Professional Learning 5
Multiplication and Division
Booklet 5.5: Solving Multiplication and Division
Word Problems

QUT
Queensland University of Technology

Australian Catholic University

Queensland Government
Education Queensland

Sustainable mathematics education capacity building: Empowering Indigenous teacher aides to enhance rural and remote Indigenous students' numeracy outcomes

This project is funded by an Australian Research Council Linkage Project grant with support from Education Queensland (EQ)

DEADLY MATHS TUTORS PROGRAM
Professional Learning 5: Multiplication and Division
SOLVING
MULTIPLICATION AND DIVISION
WORD PROBLEMS
TRIAL PACKAGE 5

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Contributing EQ Organisations

Mount Isa EQ Regional Office   Mornington Island State School
Boulia State School   Sunset State School
Burketown State School   Urandangi State School
Dajarra State School   Indigenous School Support Unit
Doomadgee State School

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Island Regional Educational Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology,
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Deadly Maths Tutor Program

PROFESSIONAL LEARNING 5: MULTIPLICATION AND DIVISION

BOOKLET 5.5

SOLVING MULTIPLICATION AND DIVISION WORD PROBLEMS

2008

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OVERVIEW

PURPOSE

These materials were designed to be used in conjunction with a Professional Learning program for teacher aides. The objective of this Professional Learning was to empower teacher aides to enhance rural and remote Indigenous students' numeracy outcomes. This document contains the materials of the fifth of five different booklets on multiplication and division.

If your school would like to receive a YuMi Deadly Maths Professional Learning program please contact the YuMi Deadly Centre (YDC) on: 07 3138 0035 or ydc@qut.edu.au.

DIRECTIONS

(1) Interviewing the students:

Pick one or more students who appear to be having trouble understanding multiplication and/or division. Interview these students using the interview schedule and the materials. Mark what they do and put their results on the Student Recording Sheet.

(2) Trialling the student activities:

Use the Recording Sheet to work out the activities the students need to do and trial these activities with the students (with each student one at a time or with a group of students). Keep a record of what happens and collect the students' work.

Interview Schedule

Materials:

Unifix cubes or counters, washable felt pens, pen, pencil, paper

Materials within this booklet: interview questions, interview cards, Student Recording Sheet

Directions:

1. Photocopy and laminate attached interview cards.

2. Gather other material (unifix or counters, paper, pens, pencils).

3. Place material in front of students. Give students pen and paper to write with.

4. Tell the students you are trying to find out what they know. Say they are not expected to know it and you will teach what is not known.

5. Give the student directions slowly – read problems. Do not give hints. If student cannot do a question, pass on to the next question, repeating that it is not important if they don't know how to do the question.

INTERVIEW MATERIALS

Questions :Multiplication and Division Problem Solving

P1	**Show Card 1** • Read the card. Ask: *Which problem is +/− and which is ×/÷?* • When student has answered, ask: *How did you work it out?*
P2	**Show Card 2** • Read the card. Ask: *Is this problem solved by× or ÷?* • When student has answered, ask: *How did you work it out?* **Ask the same questions for Card 3**
	Show Card 4 • Read the card. Ask: *Is this problem solved by× or ÷?* • When student has answered, ask: *How did you work it out?* **Ask the same questions for Card 5**
P3	**Show Card 6** • Read the card. Ask: *Is this problem solved by× or ÷?* • When student has answered, ask: *How did you work it out?* **Ask the same questions for Card 7**
P4	**Show Card 8** • Read the card. Ask: *Can you make up a problem for this that is division?* • When student has answered, ask: *How did you work it out?*
	Show Card 9 • Read the card. Ask: *Can you make up a problem for this that is multiplication?* • When student has answered, ask: *How did you work it out?*
P5	**Show Card 10** • Read the card. Ask: *Can you draw a picture that helps solve the problem?* • Ask: *Can you circle the numbers that can be used in the problem? Can you underline the words that ask for the answer?* • Ask: *Can you re-write this problem so it is easy to answer?* • Ask: *Can you solve the problem?*

Multiplication & Division PS Interview Cards

CARD 1

John bought a pie for $3 and a chocolate for $5. How much did he spend?

John bought 3 pies for $5 each. How much did he spend?

+/−   ×/÷

CARD 2

There were 6 bags each with 4 toys. How many toys?

×   ÷

CARD 3

Sue was packing 4 toys in each bag. She had 20 toys. How many bags?

×   ÷

CARD 4

Some boats went out. They each caught 7 fish. Altogether this made 42 fish. How many boats?

×   ÷

CARD 5

June had a pile of fish. She shared them amongst 8 families. Each family got 9 fish. How many fish did June have?

×   ÷

CARD 6

Bill ran 3 times as far as Jed. Jed ran 8km. How far did Bill run?

× ÷

CARD 7

Jack had to drive 6 times as far as Joe. Jack drove 72km. How far did Joe drive?

× ÷

CARD 8

John bought 4 DVDs at $29 each. They cost $116.

× ÷

CARD 9

June shared $48 dollars amongst 6 children. Each child got $8

× ÷

CARD 10

Fred bought a pie every day at work for $3. He set aside $20 for this. How much change did he have at the end of the week?

+ − × ÷

STUDENT RECORDING SHEET

Name: _____

School/Class: _____

| Interview item	Result (✓, ✗)	Comments	Activities to be completed if incorrect
P1: Defining multiplication/ division			MP1
P2: Forward, backward and factor-factor-product			MP2
P3: Comparison and factor-factor-product			MP3
P4: Constructing problems			MP4
P5: Multi-step problems and strategies			MP5

INTRODUCTION

Contents

This package contains:

- five tutoring activities (MP1 to MP5) and their student materials (games and worksheets), as well as an activity feedback sheet for each activity; and

Pedagogy

The activities in booklets 5.1 to 5.5 are based on the Rathmell Triangle relationship below. The idea is to teach students how to connect problems with language and symbols by using models.

REAL WORLD PROBLEM

MODEL — Concrete

— Pictorial

LANGUAGE ⟷ SYMBOLS
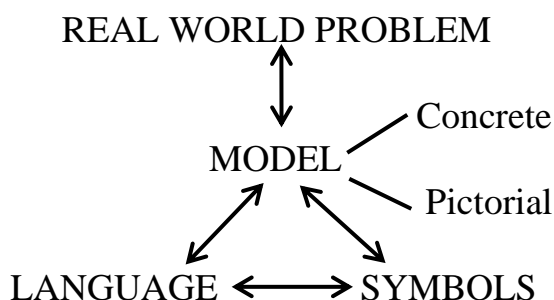
This booklet (5.5) focuses on the top (real world problems) and its relationship to the remainder of the triangle.

Relationship ×/÷ to +/−

Addition and subtraction relate to joining and separating where the parts can be different. It is mostly used when there are two different parts.

Multiplication and division relate to joining and separating into <u>equal</u> sized groups. It is mostly used when there are more than two different parts. Also, in multiplication and division, the two factors refer to different things (number of groups, number in the group)

Meanings of multiplication and division

There are three meanings which combine into an overall meaning as follows. The overall meaning of factor-factor-product will be used as the basis of problem solving for multiplication and division.

MEANING	IDEA	EXAMPLE
1. <u>Forward</u>		
Multiplication	Start with a known collection of equal groups and combine to form a product which is unknown.	There were 4 bags of 3 lollies. How many lollies?
Division	Start with a known product and partition into equal groups where either the number <u>of</u> groups is unknown or the number <u>in</u> each group is unknown.	There were 15 lollies. They were put into 5 bags. How many lollies in each bag? <u>OR</u> There were 15 lollies. They were put into bags of 5. How many bags?

