## Professional Learning 5 Multiplication and Division <br> Booklet 5.4: Division Computation Strategies



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$$
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$$

## Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

## YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".
"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Island Regional Educational Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: Growing community through education.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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CRICOS No. 00213」

This booklet was developed as part of a project which ran from 2005-2008 and was funded by an Australian Research Council Linkage grant, LP0562352: Sustainable education capacity building: Empowering teacher aides to enhance rural and remote Indigenous students' numeracy outcomes, with support provided by Education Queensland.

# PROFESSIONAL LEARNING 5: MULTIPLICATION AND DIVISION 

## BOOKLET 5.4 <br> DIVISION COMPUTATION STRATEGIES

2008

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## OVERVIEW

## PURPOSE

These materials were designed to be used in conjunction with a Professional Learning program for teacher aides. The objective of this Professional Learning was to empower teacher aides to enhance rural and remote Indigenous students' numeracy outcomes. This document contains the materials of the fourth of five different booklets on multiplication and division.
If your school would like to receive a YuMi Deadly Maths Professional Learning program please contact the YuMi Deadly Centre (YDC) on: 0731380035 or ydc@qut.edu.au.

## DIRECTIONS

## (1) Interviewing the students:

Pick one or more students who appear to be having trouble understanding division. Interview these students using the interview schedule and the materials. Mark what they do and put their results on the Student Recording Sheet.

## (2) Trialling the student activities:

Use the Recording Sheet to work out the activities the students need to do and trial these activities with the students (with each student one at a time or with a group of students). Keep a record of what happens and collect the students' work.

## INTERVIEW SCHEDULE

## Materials:

Unifix cubes, deck of playing cards, washable felt pen, pen, paper
Materials within this booklet: interview questions, cards, number tracks, number lines
Student Recording Sheet

## Directions:

1. Photocopy and laminate attached cards.
2. Gather other material (unifix, paper, pencils)
3. Place material in front of students. Give students pen and paper to write with.
4. Tell the students you are trying to find out what they know. Say they are not expected to know it and you will teach what is not known.
5. Give the student directions slowly - read problems. Do not give hints. If student cannot do a question, pass on to the next question, repeating that it is not important if they don't know how to do the question.
6. Allow students to use material and make drawings but only after they say they do not know how to do it with symbols alone.

## INTERVIEW MATERIALS

## Interview Questions

| DSA | Show and read Card 1 <br> - Ask: Can you use the top answer to answer the two underneath? <br> Repeat this for Card 2 |
| :---: | :---: |
| DSB | Show and read Card 3 <br> - Ask: Can you answer this division operation by showing hundreds tens and ones? <br> - If cannot do this, show and read Card 4 and ask: Can you answer this one? <br> - If still cannot do this, ask if they can solve it any way. |
|  | Show and read Card 5 <br> - Ask: Can you answer this division operation by working out how many $4 s$ in 512 - looking first at 100s of $4 s$ and then 10 s of $4 s$ ? <br> - If cannot do this, show and read Card 6 and ask: Can you answer this one? |
|  | Show and read Card 7 <br> - Ask: Can you answer this division operation by finding an easier operation and relating this one to anything. Say: Say the problem (have student say the problem). <br> - If cannot do this, show and read Card 8 and ask: Can you read this one? |

## Interview Cards

| $\begin{aligned} & \text { Card 1 } \\ & 6 \div 2=3 \\ & 60 \div 2= \\ & 600 \div 2==-2 . \end{aligned}$ | Card 2 $\begin{aligned} 18 \div 3 & =6 \\ 180 \div 3 & = \\ 1800 \div 3 & = \end{aligned}$ |
| :---: | :---: |
| Card 3 $\begin{aligned} & 576 \div 4 \\ & 4 \longdiv { 5 7 6 } \end{aligned}$ | Card 4 $72 \div 4$ <br> $4 \longdiv { 7 2 }$ |
| Card 5 $\begin{aligned} & 512 \div 4 \\ & 4 \longdiv { 5 1 2 } \end{aligned}$ | Card 6 <br> $76 \div 4$ <br> $4 \longdiv { 7 6 }$ |
| Card 7 $\begin{array}{r} 584 \div 4 \\ 4 \longdiv { 5 8 4 } \end{array}$ | $\begin{aligned} \text { Card } 8 & \\ & 92 \div 4 \\ & 4 \longdiv { 9 2 } \end{aligned}$ |

## STUDENT RECORDING SHEET

Name: $\qquad$

School/Class: $\qquad$

| Interview item | $\begin{aligned} & \text { Result } \\ & (\downarrow, x) \end{aligned}$ | Comments | Activities to be completed if incorrect |
| :---: | :---: | :---: | :---: |
| DSA: Higher decade division facts |  |  | DS1 |
| DSB: Separation strategy for division computation |  |  | DS2 |
| DSC: Sequencing strategy for division computation |  |  | DS3 |
| DSD: Compensation strategy for division computation |  |  | DS4 |

## INTRODUCTION

## Contents

This package contains:

- four tutoring activities (DS1 to DS4) and their students materials (games and worksheets)


## Pedagogy

The activities DS1 to DS4 are based on the Rathmell Triangle Relationship below, real world problems are related to set, array and number line models, language and symbols (and vice versa) to teach strategies for computation.

