YuMi Deadly Maths

Year 3 Teacher Resource:

NA – Cutting it up

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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QUT

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ACKNOWLEDGEMENT

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Year 3 Number and Algebra

Cutting it up

Learning goal	Students will: • represent fractions using linear materials and recognise key equivalent fractions • share collections equally to solve simple problems (halves, quarters and eighths).
Content description	Number and Algebra – Fractions and decimals • Model and represent unit fractions including ½, ¼, ⅓, ⅕ and their multiples to a complete whole (ACMNA058)
Big idea	Number – part-whole; whole-part; multiples
Resources	Rope (20 m long), fraction cards, equal strips of coloured paper; fraction discs, strips, walls and sticks; bases and overlays

Reality

Local knowledge *Where do you find halves, quarters, eighths in the local environment?* (e.g. walking through the bush and finding smooth pebbles, seeds or flowers and giving half to a friend; cutting fruit/pizzas/cakes/chocolate bars into halves, quarters, eighths to share in the family.)

Prior experience Check students' understanding of the meaning of numerator/denominator and their experience in finding halves, fourths and eighths of the whole shape/objects.

Kinaesthetic Demonstrate equivalent fractions by cutting a round cake into halves. *How many equal parts do we have? Are both sides of the cake, both halves equal?* Now cut one half into two quarters. *How many quarters are equal to one half?* [two quarters are equal to one half]. *What kind of piece would we get if the two quarters were cut again into two equal parts so that on one half there were four equal parts? If the other half was also cut into four equal pieces, how many equal parts would there be?* [8]. *So what kind of parts would the eight parts be called?* [eighths]. *We saw before that one half is equal to two quarters. If the whole were cut into eight parts, how many eighths are equal to one half?* [$^4/_8$ = $^1/_2$ and also = $^2/_4$].

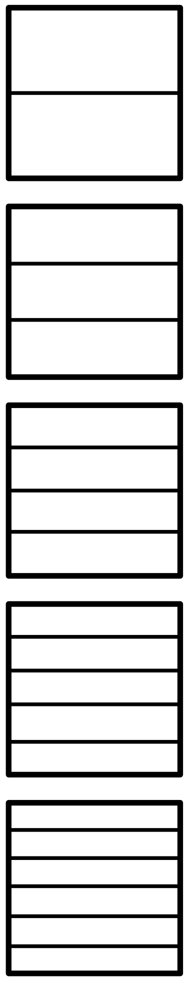
Abstraction

Body Students lay out a rope from zero (starting point, hammer in flag with "0 zero") to its full whole length (place card with "1 whole") and walk the whole length. Predict where half ($^1/_2$) will be. *How do we check the accuracy of the estimation?* Fold rope in half taking the end back to the start. Check estimate, mark rope with thin masking tape and place a card with "$^1/_2$ half" there. Unfold the rope back to the end and place a card "$^2/_2$ two halves". Walk one half. *What is one whole the same as?* [1 whole equals $^2/_2$]. Predict the quarter then halve the rope, fold the half to make a quarter. Mark the rope with thin masking tape at each quarter, fold the rope back to the half mark then full length and place cards "$^1/_4$ quarter", "$^2/_4$ two quarters", "$^3/_4$ three quarters" and "$^4/_4$ four quarters" at the respective positions. *How many quarters make a half?* [two quarters make a half]. The halfway position now has $^1/_2$ half and $^2/_4$ two quarters side by side. *What does this tell us about $^1/_2$ and $^2/_4$?* Unfold the rope back to its full length and walk one quarter. Stop and then walk another quarter, stating again that two quarters make/equal one half. Repeat the process for eighths.

Game – Relay Race to Rope Fractions (cards are left beside the rope): Four equal teams with students behind their leaders at the starting line. Teacher shows a random fraction card and first student in each team runs to that fraction and back; taps the second student's hand and goes to the end of the team. Another random fraction or equivalent fraction is called and shown. The second student runs to that fraction, goes back and taps the third student's hand. Repeat process. First team with all its members back is the winner. Use all forms of equivalent fractions in the game.