2. Backward

Multiplication	Start with an unknown product and partition into known equal groups.	There were some lollies. They were put in 7 bags. There were 2 lollies in each bag. How many lollies?
Division	Start with an unknown number of groups or an unknown number in each group and combine the groups into a known product.	There were some groups of 4. They formed 20. How many groups? <u>OR</u> There were 3 groups. They formed 18. How many in each group?

3. Comparison

Multiplication	Start with a known initial number and a known multiplier and find unknown product that is the comparison.	John has 4 times the lollies that Frank has. Frank has 6 lollies. How many lollies does John have?
Division	Start with an unknown initial number or an unknown multiplier to reach a known product or comparison; and find the unknown.	John has 4 times the lollies Frank has. John has 24 lollies. How many does Frank have? <u>OR</u> John has 24 lollies, Frank has 8 lollies. How many more times is John's lollies than Frank's lollies?

4. Factor-factor-product
   (the overall meaning)

Multiplication	Both factors are known (number of groups and number in each group) and the product in unknown.	All the above plus: $6 \times 5 = \boxed{}$
Division	The product is known while one of the factors is unknown.	All the above plus: $6 \times \boxed{} = 30$ or, $\boxed{} \times 5 = 30$

Extension

The final two sections of the booklet investigates three things:

(1) <u>Construction</u>

Mostly we interpret problems. However, an excellent way to become a better interpreter is to construct problems.

(2) <u>Strategies</u>

These are general rules of thumb that point the direction to answers that can assist in a variety of word problems. The strategies investigated are:

(i) making a drawing – acting out and doing a problem (where, of course, the doing is useful);

(ii)   identifying given, needed and wanted information – identifying what the numbers in the problem are and what the number wanted is (particularly useful in part-part-total situations); and

(iii)   restating the problem – rewriting the problem using simple words and straightforward structure.

(3)   <u>Multi-step word problems</u>

Students need to be made aware that problems can be broken into parts and done a step at a time.

Approach

The approach used in the booklets is to focus on the relationship:

form of problem   ⟷   operation and number to be used

<u>not</u> on getting the right answers. Thus many activities will end with recognising the operation.

To achieve this, activities will focus on:

(i)   interpreting problems in terms of operations; and

(ii)   constructing problems from operations.

For example, doing both these things:

(i)   circle the operation for the problem: *There were 8 times as many apples as oranges, there were 56 apples, how many oranges?* $+$ $-$ $\times$ $\div$

(ii)   construct 2 different problems for the operation: $3 \times 5 = ?$

ACTIVITIES

ACTIVITY MP1

[Defining multiplication and division]

Materials: Unifix cubes, pen, paper, material attached (worksheet, game)

Directions:

1.   State: *Addition and subtraction are joining and separating – two parts, can be different size.* Explore this: obtain 7 unifix cubes. Separate into 3 cubes and 4 cubes, join, separate again, and so on. State: *3 add 4* [as you join] *7 subtract 3* [as you separate].

    Repeat for 9 cubes, separated into 7 and 2 cubes. Say: *7 add 2, 9 subtract 7,* as you join and separate.

2.   State: *Multiplication and division are combining (or joining) and partitioning (or separating) two or more parts, all parts the <u>same</u> size.* Explore this: obtain 12 cubes. Partition (separate) into 3 groups of 4, combine (join) back to 12, partition, combine, and so on. State: *3 fours* [as combine] *12 divided by 3* [as partition].

    Repeat for 15 and 5 groups of 3. Say: *5 threes, 15 divided by 5,* as you combine and partition.

3.   Discuss similarities and differences:

<table>
<tr><td align="center"><u>Same</u></td><td align="center"><u>Different</u></td></tr>
<tr>
<td>

a. Addition is joining, so is multiplication.

b. Subtraction is separating, so is division.

</td>
<td>

c. Addition joins often different groups, but multiplication is equal groups.

d. Subtraction separates into often different groups, but division is equal groups.

e. Addition and subtraction most often involve 2 parts; while multiplication and division most often involve more than 2 parts.

f. The two numbers that are joined in addition, or into which they are separated in subtraction, refer to same things; the two numbers that describe the combining in multiplication, or which describe the partitioning in division, refer to different things (the number of groups; the number of objects in each group)

</td>
</tr>
</table>

4.   Discuss problems – identify how many parts, whether they are equal, whether same or different?

Problem	No. of parts?	Equal?	Same/different	+/− or ×/÷
Jan bought a pie for $3 and a cake for $7. How much paid? [$10]	2	No	Both $'s	+/−
Joe bought 3 cakes for $7 each.	3	Yes	One cakes,	×/÷

How much paid? [$21]			other $'s	
Jan had $24 and gave $8 to Bill. How much left? [$16]	2	No	All $'s	+/−
Jan had $24 and shared it amongst her 8 nephews. How much did each get? [$3]	8	Yes	One $'s, other nephews	×/÷

5. Complete Worksheet 1.1.

6. Play the game "Which one mix-and-match".

MP1 – Worksheet 1.1

Circle the correct option (+/− or ×/÷)!

1. Penny bought 5 pens for $4 each. How much did she pay?	+/− ×/÷
2. Frank bought a ruler for $5 and a pen for $4. How much did he pay?	+/− ×/÷
3. June planted a row of 5 tomatoes and another row of 4 tomatoes. How many tomatoes did June plant?	+/− ×/÷
4. John planted 5 rows of tomatoes. There were 5 tomatoes in each row. How many tomatoes did John plant?	+/− ×/÷
5. Peter caught 12 fish and threw back 3. How many did he keep?	+/− ×/÷
6. Jack caught 12 fish and shared them amongst the 3 families. How many fish did each family get?	+/− ×/÷
7. Roy ran 15km and then another 5km. How many kilometres did Roy run?	+/− ×/÷
8. Roy's team ran 15km. Each member of Roy's team ran 5km. How many members in Roy's team?	+/− ×/÷

MP1 – Game: Which one? Mix-and-Match Cards

Copy cards on same coloured cardboard. Cut cards into pieces along lines. Mix up and then sort into matching pairs.

There were 18 boys and 6 girls. How many children?	18 6 +/−
There were 18 boys and 6 cars. How many boys in each car?	18 6 ×/÷
There were 2 planks, one 14m and the other 7m. How many metres of wood?	14 7 +/−
There was a 14m plank. It was cut into 7 equal lengths. How long was each length?	14 7 ×/÷

Which one? Mix-and-Match Cards (cont'd)

Copy cards on same coloured cardboard. Cut cards into pieces along lines. Mix up and then sort into matching pairs.

There were 21 cases on the bus and 3 cases off the bus. How many cases?	21 3 +/−
There were 21 cases in groups of 3. How many groups?	21 3 ×/÷
There were 9 playing and 36 watching. How many people?	9 36 +/−
The 36 people were put into teams of 9. How many teams?	9 36 ×/÷

Which one? Mix-and-Match Cards (cont'd)

Copy cards on same coloured cardboard. Cut cards into pieces along lines. Mix up and then sort into matching pairs.

There were 54 kangaroos and 9 hopped away. How many kangaroos left?	54 9 +/−
The kangaroos hopped 54 times. Each kangaroo did 9 hops. How many kangaroos?	54 9 ×/÷
There was one row of 5 chairs and a second row of 7 chairs. How many chairs?	5 7 +/−
There were 5 rows of 7 chairs. How many chairs?	5 7 ×/÷

Which one? Mix-and-Match Cards (cont'd)

Copy cards on same coloured cardboard. Cut cards into pieces along lines. Mix up and then sort into matching pairs.