## REAL WORLD PROBLEM



However, the focus of the activities is to develop a repertoire of strategies for computing such examples as $36 \div 6$ and $255 \div 3$. The activities are designed to allow you to tutor students who are having difficulties with division computation for 2- and 3 -digit numbers $\div 1$-digit numbers. The activities are based on the belief that it is more important to use the algorithms to teach a variety of strategies than to get a correct answer. However, the activities also show how to get correct answers.

There are three strategies (or strategy groupings) associated with multiplication computation. These are:

## (1) Separation:

This is the traditional algorithm for long division, and it is based on sharing as can be seen in the following example $732 \div 3$. This example is considered as 7 hundreds 3 tens and 2 ones shared amongst 3 people. By separating the number into its place values, it is possible to share each place value separately. It is easily seen to be more efficient to share large place values first. So 7 hundreds are shared between 3,2 hundreds to each person, with 6 hundreds used and 1 hundred left over! This hundred makes 13 tens with the 3 tens and sharing continues until all ones are shared. This process works really well for money - $\$ 100$ and $\$ 10$ notes and $\$ 1$ coins. The algorithm on the right symbolically acts out this sharing process.
(2) Sequencing:

This is an alternative method/strategy. The number is not separated but broken into parts which become a sequence for the process of division. It is best seen in terms of arrays/area as
 on right. Multiplication of 244 by 3 is seen as the area of a $3 \times 244$ rectangle. Thus, division is seen as finding the long side of a 3 high rectangle with an area of 732 . The idea is to start to
 think of what the long side could be. Is it over 10 , over 100 ? Try 100.100 multiplied by 3 is 300 , leaving 432 as in figure on right. Try another 100 - yes, it works and leave 132 as on right. Then we can try 30 and 10 or 40 direct and finally 4 for the last 12.

This gives the algorithm on the right. It enables the division to be worked out with a sequence if parts - for example, the 244 is determined by $100,100,30,10$ and 4.
(3) Compensation:

| 100 |  |  |  |
| :---: | :---: | :---: | :---: |
| 3 | 300 | 432 |  |
|  | 100 | 100 | ? |
| 3 | 300 | 300 | 132 |

$3 \longdiv { 7 3 2 }$

| 300 | 100 |
| :---: | :---: |
| 432 |  |
| 300 | 1000 |
| 132 |  |
| 90 | 30 |
| 42 |  |
| 30 | 10 |
| 12 |  |
| 12 | 4 |
| 0 | 244 |

This is the most efficient strategy. There is no breaking into parts by separating into place values or generating sequences. Numbers are left as they are but other examples are sought which are closer to the original division example but easy to divide. For the example $732 \div 3$, an easy example would be $750 \div 3$ which is 250 , then the difference between the original and the easy example has to be compensated for. 750 is 18 too high, this is 6 threes, so subtract 6 from $250=244$, which is the answer for $732 \div 3$.

To recognise easier examples requires knowledge of (a) higher-decade division (e.g., $12 \div 4=3$ so $120 \div 4=30$ and $120 \div 40=3$ ); and (b) when a division is the inverse of an easy multiplication (e.g., $\times 25$ and $\times 50$ ). Some examples of making division easy are as follows.
(1) $581 \div 7$ : An easy example is $560 \div 7=80$, 581 is 21 more than 560 so need to add $21 \div 7=3$ to 80 , so answer is 83 .
(2) $924 \div 6$ : An easy example is $900 \div 6=150,924$ is 24 more than 900 , so need to add $24 \div 6=4$ to 150 , so answer is 154 .

Note: There is another method for making examples easier and that is to use the fact that if, for example, $6=3 \times 2$ then $24 \div 6=24 \div 3 \div 2=8 \div 2=4$.

This is particularly useful for divisors based on 2 . For example, $544 \div 8$ can be done by $54 \div 2 \div 2 \div 2$ as $2 \times 2 \times 2=8$. This is really halving, so answer half of half of half of $544=$ half of half of $272=$ half of $136=68$.

It is also useful when dividing by 2 digit numbers, e.g. $888 \div 24$ as $6 \times 4=24$, then answer is found by $888 \div 4=222$ and $222 \div 6=37$.

All the three strategies require knowledge of basic number facts (e.g., $6 \times 8=48$ ) and also higher-decade number facts (e.g., $6 \times 80=480,600 \times 80=48000$ ).
Thus, the four activities in this booklet are the higher-decade facts (Activity DS1), separation strategy (Activity DS2), sequencing strategy (Activity DS3), and compensation strategy (Activity DS4).

## Approach

The four operations of addition, subtraction, multiplication and division are important in mathematics, as they reflect everyday important actions (of joining, separation, combining and partitioning) that happen in the lives of people and that are the basis of problems that need to be solved.

For larger numbers, procedures have been developed to achieve answers using basic facts and higher-decade facts. These are based on strategies that also can be used to teach multiplication and division for, for example, mixed numbers, measures, time, and algebra. In a world of calculators and computers, computation for answers has become less important than computation for learning strategies. However, strategies when learnt will enable answers to be acquired as well as the procedures to be understood.

Thus, the sequence for the development of any of the four operations with whole number is:


Therefore, in this booklet we will focus on strategies (leaving problem solving for booklet 5.5) so as to develop mathematics ideas that will pay off in later years in other contexts (e.g. decimals). For this reason, much of the activity in this booklet is learning a repertoire of strategies (not just one strategy to get answers).