Hand

Sequences for introducing equivalent fractions:



Physical material and meaning

- Area (fraction discs). Fraction circles can be used to compare the following, by placing material on top of each other, and determine which pair is equivalent: (a) $^{1}/_{2}$ and $^{2}/_{4}$, (b) $^{3}/_{4}$ and $^{2}/_{3}$, (c) $^{4}/_{6}$ and $^{2}/_{3}$.
- Length (fraction strips). Fraction strips can be used to compare the following, by placing material beside each other, and determine which pair is equivalent: (a) $^{1}/_{3}$ and $^{2}/_{6}$, (b) $^{3}/_{4}$ and $^{4}/_{5}$, (c) $^{1}/_{2}$ and $^{3}/_{6}$.
- Set (Unifix). Use sets of Unifix to compare the following, by partitioning two sets of Unifix material and comparing the result, and determine which pair is equivalent: (a) 12 Unifix – $^{1}/_{3}$ and $^{2}/_{6}$, (b) 20 Unifix – $^{3}/_{5}$ and $^{6}/_{10}$, (c) 28 Unifix – $^{3}/_{4}$ and $^{5}/_{7}$.
- Base boards and fraction pieces. Make a set of base boards as on right and a set of fraction pieces that fit into the base boards – halves, thirds, quarters, fifths and sixths. When looking for an equivalent fraction, first find the appropriate base board and then fill it in with fraction pieces. Then the focus is on the parts – e.g. the third base board down on right fits into the first.
- Overlays and base boards. Make a set of base boards as on right and then a set of plastic overlays with the **fraction shapes turned 90 degrees**. Shade fractions on the base board and then use overlays on top of these fractions to show that $^{2}/_{3}$ is equivalent to $^{4}/_{6}$, $^{6}/_{9}$, $^{8}/_{12}$ and so on. Place and remove overlay to see relation (can be done really well with computer).
- Fold two rectangles of paper longways. Shade one half of each. Take one of the shadings and fold it the other way – into half, thirds, fourths, etc. In this way, we can develop a sequence of equivalent fractions equal to $^{1}/_{2}$: $^{1}/_{2} = {}^{2}/_{4} = {}^{3}/_{6} = {}^{4}/_{8} = \ldots$ and so on. Use the same method to develop a sequence of fractions equivalent to $^{2}/_{3}$.

Using equal paper strips of different colours, students glue a whole strip horizontally onto a blank sheet of paper and write the name/numeral beside it, *one 1*. Fold another strip (different colour) into halves, draw a black line over the fold and paste it under the whole and write *2 halves = 1 whole* beside it. Repeat for quarters and eighths, emphasising the halving strategy being used. Trace over the vertical lines common to halves, quarters and eighths noting the equivalences.

Mind

Close your eyes and in your mind see a whole cake, see $^{1}/_{2}$, $^{1}/_{4}$, $^{2}/_{4}$, $^{2}/_{8}$, $^{4}/_{8}$ (called randomly). *Visualise a chocolate block. See a half of the chocolate block. Can you break it into equivalent fractions? How many different equivalent fractions can you make so that you can share a half amongst 4 people, 6 people?*

With your eyes closed, draw half a circle (etc.) in the air. Draw half/quarter/eighth of a circle on your partner's back.

Creativity

Students draw their own shapes and show $^{1}/_{2}$, $^{1}/_{4}$, $^{2}/_{4}$ etc. of the whole shape.

Mathematics

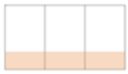
Language/ symbols

fraction, partition, equal, not equal, parts, half, halves, halving, quarter, quarters, fourths, eighths, whole, same, shape, object, benchmark, compare, share, solve, equivalent

Practice

1. Record drawings of the rope activity, naming each part.
2. Using fraction walls, find and trace the vertical lines that show equivalent fractions.

3. Using a number line, divide into whole, $^1/_2$, $^1/_4$, $^2/_4$, $^1/_8$, $^2/_8$, $^4/_8$, $^6/_8$, writing the equivalent underneath the unit fractions.
4. Shade various area models to show halves, quarters, eighths. Compare the size of fractions – bigger/smaller than, equal to.
5. Share pizzas, chocolate bars into halves/quarters/eighths. Use colour codes for distribution of various fractions to given people. Note that one half given to Sam is the same as the other half given to Jan. Draw the half slice beside each person. Compare $^2/_4$ and $^4/_8$ to the whole.
6. Investigate unnumbered models of unit fractions to develop an understanding of the fractional relationship between parts and the whole as a basis for multiplication of fractions, e.g. *Share one quarter of this shape:*

    as 

7. Use a ruler and a fraction mat (see below in Teacher's notes) to do the following: (a) find if the following fractions are equivalent: (i) $^3/_4$ and $^9/_{12}$, (ii) $^2/_5$ and $^4/_9$, (iii) $^6/_8$ and $^9/_{12}$; and (b) develop a sequence of fractions equivalent to $^1/_3$.
8. Make a set of fraction sticks (see below in Teacher's notes). Pull out the 2 and 5 stick and make $^2/_5$. Note that all equivalent fractions are shown. Look for a pattern that shows they are equivalent. Note that students tend to see this pattern additively: $^2/_3$ = $^4/_6$ = $^6/_9$ = ... is seen as adding 2 on the "top" and adding 3 on the "bottom". Students need to see this pattern multiplicatively: $^2/_3$ = $^4/_6$ = $^6/_9$ = … needs to be seen as multiplying "top" and "bottom" by the same amount – $^2/_3 \times {}^2/_2$, $^2/_3 \times {}^3/_3$, and so on.
9. Explore virtual area/linear activities to represent, compare and share halves, quarters and eighths: http://www.apples4theteacher.com/math.html ; http://au.ixl.com/math/

Connections Explore relation to division, process and symbol.

