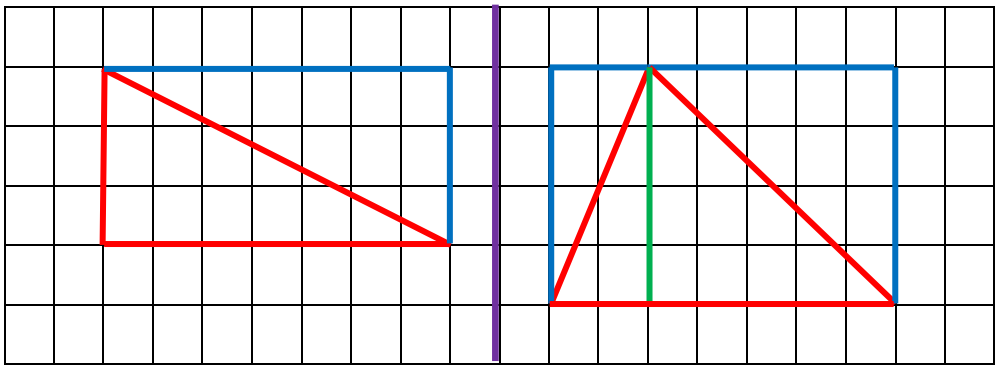


Diagram 1 Diagram 2

Hand

Students use grid paper, scissors and glue to draw parallelograms and make them into rectangles. Reverse: Draw rectangles and make them into parallelograms. Calculate the area in each case. Repeat the process with triangles.

Mind

Visualise a parallelogram whose base is twice its height. If the base is 8 m, what is the area? [32 m^2]. *See a parallelogram whose four sides are each 7 mm long. What is its area?* [49 mm^2]. *What is the special name for this parallelogram?* [rhombus]. *Visualise a triangle whose base is half its height. If the base was 6 cm, what is its area?* [36 cm^2].

Creativity

Students select their own medium to create a pattern that includes parallelograms and triangles.

Mathematics

Language/ symbols

triangle, parallelogram, base, height, perpendicular, quadrilateral, rectangle, square, rhombus, translation, reflection

Practice

1. Geoboards. Students practise the Hand activities above using rubber bands and geoboards.
2. Construct a thinkboard to solve area problems with parallelograms and triangles: write a problem; draw a solution; use a model; write the formula; solve the problem. Write a problem for your partner to solve.

Connections

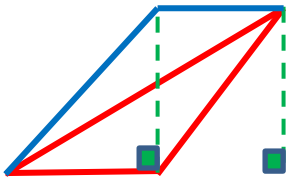
Fractions, similar figures, 3D shapes – faces and volume, sum of the angles in triangles, rectangles, parallelograms.

Reflection

Validation Students share each other's thinkboards.

Application/ problems Provide problems on area of parallelograms and triangles for students to apply to different real-world contexts independently; e.g. find the area given base and height; find the base given area and height; find the height given area and base.

Extension **Flexibility**. Students can identify and calculate the area of different types of parallelograms and triangles, e.g. obtuse scalene triangles. *Note*: An obtuse scalene triangle needs to be seen as the triangle's reflection making a parallelogram. The base must remain the same measurement and not be extended to meet the perpendicular height dropped from the top vertex (this would extend the measurement of the base).



The two equal obtuse scalene triangles can now each be seen as half the parallelogram whose area is the same as a rectangle with the same base and height.
Area of triangle remains: $\mathrm{A} = \frac{1}{2} \times \text{base} \times \text{height}$.

Reversing. Students are able to move between writing a problem on area of parallelograms and triangles ↔ drawing and calculating a solution ↔ using a model ↔ writing the formula ↔ solving the problem, starting from and moving between any given point.

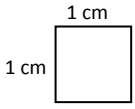
Generalising. *A parallelogram has the same area as a rectangle where the base and height are the same measurement. A parallelogram has opposite sides equal and parallel but differs from the rectangle whose angles are all right angles, as the parallelogram's opposite angles are equal but one pair is acute and the other pair is obtuse. In both the rectangle and parallelogram, the sum of the angles is 360°.*

Two congruent triangles can be joined to make a rectangle. The area of one triangle is, therefore, half the area of a rectangle that has the same base and height as the triangle.

Changing parameters. Find the area of a pentagon, hexagon, octagon.

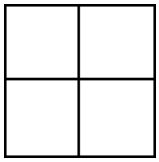
Teacher's notes

- Ensure students use the correct notation for square measure, e.g. cm × cm = square cm or cm².



1 square centimetre (1 cm²)

2 square centimetres (2 cm²)



2 centimetres squared (4 cm²)

Square centimetres vs centimetres squared

- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a triangle, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a rectangle.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that **aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.

- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