## ACTIVITIES

## ACTIVITY DS1

[Higher-decade division facts]
Materials: MAB, unifix cubes, pens, paper, calculators, attachments

## Directions:

1. Consider $8 \div 4$. Act this out by sharing 8 MAB units amongst 4 people. Ask: How many does each person get? [2] Now consider $80 \div 4$. Act this out by sharing 8 tens amongst 4 people. Ask: How many does each person get? [ 2 tens or twenty] Repeat this for $800 \div 4$.
2. Consider $15 \div 5$. Act this out with unifix by making rows of 5 . Ask: How many rows? [3] Consider $150 \div 5$. Act this out by making rows of 5 . Do it in your mind. Will 10 rows of 5 give 150, 20 rows of 5,30 rows of 5 and so on. Ask: How many rows? [30] Repeat this thinking for $1500 \div 5$.
3. Write down the results of 2 and 3. Use a calculator to work out the results of the examples under them.

| $8 \div 4=2$ | $80 \div 4=20$ | $800 \div 4=200$ |
| :--- | :--- | :--- |
| $15 \div 5=3$ | $150 \div 5=30$ | $1500 \div 5=300$ |
| $24 \div 6=$ | $240 \div 6=$ | $2400 \div 6=$ |
| $12 \div 3=$ | $120 \div 3=$ | $1200 \div 3=$ |

Ask: Can you write down a rule for dividing with tens and hundreds? If difficulty, ask students to look at relationship of other columns to left hand column.
4. Use a calculator to work out the answers for these examples:

| $9 \div 3=$ | $90 \div 30=$ | $900 \div 300=$ |
| :--- | :--- | :--- |
| $8 \div 2=$ | $80 \div 2=$ | $800 \div 20=$ |
| $14 \div 7=$ | $140 \div 70=$ | $1400 \div 70=$ |
| $18 \div 6=$ | $180 \div 6=$ | $1800 \div 600=$ |
| $32 \div 8=$ | $3200 \div 80=$ | $3200 \div 800=$ |

Ask: Can anyone think of a rule for dividing these numbers? If difficulty, ask:
Can you see how these [point to 2 right hand columns] relate to this [point to left hand column]?
5. Complete Worksheet 1.1 and 1.2.
6. Complete game "Ride them equal".

## DS1 - Worksheet 1.1

1. Complete the following using a calculator:

| $6 \div 2$ | $=$ |
| ---: | :--- |
| $60 \div 2$ | $=$ |
| $600 \div 2$ | $=$ |
| $6000 \div 2$ | $=$ |
| $60 \div 2$ | $=$ |
| $600 \div 20$ | $=$ |
| $6000 \div 20$ | $=$ |
| $600 \div 200$ | $=$ |
| $6000 \div 200$ | $=$ |
| $6000 \div 2000$ | $=$ |


| $49 \div 7$ | $=$ |
| ---: | :--- |
| $490 \div 7$ | $=$ |
| $4900 \div 7$ | $=$ |
| $49000 \div 7$ | $=$ |
| $490 \div 70$ | $=$ |
| $4900 \div 70$ | $=$ |
| $49000 \div 70$ | $=$ |
| $4900 \div 70$ | $=$ |
| $49000 \div 70$ | $=$ |
| $4900 \div 700$ | $=$ |


| $28 \div 4$ | $=$ |
| ---: | :--- |
| $280 \div 4$ | $=$ |
| $2800 \div 4$ | $=$ |
| $28000 \div 4$ | $=$ |
| $280 \div 40$ | $=$ |
| $2800 \div 40$ | $=$ |
| $28000 \div 40$ | $=$ |
| $2800 \div 400$ | $=$ |
| $28000 \div 400$ | $=$ |
| $28000 \div 4000$ | $=$ |

2. Complete these without a calculator:

3. Write down a rule that these examples follows:
4. Show how your rule can explain why if $65 \div 5=13$ then $6500 \div 50=130$ :

## DS1 - Worksheet 1.2

$$
6 \div 3=2
$$

How many zeroes in the answers?
For each question, determine the number of zeros and colour one area with the colour given by the number of zeroes - 0 - green; 1 - brown; 2 - blue; 3 - yellow. When you are finished, colour all other areas. What have you got?
(1) $600 \div 20$
(2) $6000 \div 2$
(3) $60000 \div 200$
(4) $6 \div 2$
(5) $6000000 \div 200000$
(6) $600 \div 200$
(7) $60000000 \div 2000$
(8) $6000000 \div 20000$
(9) $6000000 \div 200000$
(10) $600 \div 2$
(11) $60000 \div 2000$
(12) $600000000 \div 200000$
(13) $600000 \div 20000$
(14) $60000 \div 20$
(15) $60 \div 20$
(16) $60000000 \div 200000$
(17) $600000 \div 2000$
(18) $6000 \div 2000$
(19) $60 \div 2$
(20) $6000000 \div 2000$
(21) $6000 \div 20$
(22) $6000 \div 200$
(23) $600000 \div 200$
(24) $60000 \div 20000$


It keeps on paddling?