There was $4 and $19. How much money?	4 19 +/−
There were 4 CDs for $19 each. How much money?	4 19 ×/÷
June sharpened 15 pencils but 6 broke. How many pencils left?	15 6 +/−
There were 15 packs of 6 pencils left in the store room. How many pencils left?	15 6 ×/÷

MP1 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——————— Interesting Difficult ——————— Easy Not learning ——————— Learning
	Boring ——————— Interesting Difficult ——————— Easy Not learning ——————— Learning

2. How did you feel about trialling the activity?

    Mark the line with an X: Unconfident ——————— Very confident

3. Do you think the student was engaged in the activity? Explain.

    _____

4. What do you think the student learnt from the activity?

    _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

    _____

6. What do you think of the activity?

    _____

7. What are your suggestions for improving the activity?

    _____

    _____

8. What else do you suggest could be done to help students who have trouble with this activity?

    _____

    _____

ACTIVITY MP2
### [Forward, backward and factor-factor-product]

**Materials:**   Unifix cubes, pen, paper, material attached (worksheets, games)

Directions:

1.   Consider a simple multiplication, e.g., 3×4. Say: *Put out 3 × 4* [3 groups of 4]
     Combine the groups. Multiplication is normally combining. Ask: *How many*
     unifix in total? [12]. Say: *Think of this multiplication as a known start and an*
     unknown finish. Ask: *What is the start?* [3 lots of 4] *What is the finish?* [12]. Get
     students to show this with unifix.
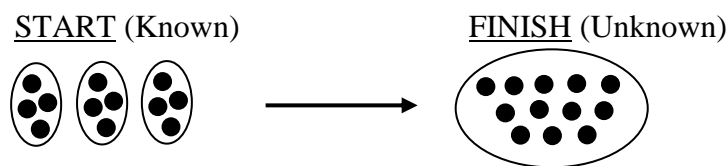
     Say: *The number of groups, 3, is called "factor". The number in each group, 4,*
     is also called "factor". The 12 is called "product".

2.   Repeat this for division example $12 \div 3$, that is, 12 counters shared amongst 3
     groups. Say: *Division is normally partitioning. Act this out with unifix.* [12
     shared among 3]. Ask: *How many in each group?* [4] Get students to show
     START and FINISH with unifix cubes. Say: *Once again we have a known start*
     and an unknown finish.
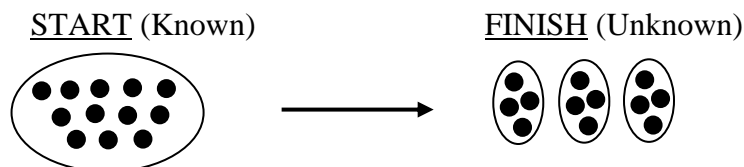
     Once again say: *The 12 is called "product"; the number of groups, 3, and the*
     number in each group, 4, are called "factors".

3.   Repeat step 1 and 2 for 5×3 and $15 \div 3$.
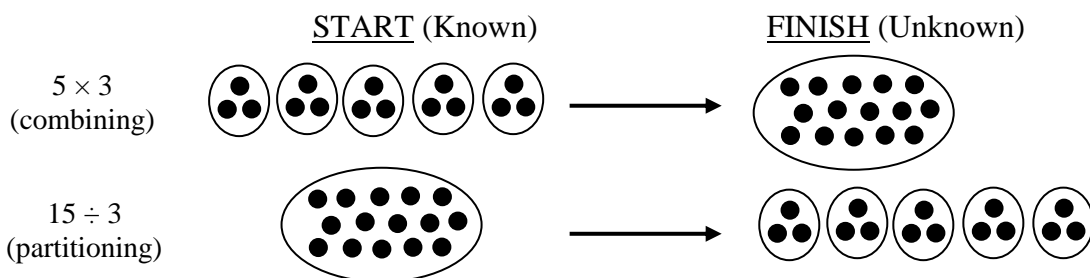
4.   Discuss the difference between multiplication and division. Ask: *Which is*
     combining and which is partitioning? Ask: *What is the start for multiplication?*
     Is it factors or product? [factors] *What is the start for division? Is it factors or*

product? [product] Ask: *What is known? What is unknown?* Point to the diagram below. Say: *Join the operations to the actions that describe them.*
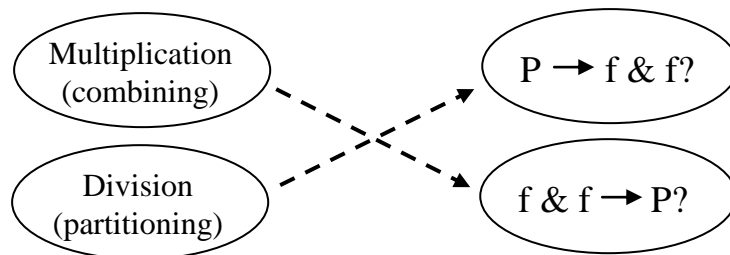

Elicit from students: "Multiplication is when you know the factors and want the product and Division is when you know the product, and one factor, and you want the other factor".

5. Say: *Look at problems and determine which of f, f, and P are known and unknown and use this to determine whether to use × or ÷.* State: *Remember that a factor is a number <u>of</u> groups and a number <u>in</u> a group while the product is the combination of the groups.* Complete Worksheets 2.1 and 2.2.

6. Look at reversing the multiplication and division situations – use examples 2×7 and $14 \div 2$. Say: *Run the stories backwards!*
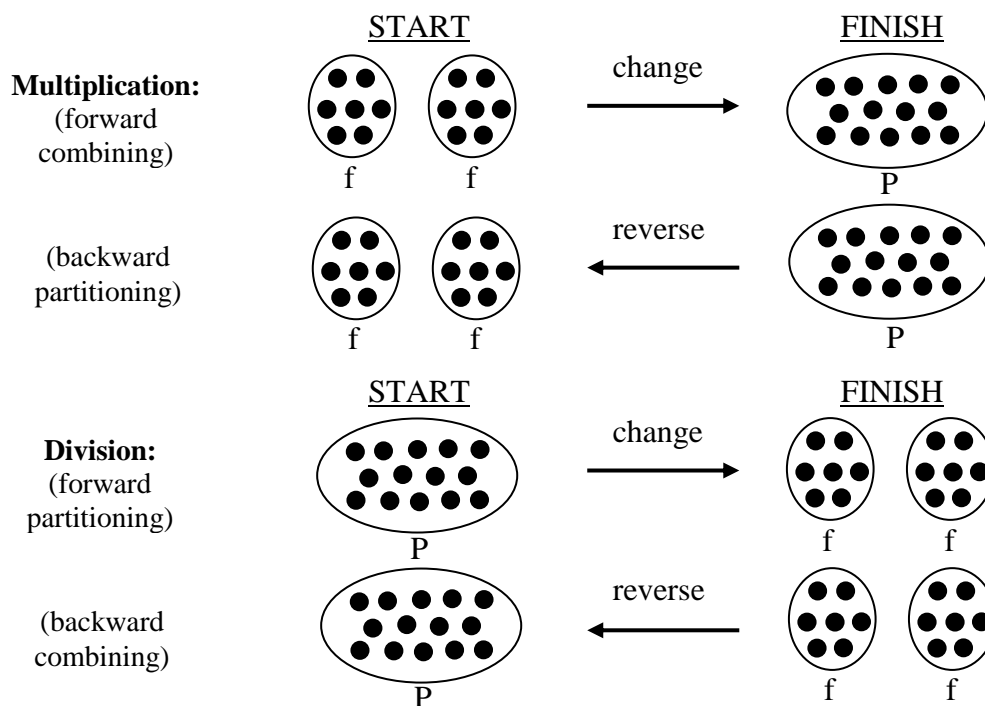
Discuss these situations. Elicit that:

(1) the reverse of multiplication is <u>backward partitioning</u>, that is, finish → start, and this is <u>division</u>; and

(2) the reverse of division is <u>backward combining</u>, that is, finish → start, and this is <u>multiplication</u>.