Reflection

Validation Students discuss when they use or see fractions in real life, e.g. getting only half your pocket money for not doing your jobs at home; a cake/pizza being cut into eighths.

Application/ problems Provide applications and problems for students to apply to different real-world contexts independently; e.g. *How much pizza would you get if there were three people to share it? How much if there were five or eight people to share it? Draw the whole pizza and then draw the shares: third, fifth, eighth. What fraction gives you the most pizza, the least pizza?* Use the thinkboard for students to solve other real-life problems.

Extension **Flexibility**. Students work with different materials and models (line, area and volume) to gain a rich knowledge of the fraction concept with particular reference to the halving strategy.

Reversing. Students are able to cut the whole into unit fractions and reverse when given a unit fraction to make the whole; e.g. line segments on paper showing $^1/_2$, $^1/_4$, $^1/_8$ (or multiple). *Make the whole*.

Generalising. *When the whole is divided into fractions, each part is of equal size. The bottom number tells the number of equal parts the whole is being divided into. The greater the number of parts, the smaller each part becomes. Fractions may be re-divided to make an equivalent amount to the initial fraction*.

Changing parameters. Explore and find equivalent fractions with thirds extending to twelfths, and fifths going to tenths. Provide further opportunities for students to reinforce the halving strategy using real-life objects or concrete materials.

Teacher's notes

- Ensure that students know that halves are made when we have two **equal** pieces made from the whole and similarly for other fraction types.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a kookaburra, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different bird.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Useful websites for resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that **aligns with students' understanding** is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters.

Fraction mat

1 whole									
1 half $\frac{1}{2}$	1 half $\frac{1}{2}$								
1 third $\frac{1}{3}$	1 third $\frac{1}{3}$	1 third $\frac{1}{3}$							
1 fourth $\frac{1}{4}$	1 fourth $\frac{1}{4}$	1 fourth $\frac{1}{4}$	1 fourth $\frac{1}{4}$						
1 fifth $\frac{1}{5}$	1 fifth $\frac{1}{5}$	1 fifth $\frac{1}{5}$	1 fifth $\frac{1}{5}$	1 fifth $\frac{1}{5}$					
1 sixth $\frac{1}{6}$	1 sixth $\frac{1}{6}$	1 sixth $\frac{1}{6}$	1 sixth $\frac{1}{6}$	1 sixth $\frac{1}{6}$	1 sixth $\frac{1}{6}$				
1 seventh $\frac{1}{7}$	1 seventh $\frac{1}{7}$	1 seventh $\frac{1}{7}$	1 seventh $\frac{1}{7}$	1 seventh $\frac{1}{7}$	1 seventh $\frac{1}{7}$	1 seventh $\frac{1}{7}$			
1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$	1 eighth $\frac{1}{8}$		
1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	1 ninth $\frac{1}{9}$	
1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$	1 tenth $\frac{1}{10}$

Fraction sticks

The fraction sticks are an interesting material. A set is like below:

1	2	3	4	5	6	7	8	9	10

2	4	6	8	10	12	14	16	18	20

3	6	9	12	15	18	21	24	27	30

4	8	12	16	20	24	28	32	36	40

5	10	15	20	25	30	35	40	45	50

6	12	18	24	30	36	42	48	54	60

7	14	21	28	35	42	49	56	63	70

8	16	24	32	40	48	56	64	72	80

9	18	27	36	45	54	63	72	81	90

10	20	30	40	50	60	70	80	90	100

A fraction is made by putting two rows together, for example, $^2/_5$:

2	4	6	8	10	12	14	16	18	20

5	10	15	20	25	30	35	40	45	50

This can be used to show the pattern for equivalent fractions. Two of these can be used to compare, add or subtract fractions (e.g. $^3/_7$ and $^2/_5$) by aligning the "common denominator" (in this case, 35):

3	6	9	12	15	18	21	24	27	30
7	14	21	28	**35**	42	49	56	63	70

2	4	6	8	10	12	14	16	18	20
5	10	15	20	25	30	**35**	40	45	50