## DS1 - Game: "Ride Them Equal"

Materials:
A marker (coloured unifix) for each player (start at "START"), calculator, pen.
Number of players: 2

## Directions:

1) Pick on exercise from each cloud. Cross out the examples chosen.
2) Use a calculator to work out the answers.
3) If the number of zeros are the same, move marker one step towards finish line.
4) Wild can be used to make your own problem.
5) First to finish wins.


| START |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| START |  |  |  |  |  |  |

## DS1 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

| NAME | STUDENTS' REACTIONS |  |
| :--- | :--- | :--- |
|  | Boring $\quad$ | Interesting |
|  | Difficult $\quad$ | Easy |
|  | Not learning $\quad$ | Learning |
|  | Boring $\quad$ | Interesting |
|  | Difficult $\quad$ | Easy |
|  | Not learning | Learning |

2. How did you feel about trialling the activity?

Mark the line with an $\mathrm{X}: \quad$ Unconfident $\longrightarrow$ Very confident
3. Do you think the student was engaged in the activity? Explain.
$\qquad$
4. What do you think the student learnt from the activity?
$\qquad$
5. Do you think the student has gained an understanding of the concept being taught? Explain.
$\qquad$
6. What do you think of the activity?
$\qquad$
7. What are your suggestions for improving the activity?
$\qquad$
$\qquad$
8. What else do you suggest could be done to help students who have trouble with this activity?

## ACTIVITY DS2

[Separation strategy for division computation]

Materials: $\quad \operatorname{MAB}($ for $100, \mathrm{I}$ for 10, o for 1), plates, place value chart (100, 10, 1), pen, paper, attachments

## Directions:

1. Consider the division example $92 \div 4$. From booklet 5.2 , this can be considered as " 92 shared amongst 4 " or "how many groups of 4 in 92 ". Discuss this with students. Ask: How can we think about $92 \div 4$ ? What does $92 \div 4$ mean? Also ask: Is there another way to write it? Lead students to remember that $92 \div 4$ can also be written as $4 \longdiv { 9 2 }$. State: For computation we use this second way! [point to $4 \longdiv { 9 2 }$ ]
2. State: The first strategy is based on separating the number being divided [quotient] into place values and sharing! Ask: How many tens in 92? [9] How many ones left over? [2] State: Show this with MAB. [put out 9 tens and 2 ones on place value chart] Show the 4 to share amongst. [put out 4 plates]

| $\mathbf{H}$ | $\mathbf{T}$ | $\mathbf{0}$ |
| :---: | :---: | :---: |
|  | $\\|\\|\\|\\|\\|$ | 00 |
|  | $\\|R\\| \\|$ |  |



Ask: What shall we share first? [let students discuss but encourage them to see that in real life it is easier to share the larger notes first - the tens] State: Share the tens. ["a ten to you, a ten to you" and so on] Ask: How many tens did each plate get? [2] How many tens used all together? [8] How many tens left over? [1]

| $\mathbf{H}$ | $\mathbf{T}$ | $\mathbf{0}$ |
| :---: | :---: | :---: |
|  | $\mathbf{1}$ | 00 |
|  |  |  |



Ask: What can we do with the left over 10 to keep sharing? [discuss but encourage students to see need to break/trade it for ones] Ask: How many ones does this give us? [12] State: Share these ones.

Ask: How many ones does each plate get? [3] How many ones are used? [12] How many ones are left over? [0] What is the answer? [2 tens and 3 ones - 23]
3. Repeat direction 2 but record with a pen and paper as go

Step 1 Setting up the sharing.

| H | $\mathbf{T}$ | $\mathbf{0}$ |
| :---: | :---: | :---: |
|  | $\\|\\|\\|$ | 00 |
|  | $\\|\\|$ |  |
|  | $\\|I\\|$ |  |
|  |  |  |




How much do we have?

Step 2 Sharing the larger place value - the tens.



How many
used?
How many tens
$\downarrow$ to each plate?

Step 3 Regrouping.

| H | T | $\mathbf{O}$ |
| :---: | :---: | :---: |
|  |  | 0000 |
|  |  | 000 |
|  |  | 00 |
|  |  |  |


$4 \longdiv { 2 }$
$\frac{8}{92}$
$\mu^{12}$
How many ones?

Step 4 Sharing the ones.


Step 5 State the answer. [23]


How many left over?
4. Repeat for $771 \div 3$. State: Act this out with MAB and 3 plates. State: Point at numbers and material as you ask the questions.

Step 1 Set up.

$3 \longdiv { 7 7 1 }$
How many hundreds?
How many tens left over?
How many ones left over?
How many to share amongst?

Step 2 Sharing the hundreds.


How many hundreds to each plate?
How many hundreds used?
How many hundreds left over?

Step 3 Regrouping 100s $\rightarrow 10 \mathrm{~s}$.


How many tens now?

Step 4 Sharing the tens.


Step 5 Regrouping 10s $\rightarrow 1 \mathrm{~s}$.


How many ones now?

Step 6 Sharing the ones.


How many to each plate?
How many used?
How many ones left over?

Step 7 State the answer. [257]
5. Complete worksheet 2.1.
6. Go through separation strategy without material for example $936 \div 4$. Point to numbers as ask questions.