Point to the diagram below. Say: *Join the backward operations to their actions and to the operation they actually give:*

Note: that the rule for P and f remains the same:

- Multiplication – f ✓ & f ✓   P?
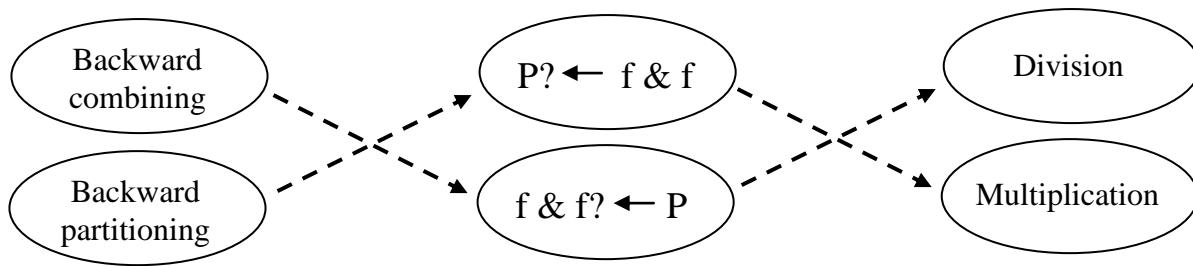- Division –   P ✓   f ✓ & f?

7. Say: *Look at backward problems and determine which of f, f, and P are known and unknown and use this to determine whether to use × or ÷.* State: *Remember that f is a number <u>of</u> groups or a number <u>in</u> each group while P is the combination.*

8. Complete Worksheets 2.3, 2.4 and 2.5.

9. Play the game "Forward/backward Mix and Match".

MP2 – Worksheet 2.1

For the following problems, (circle) the factors and <u>underline</u> the product (if they are given).

---

1. There were 6 boys each with 3 fish. This gave 18 fish altogether.

---

2. There were 12 trees being planted in 3 rows. This meant 4 trees in each row.

---

3. There was $36 shared amongst 4 boys. Each boy got $9.

---

4. There were 48 eggs to be put in packets of 12. This made 4 packets.

---

5. 7 chickens each laid 4 eggs. This made 28 eggs.

---

6. 23 bags were made each with 6 lollies. How many lollies altogether?

---

7. There were 7 trains to pull the carriages. There were 63 carriages. This meant each train had 9 carriages.

---

8. The relay team ran the 56km. There were 8 in the team. How many kilometres each?

MP2 – Worksheet 2.2

Forward stories:

For each story, determine f, f and P. Then circle whether × or ÷. The first two have been done for you!

1.	Frank bought 6 chocolates for $8. How much did he pay?	f 6 f 8 P ?	(×) ÷
2.	Jack shared $15 amongst his 3 daughters. How much did each get?	f 3 f ? P 15	× (÷)
3.	Fred had 4 drums each with 6 litres. How many litres?	f f P	× ÷
4.	June had 20m of wood which she cut into 4m lengths. How many pieces of wood?	f f P	× ÷
5.	Sue put out the chairs in 6 rows. Each row had 30 chairs. How many chairs?	f f P	× ÷
6.	Jenny put out 6 rows of chairs. She only had 30 chairs. How many in each row?	f f P	× ÷
7.	Bill had 4 pieces of wood each of 5m. What was the total length of the wood?	f f P	× ÷
8.	Anne was putting 72 litres of fuel in 8 drums. How many litres in each drum?	f f P	× ÷

MP2 – Worksheet 2.3

Backward stories:

For each story, determine f, f and P. Then circle whether × or ÷. The first two have been done for you!

1.	Mary had 4 boxes of books. Each box had the same number of books in it. There were 72 books overall. How many books in each box?	f 4 f ? P 72	× (÷)
2.	Jack shared his money amongst his 5 friends. He gave each friend $24. How much money did he have to share?	f 5 f 24 P ?	(×) ÷
3.	Sue had a long plank. She cut it into 7 pieces. Each piece was 2m long. How long was the original plank?	f f P	× ÷
4.	Jenny planted 6 rows of beans. Overall she planted 96 beans. How many in each row?	f f P	× ÷
5.	The football teams gathered. Each team had 13 players. There were 104 players. How many teams?	f f P	× ÷
6.	John put the apples in packets of 12. He had 156 apples. How many packets?	f f P	× ÷
7.	The water was shared out. Each person got 11 litres. There were 23 people. How many litres to start with?	f f P	× ÷
8.	John put the apples into 12 packets. Each packet finished with 7 apples. How many apples at the start?	f f P	× ÷

MP2 – Worksheet 2.4

Mixture of stories:

For each story, determine f, f and P. Then circle whether × or ÷.

1.	There were 5 envelopes. Each contained $45. How much money in total?	f f P	× ÷
2.	Jack planted 468 trees in rows of 18. How many rows?	f f P	× ÷
3.	There were 5 envelopes. The $45 was shared amongst them. How much money in each envelope?	f f P	× ÷
4.	The money was shared amongst the 5 envelopes. Each ended up with $45. How much money was shared?	f f P	× ÷
5.	Jack planted 468 trees in each row. There were 18 rows. How many trees?	f f P	× ÷
6.	Vi ran 15km every day. She needed to run 195km. How many days?	f f P	× ÷
7.	Each envelope received the same amount of money. There were 5 envelopes. This made $45. How much in each envelope?	f f P	× ÷
8.	Vi needed to run 195km. She broke this up across 13 days. How much did she run each day?	f f P	× ÷

MP2 – Worksheet 2.5

Some words usually mean either multiplication or division. For example, "lots of" usually means multiplication, and "share" and "group" usually mean division. However, by going backwards, we can have "sharing" multiplication and "lots of" division.

Choose multiplication and division words and write them in the left column, then create the opposite problem. The first and fourth have been done for you.

Multiplication Word	Division problem / Backward multiplication
1. Lots of	John had 4 lots of chocolates. Overall, there were 24 chocolates. How many in each lot if all lots were equal? f = 4 f = ? P = 24 So ÷
2.	 f = f = P = So
3.	 f = f = P = So
Division Word	**Multiplication problem / Backward division**
4. Share	Sue shared her chocolates between 3 children. Each got 6 chocolates. How many did Sue have to start? f = 3 f = 6 P = ? So ×
5.	 f = f = P = So
6.	 f = f = P = So

MP2 – Game: Forward/Backward Mix and Match

Copy cards on same coloured cardboard. Cut cards into pieces along lines. Mix up and then match back together.

Jack bought 5 shirts at $32. How much did he pay?

f   5
f   32
P   ?

×

Sue shared her money amongst her 5 nieces. Each got $32. How much did Sue share?

Henry ran each night for 6 nights. Each night he ran 11km. How far did he run overall?

f   11
f   6
P   ?

×

Barbara was cutting the ribbon into 11m lengths. She ended up with 6 lengths. How many metres of ribbon did she start with?

Forward/Backward Mix-and-Match Cards (cont'd)

Barbara had 66m of ribbon and she cut it into 11m lengths. How many pieces did she end up with?

