## YuMi Deadly Maiths

Year 6 Teacher Resource:
MG - Angle strength

Prepared by the YuMi Deadly Centre Faculty of Education, QUT

## ACKNOWLEDGEMENT

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## Year 6 Measurement and Geometry

## Angle strength

Learning goal Students will calculate unknown angles using angle relationships.

Content description

Measurement and Geometry - Geometric reasoning

- Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles (ACMMG141)

Big idea
Geometry - interpretation vs construction
Resources Elastics, rope, markers 1-12, clock face paper templates, protractors, geoboards, string or rubber bands, circle tools, paper, triangular polydrons

## Reality

Local knowledge Discuss where students find angles on a straight line, e.g. signpost on a straight road $-90^{\circ}$ either side of the post; vertically opposite angles - chopsticks or two drinking straws crossing each other; angles at a point - a number of roads meeting at a centre (sometimes a roundabout), petals on a daisy.

Prior experience Check knowledge of angles on a straight line, vertically opposite angles and angles at a point.
Kinaesthetic Students put a marker on the ground and select a point that is directly in front of the marker. They place an elastic on the ground so that it makes a straight line from the marker to the direction of the selected point. Students stand directly behind the elastic line and face the selected point with their arms stretched out from their shoulders so that a straight line is made across their bodies (that should be perpendicular to the line of the elastic). What is the angle from the elastic to your right arm, left arm? [both $90^{\circ}$ ]. Angles on a straight line add up to $180^{\circ}$.

Students cross their arms straight in front of them to see vertically opposite angles they have created. Describe the four angles you have made. [The angles that are vertically opposite look to be equal.] Replicate their arm positions with elastics on the ground that cross each other in many ways making different lots of vertically opposite angles.

Angles at a point: Groups of five to six students lie on the ground so that their feet are touching at a centre point and their heads are spaced around forming a circle. Another student starts at one person and walks around the circumference making a full turn.

## Abstraction

Body Note: All illustrated clock faces are to be made on the ground by the students using rope, markers and elastics.
Students take a rope and make a circle. They place marker strips to represent a clock face. In groups, students make angles around the clock face with elastics. One student stands at the 12 position and holds one end of the elastic there to form the baseline to another student standing and holding the elastic at centre point, the pivotal point. Another student moves the elastic firstly to the three, 15 minute position, quarter turn. What kind of an angle is this? [Right, $90^{\circ}$ ] (Check the other three quadrants here to establish that there are four right angles in the clock face all adding to $360^{\circ}$ - angles around a point equal $360^{\circ}$.)


How many spaces (hours) are occurring in the clock face when an angle of $90^{\circ}$ is made? [three: 12-1, 1-2, 2-3]). Reverse: make other $90^{\circ}$ angles, e.g. elastic from 1-4, 2-5 positions etc.


Name the angles: $\mathrm{c}=$ centre (e.g.|1c4), rays $\overrightarrow{1 \mathrm{c} \text { and } \mathrm{c} 4 ;}$ | $2 \mathrm{c5}$, rays $\overrightarrow{2 \mathrm{c} \text { and } \mathrm{c5} \text {, etc. }}$
The student then moves the elastic to the 1 o'clock, 5 minute position. What fraction of the quarter turn has been made? [ $1 / 3$ of $90^{\circ}$, or $1 / 12$ of $360^{\circ}$, so equal to $30^{\circ}$.

$60^{\circ}$


If that is $1 / 3$ of $90^{\circ}$, or $1 / 12$ of $360^{\circ}$, what number on the clock face will give us $60^{\circ}$ in the first quadrant? [from the 12 to 2 position]. Where should the elastic be placed to show $45^{\circ}$ ? [from the 12 to halfway between the 1 and 2 positions]. Name the angles and rays.

Going from any number on the clock face directly across to its opposite, what kind of angle is being formed? e.g. from 12 to 6, from 1 to 7, from 2 to 8, etc. all making straight lines. Straight angles are equal to $180^{\circ}$.




 are supplementary, i.e. equal to $180^{\circ}$.

This may also be shown as two $90^{\circ}$ angles using two differently coloured elastics, 12 to 6 and 3 to $9,90^{\circ}$ plus $90^{\circ}=180^{\circ}$. What happens when we move to various points around the clock face and complete a full turn? (Note that every 5 minute sector is equal to $30^{\circ}$, $12 \times 30^{\circ}=360^{\circ}$, so the angle formed with two elastic radii from, say, 7 to centre to $11=$ $120^{\circ}, 4 \times 30^{\circ}$.) How many degrees are there in a full turn or revolution? [ $360^{\circ}$ ].

Using two differently coloured elastics, make a number of different vertically opposite angles, e.g. (a) 11 to 5 and 9 to 3 , and (b) 1 to 7 and 2 to 8 . Calculate the number of degrees in each angle to show vertically opposite angles are equal (remember each 5 minute sector equals $30^{\circ}$ ). Name the angles remembering the vertex is the common end point.