Step 1 Set up.

| $4 \longdiv { 9 3 6 } \quad$ | How many to share? |
| :--- | :--- |
|  | Write this down. |
|  | How many to share amongst? |
|  | Write this down. |
|  | Draw $\Gamma$ to show division. |

Step 2 Divide the 100s.

| $\frac{2}{4}$ |
| :---: |
| $\frac{936}{8}$ |
| 1 |

How many 100s to divide? [9]
How many 100s to each group? [ $9 \div 4$ gives 2 to each group]
How many 100s left? [1]

Step 3 Regroup and divide the 10 s.

| 23 | Change the 100s to 10s. |
| :---: | :---: |
| $4 \longdiv { 9 3 6 }$ | How many 10s to divide? [13] |
| 8 |  |
| $\begin{aligned} & \hline 13 \\ & 12 \\ & \hline \end{aligned}$ | How many 10s to each group? [13 $\div 4$ gives 3 to each group and 1 left over] |
| 1 | How many 10s left? [1] |

How many 10s left? [1]
Step 4 Regroup, divide the 1s and state answer.

| 234 | Change the 10s to 1s |
| :---: | :---: |
| $4 \longdiv { 9 3 6 }$ | How many ls to divide? [16] |
| 13 | How many 1s to each group? [4] |
| 12 | How many 1s left? [0] |
| 16 16 | What is the answer? [234] |
| 0 |  |

7. Repeat direction 6 for $4 \longdiv { 8 2 8 }$. Discuss the zero for tens.
8. Complete Worksheet 2.2.
9. Play games: "Separation Division Snap", "Separation Division Rummy", "Separation Division Concentration", "Separation Division Cover-the-board", "Separation Division Mix-and-Match" and "Separation Division Bingo".

DS2 - Place Value Chart


## DS2 - Worksheet 2.1

Use MAB and Place Value Chart to complete the following. (Put letters on top of answers in the box below).
(1)
$3 \longdiv { 8 1 }$
G
(2)
$4 \longdiv { 9 6 }$
N
(3)
$6 \longdiv { 7 8 }$
D
(4)
$5 \longdiv { 8 5 } \quad \mathrm { M }$
(5)
$7 \longdiv { 9 8 }$
O (6)
$3 \longdiv { 9 6 }$
L
(7)
$3 \longdiv { 6 2 7 }$
I
(8)
$4 \longdiv { 7 8 4 }$
T (9)
$6 \longdiv { 8 0 4 }$
A
(10)
$5 \longdiv { 9 4 0 }$
R
(11)
$4 \longdiv { 9 1 2 }$
S

## What am I?

$$
\begin{array}{llllllllll}
\overline{228} & \overline{14} & \overline{32} & \overline{209} & \overline{13} & , & \overline{228} & \overline{17} & \overline{134} & \overline{188}
\end{array} \overline{196}
$$

$$
\begin{array}{llllllll}
\overline{134} & \overline{24} & \overline{13} & \overline{228} & \overline{196} & \overline{188} & \overline{14} & \overline{24}
\end{array}
$$

Complete without MAB. (Put letters on top of answers in the box below).
(1)
$6 \longdiv { 8 6 4 }$
E
(2)
$7 \longdiv { 9 9 4 }$
R (3)
$3 \longdiv { 9 2 1 }$
L
(4)
$3 \longdiv { 8 6 7 }$
U (5)
$5 \longdiv { 8 6 5 }$
N (6)
$7 \longdiv { 8 1 2 }$
Y
(7)
$6 \longdiv { 6 2 4 }$
P (8)
$3 \longdiv { 6 2 1 }$
O (9)
$7 \longdiv { 7 3 5 }$
A
(10)
$4 \longdiv { 8 3 6 }$
D

| How do I act? |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\overline{104}$ | $\overline{272}$ | $\overline{207}$ | $\overline{289}$ | $\overline{209}$ |  | $\overline{105}$ | $\overline{173}$ |
| 209 | $\overline{209}$ |  |  |  |  |  |  |
| $\overline{209}$ | $\overline{144}$ | $\overline{105}$ | $\overline{209}$ | $\overline{307}$ | $\overline{116}$ |  |  |

## DS2 Games

## Snap, Rummy and Concentration

Materials:
Four pages of pictures following these instructions (equations, algorithms, array, partitioned array).
Number of players: 2-4

## Directions:

1. Print the four pages in 4 different colours, laminate and cut each page into 12 cards (making 48 cards of 4 different colours). Shuffle the cards. Follow instructions for these games:
2. Snap: Two players, cards divided equally between players. Together, the two players play the top card of their deck face up in front of them. First to call snap when two cards who the same number wins a point. The player with the most points wins when all cards played.
3. Rummy: Two to four players, deal out 7 cards to each player. Remaining cards face down in middle with one card face-up beside deck. Players put out any doubles or triples. Players in turn pick up a card (either the top face-up card or face-down card) and place a card face-up on the face-up pile. Doubles and triples are put out as they are formed. The winner is the first player to put out all their cards as doubles and triples.
4. Concentration: Place all cards face down on table. Players take turns selecting 2 cards. If they are the same, keep the pair and take another turn. The winner is the player with the most pairs when all cards used.

## Cover-the-Board Game

Materials: $\quad$ Same materials as for card games except the symbols page is kept as a base board, while the other 3 pages are cut into 12 picture cards.
Number of players: 2-4

## Directions:

1. Print the four pages from the card games, each on different colour paper or light card (laminate if possible).
2. Use the symbols as a base board.
3. Cut the other pages into 12 cards each.
4. Each player gets a set of cards.
5. In turn, each player places a card correctly on base board (card and board have to display same number) or on top of another card already placed.
6. At the end, the player with most cards on top wins.