÷

f   11
f   ?
P   66

Henry ran 11km each night. After some nights he worked out he had run 66km. How many nights had he been running?

Sue shared $160 amongst 5 nieces. How much did each get?

÷

f   5
f   ?
P 160

Jack bought 5 shirts for $160. How much did each shirt cost?

Forward/Backward Mix-and-Match Cards (cont'd)

There was $165. It was shared amongst the 15 families. How much money did each family get?

÷

f   15
f   ?
P 165

15 boxes were filled with the same number of cans. 165 cans were used. How many boxes?

The 364 people were shared amongst the buses. Each bus took 28. How many buses?

÷

f   ?
f   28
P 364

Vi planted rows of trees, 28 in each row. She used 364 trees. How many rows?

Forward/Backward Mix-and-Match Cards (cont'd)

There were 15 boxes each with 11 cans. How many cans?

×

f   15
f   11
P   ?

There was a pile of money. The 11 men each took their share. They each got $15. How much money was in the pile?

Vi planted 13 rows of trees with 28 in each row. How many trees?

÷

f   13
f   28
P   ?

The people were shared amongst the 13 buses. Each bus had 28 people. How many people?

MP2 Activity Feedback Sheet

1.   How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——————————— Interesting Difficult ——————————— Easy Not learning ——————————— Learning
	Boring ——————————— Interesting Difficult ——————————— Easy Not learning ——————————— Learning

2.   How did you feel about trialling the activity?

     Mark the line with an X: Unconfident ——————————— Very confident

3.   Do you think the student was engaged in the activity? Explain.

    _____

4.   What do you think the student learnt from the activity?

    _____

5.   Do you think the student has gained an understanding of the concept being taught? Explain.

    _____

6.   What do you think of the activity?

    _____

7.   What are your suggestions for improving the activity?

    _____

    _____

8.   What else do you suggest could be done to help students who have trouble with this activity?

    _____

    _____

ACTIVITY MP3
[Comparison and factor-factor-product]

Materials: Unifix cubes, pen, paper, material attached (worksheets, games)

Directions:

1.   Say: *Up to now, we have learnt that multiplication/division are combining equal groups/partitioning into equal groups and that forward and backward stories relate to factor-factor-product as follows:*

| $\times$ | forward combining | —— | backward partitioning | —— | f f known | P unknown |

| $\div$ | forward partitioning | —— | backward combining | —— | f f one unknown | P known |

State: *We are now going to look at a third meaning, comparison.*

2.   Direct: *Put out 2 counters and 6 counters. Look at them and compare.* Ask: *Which is bigger?* [6] *How many more than 2 is 6?* [4] *How many times larger than 2 is 6?* [3×]. Discuss how comparison can be:
   (1)   order – 6 is bigger than 2
   (2)   addition/subtraction – 6 is 4 more than 2 (i.e. $6 - 2 = 4$, $4 + 2 = 6$)
   (3)   multiplication/division – 6 is 3 times larger than 2 (i.e. $6 \div 2 = 3$, $2 \times 3 = 6$)

Repeat this for 3 counters and 15 counters.

3.   State: *We are going to look at multiplication/division comparison.* State a problem: *Jack has 4 times as much as Fred. Fred has $7. This means Jack has $28.* Ask: *We have three numbers, 4, 7 and 28. Which are factors (f)?* [4 and 7] *Which is the product (P)?* [28]

Look at situations where the Product is unknown:

	Problem	Known/		Operation
(a)	Jan bought 5 times as many cans as Sue. Sue bought 8 cans. How many cans did Jan buy?	f	5	$\times$
		f	8	$\div$
(b)	Dad weighed 4 times as much as his son. The son weighed 22kg. How much did dad weigh?	f	4	$\times$
		f	22	$\div$

Discuss that one factor is the "how many times", the other factor is the first thing (the amount to be "timesed"), while the product is the second thing (the amount that is "times bigger"). Then look at situations where the Product is known (that is, the "times bigger" thing is known):

	Problem	Known/	Operation
(a)	Sid ran 3 times as far as Jack. Sid ran 18km. How far did Jack run?	f 3	×
		f ?	÷
(b)	The Warriors scored 30 points. The Falcons scored 6 points. How many times larger was the Warriors' score than the Falcons'?	f ?	×
		f 6	÷

Discuss again that the product is the second thing – the amount that is times bigger.

4.   Complete Worksheet 3.1.

5.   State: *Although there are three specific meanings for multiplication and division (forward, backward and comparing), everything can be solved by considering factor-factor-product.* State: *Multiplication is factors known, Product unknown. Division is one factor not known, Product known.* Ask: *Join the things that are all the same in the diagram below.*
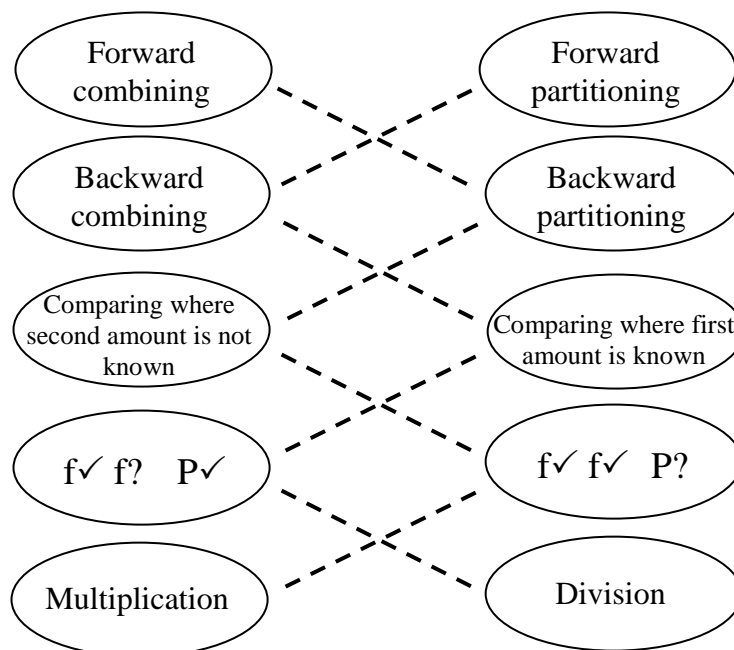
6.   Complete Worksheets 3.2 and 3.3.

7.   Play the games: "Problem Mix and Match" and "Place the Problem".

MP3 – Worksheet 3.1

Comparison problems:

Consider the following problems. Fill in factor-factor-product and circle the operation. Do not work out the answer. The first two have been done for you.

Problem	factor-factor-product	Operation
1. Henry had 5 times as much money as Kate. Kate had $21. How much did Henry have?	f 5 f 21 P ?	⊗ ÷
2. Jack has 7 times as much land as Joan. Jack has 21 hectares. How many hectares has Joan?	f 7 f ? P 21	× ⊘
3. Bill has 3 times as much money as Frank. Frank has $24. How much does Bill have?	f f P	× ÷
4. Mary travels 6 times longer than Chrissie. Mary travels 42km. How far does Chrissie travel?	f f P	× ÷
5. There are 9 times as many Cokes as lemonades. There are 6 crates of lemonade. How many crates of Coke?	f f P	× ÷
6. June has 5 bottles and Peta has 35 bottles. How many times as much does Peta have?	f f P	× ÷
7. There are 9 times as many Cokes as lemonades. There are 63 crates of Coke. How many crates of lemonade?	f f P	× ÷
8. June has some bottles and Peta has 5 times as much. Peta has 35 bottles. How many does June have?	f f P	× ÷

MP3 – Worksheet 3.2

Mixed problems:

Consider the following problems. Fill in factor-factor-product and circle the operation. Do not work out the answer. The first two have been done for you.