Make a number of angles within the circle, e.g. 12 to 2,2 to 5,5 to 7,7 to 8 and 8 to 12 :


Name and calculate the measure of all angles. Check with a protractor.
Reverse: Give the type of angle, students make it with elastics and make many different angles on the clock face with elastics equal to $30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}, 180^{\circ}, 360^{\circ}$. Relate to above activity: one space between $=30^{\circ}$, one and half spaces $=45^{\circ}$, two spaces $=60^{\circ}$, three spaces $=90^{\circ}$, four spaces $=120^{\circ}$, straight line $=180^{\circ}$.

Students draw, measure and name the angles that have been made in the above activities, angles on a straight line, vertically opposite angles, angles around a point. Use a protractor to measure the angles.

Mind $\quad$ Close your eyes and see a chain fence with small vertically opposite angles in the panels that strengthen the fence and also in the construction of Brisbane's Story Bridge. See the vertically opposite angles that indicate a railway crossing. See all the angles that are made around a point in a dartboard, a windmill, and the angles that can be made depending on the direction of the wind in a weather vane. See all the variety of angles that can be made on a protractor totalling $180^{\circ}$.

Creativity Create a design or pattern using vertically opposite angles, straight angles or angles around a point that includes either all, one or two of these types of angles in your art pattern.

## Mathematics

Language/ symbols
angle, vertex, ray, labelling angles $A B C, A B / B C$ rays, protractor, acute, right angle, obtuse, straight, reflex, revolution, vertically opposite, supplementary, intersecting lines, angles at a point

1. Geoboards, use string or rubber bands to:
(a) make a right angle
(b) make an acute angle
(c) make an obtuse angle
(d) make a straight angle
(e) make a reflex angle
(f) make pairs of vertically opposite angles
(g) make supplementary angles (angles on a straight line)
(h) make angles around a point
(i) make an angle from a starting point near the edge of the geoboard; name the angle you have made
(j) make an angle from a starting point near the centre of the geoboard; name the angle you have made.
2. Circle tool: Make and measure (with a protractor) as many different types of angles as you can. Draw them using a protractor.
3. Paper folding: Fold a paper circle in halves starting from many different points. Open the circles to show all the angles you have made. Measure them all and record the number of degrees in each sector. Check that they all add up to $360^{\circ}$.
4. Polydrons: Explore making supplementary angles, vertically opposite angles and angles around a point.
5. Worksheets: Naming angles and rays and calculating the number of degrees, with and without protractors.
6. For virtual activities, see the following websites:
http://www.math-aids.com/Geometry/Angles/
https://www.tes.co.uk/teaching-resource/angles-crossnumber-6293169
http://www.mathsisfun.com/geometry/vertically-opposite-angles.html
http://www.mathworksheets4kids.com/angles.html

## Connections

Relate to angles on parallel lines and angles in 2D and 3D shapes.

## Reflection

Application/ problems

Students check where the above angles are found in their world: construction (e.g. bridges, Eiffel Tower), art, patterns and design, dressmaking.

Provide applications and problems for students to apply to different real-world contexts independently; e.g. Use connector straws, construction sticks, Zaks to construct a bridge that is strengthened by using vertically opposite angles, supplementary angles in triangles and angles around a point.

Flexibility. Students are able to see, name and measure straight, supplementary, vertically opposite angles and angles around a point in many different contexts.

Reversing. Students are able to move between describing angles $\leftrightarrow$ modelling angles $\leftrightarrow$ drawing angles $\leftrightarrow$ naming angles, starting from and moving between any given point.

Generalising. The vertex of the angle is always indicated by the letter in the middle of the angle, e.g.

Vertex: B


The vertex is the common end point of two intersecting rays.
Changing parameters. Students explore the application of supplementary angles in the external angles in triangles, vertically opposite angles in parallel lines and angles around a point in 2D and 3D shapes.

## Teacher's notes

- Angles:
- Acute: less than $90^{\circ},<90^{\circ}$
- Right: equal to $90^{\circ},=90^{\circ}$
- Obtuse: more than $90^{\circ}$ but less than $180^{\circ},>90^{\circ}<180^{\circ}$
- Straight: equal to $180^{\circ},=180^{\circ}$
- Reflex: more than $180^{\circ},>180^{\circ}$
- Revolution: full turn, equal to $360^{\circ}$, $=360^{\circ}$
- Steps for using a protractor:
- Place the midpoint of the baseline of the protractor on the vertex of the angle so that the baseline coincides with one arm of the angle.
- The other arm points to the number of degrees contained in the amount of turn.
- Supplementary angles are angles whose sum adds up to $180^{\circ}$. Angles on a straight line are supplementary as together they add up to $180^{\circ}$, as are the sum of the angles in a triangle since the three add up to $180^{\circ}$.
- Complementary angles are angles whose sum is $90^{\circ}$.
- Circle tool: Cut two congruent circles, one white and the other a different colour. Place one circle under the other and cut a radius through both. One part of the bottom circle can now be lifted over the top of the upper circle and turned to form angles.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a kookaburra, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different bird.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