## DS2 Games Continued

## Mix-and-Match Cards

Materials: $\quad 12$ mix and match cards (two per page).
Number of players: 1 (though can be a group)
Directions:

1. Print all cards in same colour.
2. Cut cards along lines into pieces.
3. Mix pieces together.
4. Students put pieces back together to form cards.

## Bingo

Materials: $\quad 7$ pages of material - one set of symbol flash cards and six bingo base boards, unifix cubes.
Number of players: 2-6

## Directions:

1. Print the flash cards (symbols) on white paper or card, cut out and laminate if possible.
2. Print the six bingo base boards, each on different coloured paper or card (laminate if possible).
3. One player (caller) takes the flash cards and shuffles them, the rest take a base board and unifix cubes.
4. Caller shows cards one at a time.
5. Players cover same operation on their board with unifix cube.
6. First player to get 3 in a row (across, down or diagonal) is the winner (calls "bingo") and becomes caller in next game.

DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board

| $196 \div 7=28$ | $128 \div 2=64$ | $693 \div 9=77$ |
| :--- | :--- | :--- |
| $432 \div 6=72$ | $85 \div 5=17$ | $464 \div 8=58$ |
| $124 \div 4=31$ | $752 \div 8=94$ | $441 \div 7=63$ |
| $215 \div 5=43$ | $201 \div 3=67$ | $768 \div 8=96$ |
|  |  |  |

## DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

| $7 \longdiv { 1 9 6 }$ $\left.\begin{array}{c}14 \\ \hline 56 \\ 56 \\ \hline 0\end{array}\right)$ | $\begin{array}{r}6 \\ \hline 128 \\ \hline 12 \\ \hline 08 \\ 8 \\ \hline 0\end{array}$ | 779693 <br> 63 <br> 63 <br> 63 <br> 0 |
| :---: | :---: | :---: |
| $\begin{array}{r} 72 \\ 6 \longdiv { 4 3 2 } \\ \frac{42}{12} \\ \frac{12}{0} \end{array}$ | 5 <br> 17 <br> 85 <br> $\frac{5}{35}$ <br> 35 <br> 0 | $\begin{array}{r}8 \\ \hline 464 \\ 40 \\ \hline 64 \\ 64 \\ \hline 0\end{array}$ |
| $\begin{array}{r}41 \\ 4124 \\ \hline 12 \\ \hline 04 \\ 4 \\ \hline 0\end{array}$ | 94 $\left.8 \begin{array}{r}752 \\ 72 \\ \hline 32 \\ 32 \\ \hline 0\end{array}\right)$ | 63 $\left.7 \begin{array}{\|c}441 \\ 42 \\ \hline 21 \\ 21 \\ \hline 0\end{array}\right)$ |
| $\begin{array}{r} 43 \\ 5 \longdiv { 2 1 5 } \\ 20 \\ \hline 15 \\ 15 \\ \hline 0 \end{array}$ | $3 \longdiv { 2 0 1 }$ <br> $\frac{18}{21}$ <br> $\begin{array}{r}21 \\ \hline 0\end{array}$ | 96 $\left.8 \begin{array}{r}768 \\ 72 \\ \hline 48 \\ 48 \\ \hline 0\end{array}\right)$ |

DS2 - Separation Division Snap, Rummy,
Concentration and Cover-the-Board Cards/Board Continued


## DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

| One hundred <br> and ninety-six <br> divided by <br> seven | One hundred <br> and twenty- <br> eight divided <br> by two | Six hundred <br> and ninety <br> three divided <br> by nine |
| :---: | :---: | :---: |
| Four hundred <br> and thirty-two <br> divided by six | Eighty-five <br> divided by five | Four hundred <br> and sixty-four <br> divided by <br> eight |
| One hundred <br> and twenty- <br> four divided by <br> four | Seven hundred <br> and fifty-two <br> divided by <br> eight | Four hundred <br> and forty-one <br> divided by <br> seven |
| Two hundred <br> and fifteen <br> divided by five | Two hundred <br> and one divided <br> by three | Seven hundred <br> and sixty-eight <br> divided by <br> eight |

## DS2 - Separation Division Mix-and-Match Cards



## DS2 - Separation Division Mix-and-Match Cards Continued



## DS2 - Separation Division Mix-and-Match Cards

 Continued

## DS2 - Separation Division Mix-and-Match Cards Continued

|  | $\underline{H}$ | II | 0 <br> $\because 0$ <br> $\square$ |
| :---: | :---: | :---: | :---: |



## DS2 - Separation Division Mix-and-Match Cards

 Continued

## DS2 - Separation Division Mix-and-Match Cards Continued



DS2 - Separation Division Bingo Flashcards

| $196 \div 7=28$ | $128 \div 2=64$ | $693 \div 9=77$ |
| :--- | :--- | :--- |
| $432 \div 6=72$ | $85 \div 5=17$ | $464 \div 8=58$ |
| $124 \div 4=31$ | $752 \div 8=94$ | $441 \div 7=63$ |
| $215 \div 5=43$ | $201 \div 3=67$ | $768 \div 8=96$ |