Problem	factor-factor-product	Operation
1. Mary has 4 times the money Mack has. Mack has $16. How much does Mary have?	f 4 f 16 P ?	(×) ÷
2. Mary shares her money among 4 students. Mack gets $16 and wonders how much there was. How much did Mary have to share?	f 4 f 16 P ?	(×) ÷
3. Sue shares $120 amongst 10 mates. How much does each get?	f f P	× ÷
4. There are 7 bikes. There are 8 times as many cards as bikes. How many cars are there?	f f P	× ÷
5. Sue shares her money between some mates. Each gets $120. If there were 10 mates, how much did Sue share?	f f P	× ÷
6. Each CD cost $32. There were 4 bought. How much did they all cost?	f f P	× ÷
7. Each CD cost $32. The CD player cost $64. How many times more costly was the CD player?	f f P	× ÷
8. Each CD cost $32. Four students shared the cost. What did they each pay?	f f P	× ÷

MP3 – Worksheet 3.3

Tricksy problems:

Consider the following problems. Fill in factor-factor-product and circle the operation. Do not work out the answer. The first has been done for you.

Problem	factor-factor-product	Operation
1. Danny paid 5 times as much as $14. How much did he pay?	f　　5 f　　14 P　　?	⊗ ÷
2. Shares sell for 4 times their previous value. If they are $44 now, what were they?	f f P	× ÷
3. The land speed record went up 3 times as fast to 600km/h. What was the previous record?	f f P	× ÷
4. Bob gave back $64 to Fred. Fred shared it amongst 4 people including Sue. What did Sue get?	f f P	× ÷
5. Benjamin had some money. He shared it amongst 4 people. They were all happy with what they got. Benjamin found later he had given away $48. How much had each person got?	f f P	× ÷
6. The gang gathered and split up into 7 equal groups. Each group rejoined and 56 gang members were present. How many in each group?	f f P	× ÷
7. The cost of perfume is 20 times the mL bought. This means 20mL costs $400. How many mL cost $600?	f f P	× ÷
8. Mum wanted to know how many small cakes to buy. They came in packets of 6 and 102 were needed. How many packets should Mum buy?	f f P	× ÷

MP3 – Game: Problem Mix-and-Match Cards

Copy cards on same coloured cardboard. Cut cards into pieces along lines. Mix up and then match back together.

June had 7 bags with 8 balls in each. How many balls?

Sue shared the lollies amongst the 7 boys. Each got 8 lollies. How many lollies?

John had $7. Fred had 8 times as much as John. What did Fred have?

×

f   7
f   8
P   ?

John had $56 which he shared amongst 8 friends. How much money did each friend get?

June had 7 bags each with the same number of balls. Overall there were 56 balls. How many in each bag?

Sue had 7 times the lollies of the boy. Sue had 56 lollies. How many did the boy have?

÷

f   7
f   ?
P   56

Problem Mix-and-Match Cards (cont'd)

There were 60 cases with 4 cameras in each. How many cameras?

The four friends shared the cost of everything. There were 4 chairs to buy. They all put in and paid $60 per chair. How much did they spend in total?

The value of the doll increased by 4 times. It was $60, what is its price now?

×

f   60
f   4
P   ?

The collector spent $240 on 4 dolls. How much for each doll?

There were 4 cases. They cost $240. How much for each case?

÷

f   ?
f   4
P 240

The chairs' price went up 4 times. They were now costing $240. How much had they cost before?

Problem Mix-and-Match Cards (cont'd)

The 8 t-shirts cost $24 each. How much was spent?

The money was spent on 8 t-shirts. Sam worked out this meant each t-shirt cost $24. How much money was there?

×

f   8
f   24
P   ?

The coats were 8 times the value of the t-shirts. How much did the coats cost if the t-shirts were worth $24?

$192 was put aside for t-shirts. 24 were bought. How much did each cost?

24 t-shirts were bought and they each cost the same. Spending $192 meant that the t-shirts were cheap. How cheap were they?

÷

f   ?
f   24
P 192

Since 1947 the cost of a t-shirt has gone up 24 times. What was the cost in 1947 for a $192 t-shirt now?

Problem Mix-and-Match Cards (cont'd)

I bought 27 DVDs for $28 each. How much was spent?

I planted all the trees. It made 27 rows of 28. How many trees to start with?

We had to increase the 27kg statue in size 28 times. How much does it weigh now?

×

f 27
f 28
P ?

I paid $756 for 27 DVDs. How much did each cost?

I planted 27 rows. I used all the 756 trees. How many trees in each row?

We increased the statue 28 times and it weighed 756kg. How much did it weigh at the start?

÷

f 27
f ?
P 756

MP3 – Game: Place the Problem

Materials: Gameboard (below), pen, paper, unifix or counters, answer sheet (attached)

Number of players: 2 playing, 1 umpire

Directions:

1) Umpire takes the answer sheet.

2) Players, in turn, choose a statement from the cloud. Cross it out. Place it in multiplication or division box (point to box).

3) If umpire says correct, move counter forward one space.

4) First to finish wins.

f✓ f✓ P?

Comparing when 1st thing is unknown

Backward partitioning

P is known

Backward combining

Comparing when 2nd thing is known

P is unknown

f? f✓ P✓

Forward partitioning

One f is unknown

Forward combining

f✓ f? P✓

Comparing when 2nd thing is unknown

Comparing when times unknown

Both f's are known

Comparing when 1st thing/times is known

MULTIPLICATION	DIVISION

START						FINISH
START						FINISH

MP3 – *Place the Problem Answer Sheet*

MULTIPLICATION	*DIVISION*
f✓ f✓ P?	*f? f✓ P✓*
	f✓ f? P✓
P is unknown	*P is known*
Both f's are known	*One f is unknown*
Forward combining	*Backward combining*
Backward partitioning	*Forward partitioning*
Comparing when first thing and times known	*Comparing when first thing unknown*
Comparing when 2nd thing unknown	*Comparing when 2nd thing known*
	Comparing when times unknown

MP3 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——————— Interesting Difficult ——————— Easy Not learning ——————— Learning
	Boring ——————— Interesting Difficult ——————— Easy Not learning ——————— Learning

2. How did you feel about trialling the activity?

    Mark the line with an X: Unconfident ——————— Very confident

3. Do you think the student was engaged in the activity? Explain.

    _____

4. What do you think the student learnt from the activity?

    _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

    _____

6. What do you think of the activity?

    _____

7. What are your suggestions for improving the activity?

    _____

    _____

8. What else do you suggest could be done to help students who have trouble with this activity?

    _____

    _____

ACTIVITY MP4
[Constructing problems]

Materials: Unifix cubes, pen, paper, material attached (worksheets, games)

Directions:

1. Say: *Now we are going to construct problems. It is a good way to learn how to solve problems.* Say: *Consider the following combining problem: "I bought 3 bottles, each cost $8. I spent $24."* Say: *From this story write 3 stories with one number unknown.*

   (a) The 3 is unknown – "I bought some bottles, each cost $8. I spent $24. How many bottles?"

   (b) The 8 is unknown – "I bought 3 bottles. They all cost the same. I spent $24. How much did each bottle cost?"

   (c) The 24 is unknown – "I bought 3 bottles, each cost $8. How much did I spend?"