## DS2 - Separation Division Bingo Boards

| One hundred and ninetysix divided by seven | Four hundred and thirtytwo divided by six |  | One hundred and twentyfour divided by four |  | Two hundred and fifteen divided by five |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 64 \\ 2 \longdiv { 1 2 8 } \\ \frac{12}{08} \\ \frac{8}{0} \end{gathered}$ | 5 5 $\left.\begin{array}{c}17 \\ 85 \\ \frac{5}{35} \\ 35 \\ \hline 0\end{array}\right]$ |  | $\begin{array}{r}94 \\ 8752 \\ 72 \\ \hline 32 \\ 32 \\ \hline 0\end{array}$ |  | $\begin{array}{r}3 \\ 67 \\ 18 \\ \hline 201 \\ 21 \\ \hline 20\end{array}$ |  |
|  | H ${ }^{\text {T }}$ | 0 | H ${ }^{\text {T }}$ | 0 | H ${ }^{\text {T }}$ | 0 |
|  |  | : | - 1111 |  |  | : |



## DS2 - Separation Division Bingo Boards Continued



| One hundred and twentyeight divided by two |  |  | $\begin{array}{r} 77 \\ 9 \begin{array}{r} 693 \\ 63 \\ \hline 63 \\ 63 \end{array} \\ \hline 0 \end{array}$ |  |  | H | T | 0 | H | T | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | - | IIII | $\cdots$ | 國 | 11111 | :\% |
| H $\mathbf{T}$ $\mathbf{O}$ |  |  |  |  |  | $\begin{array}{r} 58 \\ 8 \longdiv { 4 6 4 } \\ \frac{40}{64} \\ \hline 64 \\ \hline 0 \end{array}$ |  |  | Two hundred and one divided by 3 |  |  | Eighty-five divided by five |  |  |
| - | 1 | : |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{array}{r} 96 \\ 8 \longdiv { 7 6 8 } \\ 72 \\ \hline 48 \\ 48 \end{array}$ |  |  | H |  | 0 | Seven hundred and fifty-two divided by eight |  |  | $\begin{gathered} 63 \\ 7 \longdiv { 4 4 1 } \\ \frac{42}{21} \\ \frac{21}{0} \end{gathered}$ |  |  |  |  |  |
|  |  |  |  | $\because$ |  |  |  |  |  |  |  |  |  |
|  |  |  | II | : |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

## DS2 - Separation Division Bingo Boards Continued

| H | T | 0 | Four hundred and forty-one divided by seven |  |  | Four hundred and sixtyfour divided by eight |  |  | $\begin{array}{r} 28 \\ 7 \begin{array}{r} 196 \\ \frac{14}{56} \\ \frac{56}{0} \end{array} \end{array}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ■ | II | \% \% |  |  |  |  |  |  |  |  |  |  |
| Six hundred and ninetythree divided by nine |  |  | $\begin{array}{r} 72 \\ 6 \longdiv { 4 3 2 } \\ \frac{42}{12} \\ \frac{12}{0} \end{array}$ |  |  | $\begin{gathered} 43 \\ 5 \longdiv { 2 1 5 } \\ \frac{20}{15} \\ \hline 15 \\ \hline 0 \end{gathered}$ |  |  | $\begin{gathered} 31 \\ 4 \longdiv { 1 2 4 } \\ \frac{12}{04} \\ \frac{4}{0} \end{gathered}$ |  |  |  |
| Seven hundred and sixty-eight divided by eight |  |  | H | T | 0 | H | T | 0 | H | T |  | 0 |
|  |  |  |  | IIIII | : | - |  | - | V | IIIII |  | - |



## DS2 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

| NAME | STUDENTS' REACTIONS |  |
| :--- | :--- | :--- |
|  | Boring $\quad$ | Interesting |
|  | Difficult $\quad$ | Easy |
|  | Not learning $\quad$ | Learning |
|  | Boring $\quad$ | Interesting |
|  | Difficult $\quad$ | Easy |
|  | Not learning | Learning |

2. How did you feel about trialling the activity?

Mark the line with an X: Unconfident - Very confident
3. Do you think the student was engaged in the activity? Explain.
$\qquad$
4. What do you think the student learnt from the activity?
$\qquad$
5. Do you think the student has gained an understanding of the concept being taught? Explain.
$\qquad$
6. What do you think of the activity?
$\qquad$
7. What are your suggestions for improving the activity?
$\qquad$
$\qquad$
8. What else do you suggest could be done to help students who have trouble with this activity?

## ACTIVITY DS3

## [Sequencing strategy for division computation]

Materials: Unifix, dot paper, calculator, pen, paper, attachments

## Directions:

1. Use unifix and dot paper to make arrays. Ask: How many rows for the following?
(a) 18 cubes, 3 per row
(b) 32 cubes, 4 per row
(c) 54 cubes, 9 per row

Ask: What do you notice? Elicit that, for example (a), 6 rows, $6 \times 3=8$ and $18 \div 3=6$. State: By finding how many in each row, we are really finding division.
2. State: We are going to learn a new method for division based on sequencing. It uses rows and columns. It thinks about division like $18 \div 3=6$ as "how many threes in 18 ?" Ask: Look at $72 \div 4$. State: We think 72 cubes, 4 per row, how many rows? Ask: Look at drawing of this [on right]. Think! How many 4s in 72? Is there 10? 20? For each answer, work out how many would be used.

$$
10 \times 4=40 ; \text { OK } \quad 20 \times 4=80 ; \text { too many }
$$

State: Need to be less than 72. State: Redraw the diagram to remove the 10 rows of 4. Ask: How many used? [40] How many left? [72-40