   Ask: *Which stories are multiplication? Which are division?*

2. Repeat for another combining story. For example, "I planted 9 rows of 16 plants. This used 144 plants."

3. Repeat steps 1 and 2 for two partitioning and two comparing stories:

Partitioning	Comparing
Problem 1:	Problem 1:
I shared $104 amongst 13 people. Each person got $8	Frank had 6 times as much money as Bill. Bill had $11. Frank had $66.
Problem 2:	Problem 2:
I allocated the 168 staff so that there was 12 to a minibus. How many minibuses?	The land increased in value 5 times. It was $105 per section. Now it was $525 a section.

   Determine which stories are × and which are ÷. Ask: *Are there more ÷ than × stories?*

4. Complete Worksheets 4.1, 4.2 and 4.3.

5. Play the games "Mult/Divide Problem Race Track" and "Build a Mult/Divide Problem".

MP4 – Worksheet 4.1 (combining problems)

Create 3 different problems with one number unknown from the information in the first column. Determine what is known/unknown re factor-factor-product. Circle the operation.

COMBINING STORIES	PROBLEMS WITH ONE NUMBER UNKNOWN	
A. Frank bought 6 cases of Coke. Each case cost $24. He paid $144.	A1.	f f P
	A2.	f f P
	A3.	f f P
B. There were 7 relay runners. They each ran 5km. In total, the group ran 35km.	B1.	f f P
	B2.	f f P
	B3.	f f P

MP4 – Worksheet 4.2 (partitioning problems)

Create 3 different problems with one number unknown from the information in the first column. Determine f, f & P and circle the operation.

PARTITIONING STORIES	PROBLEMS WITH ONE NUMBER UNKNOWN	
C. Bruno made 48 pies. He shared them amongst his 6 children. Each child got 8 pies.	C1.	f f P
	C2.	f f P
	C3.	f f P
D. Joan had 54 lollies. She put them in bags of 6. She made 9 bags of lollies.	D1.	f f P
	D2.	f f P
	D3.	f f P

MP4 – Worksheet 4.3 (comparison problems)

Create 3 different problems with one number unknown from the information in the first column. Determine f, f and P and circle the operation.

| COMPARING STORIES	PROBLEMS WITH ONE NUMBER UNKNOWN	
E. Janet had 4 times the money Lucy had. Lucy had $16. Janet had $64.	E1.	f f P
	E2.	f f P
	E3.	f f P
F. The balloon increased in size 7 times. It was 4m high. It became 28m high.	F1.	f f P
	F2.	f f P
	F3.	f f P

MP4 – Game: Mult/Divide Problem Race Track

Materials: Enlarged Mult/Divide Problem Race Track Gameboard, dice, different coloured counters for each player.

**Number of players:**   2-4

Directions:

1)   Obtain a selection of problems with one number unknown from what the students have written in MP4 Worksheets 4.1, 4.2 and 4.3. Cut these into cards.

2)   Shuffle these cards and place in "Problems" position on Race Track board.

3)   Play the game:

- Each player, in turn, throws die and moves along the race track the number shown on the die.
- If a player lands on ⭐ he/she picks up a problem.
- If give correct operation, move two steps forward.
- If give incorrect operation, move two steps back.
- First player to reach Finish wins.
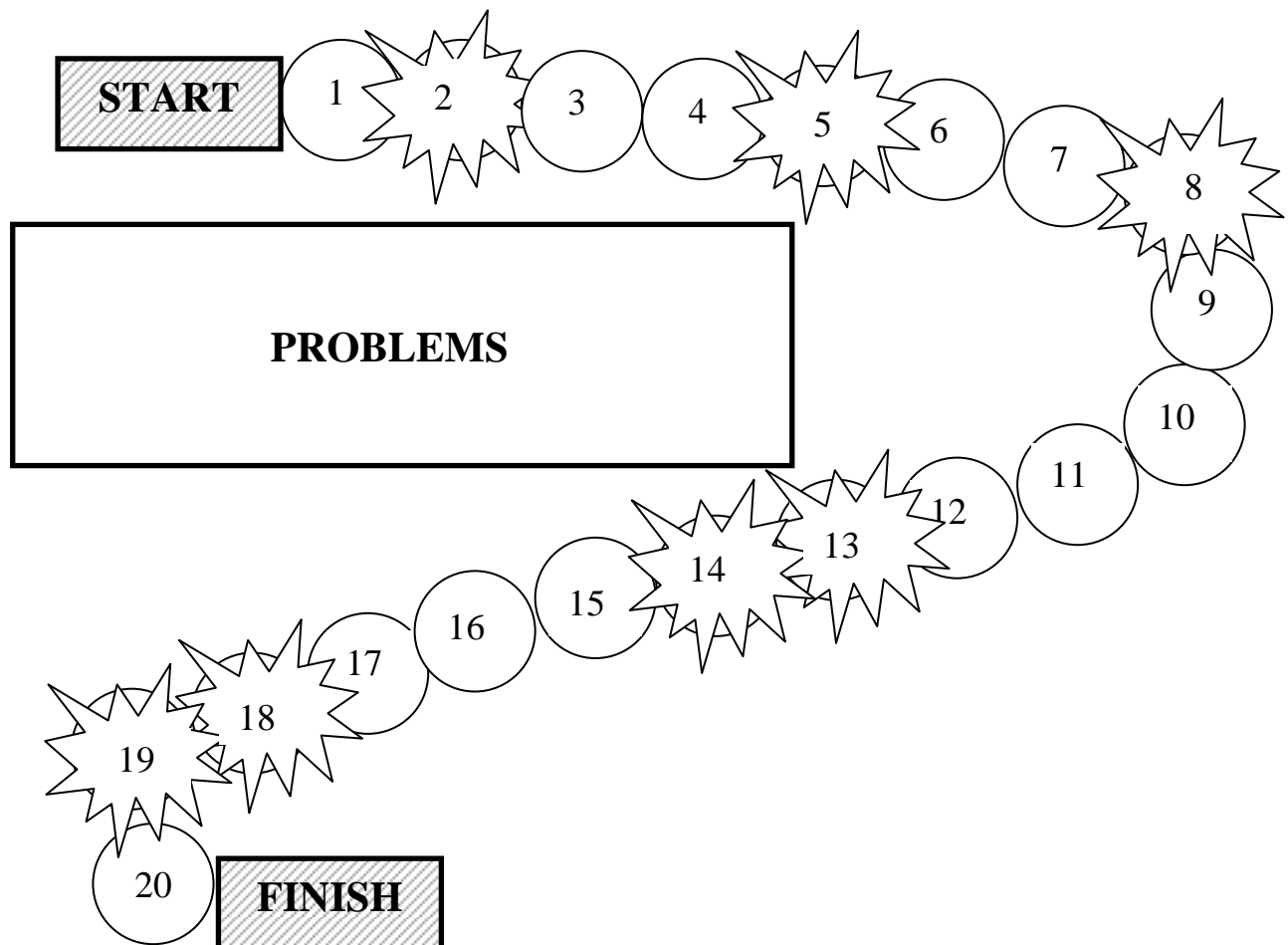
MP4 – Game: Build a Mult/Divide Problem

Materials: Pen, paper, disc of wood (or a coin) with multiplication on one side and division on the other side, OR a coin (heads – multiplication, tails – division).

Multiplic-ation ×
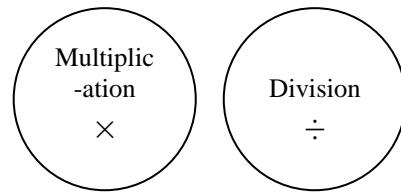
Division ÷

Directions:

Play one of the versions of the game: "Build a Multiplication/Division Problem".

VERSION 1:

1) Two (2) player game.

2) Each player in turn throws disc/coin to select problem type.

3) Opponent starts a problem giving context (e.g., some boys were carrying bags).

4) Player continues problem, giving the first number.

5) Opponent continues problem, giving what this number does.

6) Player completes problem, giving the second number and the question.

7) Score 1 point if correct problem.

8) First player to 3 points wins.

VERSION 2:

1) Four (4) players – opposites work together.

2) Each pair throws disc/coin to determine problem type.

3) Each pair works together to tell a problem:

   1st person states context

   2nd person states 1st number and what it is doing

   1st person states 2nd number and what its relation is to 1st number

   2nd person gives question.

4) If correct, pair scores 1 point.

5) First pair to 4 points wins.

MP4 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——————————— Interesting Difficult ——————————— Easy Not learning ——————————— Learning
	Boring ——————————— Interesting Difficult ——————————— Easy Not learning ——————————— Learning

2. How did you feel about trialling the activity?

    Mark the line with an X: Unconfident ——————————— Very confident

3. Do you think the student was engaged in the activity? Explain.

    _____

4. What do you think the student learnt from the activity?

    _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

    _____

6. What do you think of the activity?

    _____

7. What are your suggestions for improving the activity?

    _____

    _____

8. What else do you suggest could be done to help students who have trouble with this activity?

    _____

    _____

ACTIVITY MP5

[Multi-step problems and strategies]

Materials: Pen, paper, calculator, material attached (worksheets, games)

Directions:

1. Ask: *Consider this problem. "I bought 4 CDs for $19 each. How much change from $100?"* State: *This problem has more than one step. The first sentence is multiplying 4 × 19 = 16. The second sentence is change or subtraction 100 − 76 = 24. We are going to look at how to work these out.* State: *Consider problem "Jan shared $36 amongst 4 children. One of the children, Vince, already had $6. How much does he have now?"* Say: *There are 3 ways to help work this out.*

2. **Way 1** Say: *The first step is Polya's 4 stages. Look at this poster:*

 | POLYA'S 4 STAGES | |
   | --- | --- |
   | SEE | – work out what you have to do |
   | PLAN | – make a plan to do it (work out a way) |
   | DO | – do the plan (work the problem out) |
   | CHECK | – check your answer (does it make sense?) See what you can learn from doing it (what worked?) |

   Let's work through the problem.

   (a) SEE   Ask: *What is going on here? Do you see two parts? What is the first part? What is the second? Does the first affect the second?*

   (b) PLAN   State: *Break the problem into parts.* Ask: *What parts are there? Which should you do first? What does each part mean?* [Sharing means ÷. How much means +].

   (c) DO   Say: *Do the first part. Do the second part. Put them together* [36 ÷ 4 = 9, 9 + 6 = 15, Answer $15]

   (d) CHECK   Ask: *Is the answer correct? Can you reverse what you are doing? Does it turn out OK? What did you learn?*

   Discuss the 4 stages. What do the students think of them? Useful?

3. **Way 2** Say: *Use strategies to help in the SEE and PLAN part of Polya's stages. There are 4 strategies that really help.* Introduce these strategies: (1) Break problem into parts; (2) Make a drawing or diagram; (3) Find what is given, needed and wanted; and (4) Restate the problem. Say: *We will go through each in turn for the problem "Jan shared $36 amongst 4 children. One of the children, Vince, already had $6. How much does Vince have now?"*

(a) Break into parts – get students to see that the sharing (division) and 'how much with $6' (addition) parts should be done separately, with the sharing first.

(b) Drawing – Say: *Look at these 2 drawings.* Ask: *Which helps the most?*
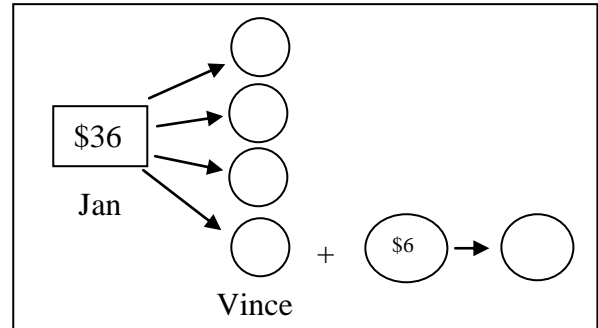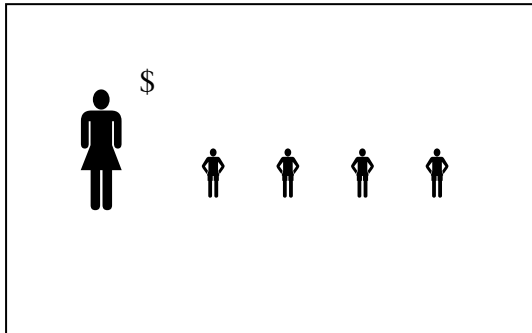
(c) Given, needed and wanted – Get students to circle each number. Ask: *How is this number used in the problem. What is it?* Get students to underline the phrase that asks for the answer. Ask: *What are we required to find? Do we have to worry about the other 3 children?*

(d) Restate – Say: *Try to rewrite/resay the problem so it is easy?* Ask: *Which of the two below makes it easier?*

Restate 1	Restate 2
Share $36 into 4. Add $6 to a share.	Sharing gives everyone something and then there is the $6 that Vince has.

Discuss the 4 strategies. Which make sense? Why?

4. **Way 3** Look at language. The restating shows that it is often the way things are put that make a problem hard. Give problems with easy language:

  e.g. John got 1/4 of the $36. He had $6 before. How much does he have now?

Get familiar with all the methods. Use the "Problem Strategy Template" attached.

5. Complete Worksheet 5.1.

MP5 – Problem Strategy Template

PROBLEM

DRAWING	**GIVEN**
	NEEDED
	WANTED
	REWRITE

WORKING

1ˢᵗ **step:**

2ⁿᵈ **step:**

Extra steps:

ANSWER:

MP5 – Worksheet 5.1

Use the template and steps to solve these problems. The first 5 are all 2-step problems. Use a calculator!

---

1. Sue bought 6 pies for $4 each. How much change from $50?

---

2. Frank received his share of $60. There were 5 sharing. He already had $13. How much does he have now?

---

3. Bill bought 5 cakes for $6 and a cheese tray for $23. How much did he pay?

---

4. Frank had 6 times the money Greg had. Greg had $7. How much more money does Frank have to find to pay his $50 fee?

---

5. There were 7 cars on the road. Inside the parking station there were 6 times the cars that were on the road. How many cars in total?

---

6. Bill bought 7 shirts for $18 each and 6 pairs of pants for $33 each. How much did he pay?

---

7. Bill had 3 times the money Joe had. Joe had 7 times the money I had. I had $11. How much did Bill have?

---

8. Seven boats all caught 18 fish. The eighth boat caught 3 times this many fish. How many fish did all 8 boats catch?

MP5 Activity Feedback Sheet

1.   How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——————— Interesting Difficult ——————— Easy Not learning ——————— Learning
	Boring ——————— Interesting Difficult ——————— Easy Not learning ——————— Learning

2.   How did you feel about trialling the activity?

   Mark the line with an X: Unconfident ——————— Very confident

3.   Do you think the student was engaged in the activity? Explain.

   _____

4.   What do you think the student learnt from the activity?

   _____

5.   Do you think the student has gained an understanding of the concept being taught? Explain.

   _____

6.   What do you think of the activity?

   _____

7.   What are your suggestions for improving the activity?

   _____

   _____

8.   What else do you suggest could be done to help students who have trouble with this activity?

   _____

   _____