Look at 32. Ask: How many 4s in 32? [8] State: Recheck the diagram again. Redraw to include $8 x 4=32$. Ask: What is the answer? [18]
3. State: This new method has a special algorithm which makes it really easy. Let's look at it for $96 \div 3$ and $972 \div 4$. Check answers with calculators.
Problem Drawing $\quad$ Algorithm $\quad \frac{\text { Questioning/ }}{\underline{\text { Directions }}}$

Step 1 Set up.
$9 6 \div 3 \quad 3 \begin{array} { l } { 3 } \\ { 9 6 } \end{array} \quad 3 \longdiv { 9 6 } \begin{array} { l } { \text { How many } 3 \operatorname { s i n } 9 6 ? \text { Is } } \\ { \text { there 10, 20? } } \end{array}$

Step 2 Try tens that are less than needed.

$3 \longdiv { 9 6 }$
$-\left.\frac{60}{36}\right|^{20}$
Try 20. $3 \times 20=60$.
Redraw. How many used? [60] How many left? [9660=36]

Step 3 Keep going.

| 3 |  |  | 20 | How many 3s for 6? [2] How many all together?$[20+10+2=32]$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| 60 | 20 | 36 |  |  |
| 30 | 10 | -30 | 10 |  |
| 6 | ? |  |  |  |

Step 4 Finish off the ones and work out answer.


| $3 \longdiv { 9 6 }$ |  | Is there another 10? [Yes] |
| :---: | :---: | :---: |
| -60 | 20 | Remove it. How much |
| 36 |  | removed? [30] How much |
| -30 | 10 | left? [6] |
| 6 |  |  |
| -6 | 2 |  |
|  | 32 |  |

Problem
Drawing
Algorithm Questioning /
Directions
Step 1 Setting up.
$972 \div 4$

$4 \longdiv { 9 7 2 }$
Start thinking! How many 4s in 972? Is there 10? Is there 100? Many 10s?
Many 100s?
Step 2 Start removing and keep going.

$\left.\begin{array}{r}4 \longdiv { 9 7 2 } \\ \frac{-400}{572}\end{array} \right\rvert\, 100$
Remove the 100! How much is this? $[4 \times 100=$ 400] How much left? [972$400=572$ ] Is there another 100? Many 10s?


Step 3 Keep going through tens and ones.


| $4 \longdiv { 9 7 2 }$ |  |
| ---: | ---: |
| $\frac{-400}{572}$ | 100 |
| $\frac{-400}{172}$ | 100 |
| $\frac{-120}{52}$ | 30 |



| $4 \longdiv { 9 7 2 }$ |  |
| ---: | :---: |
| $\frac{-400}{572}$ | 100 |
| $\frac{-400}{172}$ | 100 |
| $\frac{-120}{52}$ | 30 |
| $\frac{-40}{12}$ | 10 |
| $\frac{-12}{0}$ | $\frac{3}{243}$ |

Remove 30: How many what's left? Any more? Ok remove 10? What is 30x4?
[120] What about the 12. How many 4s? Yes 3!

Answer? $[100+100+30+10+3=$ 243]

Note: State: The example could be completed by working out the 100s, 10s and 1 s more accurately. e.g.

$4 \longdiv { 9 7 2 }$
$4 \longdiv { 9 7 2 }$
$\frac{-800}{172}$ 200
$4 \longdiv { 9 7 2 }$

$$
\begin{array}{r|r}
\frac{-800}{172} & 200 \\
-160 \\
\hline 12 & 40
\end{array}
$$

$$
\begin{array}{r|r}
4 \lcm{972} \\
-800 \\
172 \\
-160 & 200 \\
\hline 12 & 40 \\
-12 & \begin{array}{r}
3 \\
\hline 0
\end{array} \\
\hline 243
\end{array}
$$

## 4. Complete Worksheet 3.1.

5. State: Now we are going to do this method without a drawing (though we can think of a drawing). (Check answers with a calculator.)

Look at examples $657 \div 9,715 \div 5$ and $836 \div 4$ on the next page.

## Example

## Algorithm

$9 \longdiv { 6 5 7 }$
$9 \longdiv { 6 5 7 }$

| $\frac{-450}{207}$ | 50 |
| ---: | ---: |
| -180 |  |
| 27 | 20 |


| $9 \longdiv { 6 5 7 }$ |  |
| :---: | :---: |
| -450 | 50 |
| 207 |  |
| - 180 | 20 |
| 27 |  |
| -27 | 3 |
| 0 | 73 |

$715 \div 5$
$657 \div 9$

20

$9 \longdiv { 6 5 7 }$
$\frac{180}{27} 20$
$\begin{array}{r}-27 \\ \hline 73\end{array}$






Finish the 1s! How many 9s in 27? [3]
How much left? [0] So answer is? $[50+20+3=73]$

How many 9 s in 657? 10, 100? Not 100, so try a fair number of 10 s! What about 5 10 s or 50 ? [ $50 \times 9=450$ ]

Not there yet? Try another 210 s or 20 !
[ $20 \mathrm{x} 9=180$ ]

## Thinking

$[20 \times 9=180]$

| $836 \div 4$ | $\begin{array}{r} 4 \longdiv { 8 3 6 } \\ -400 \\ \hline 436 \\ -400 \\ \hline 36 \\ -36 \\ \hline 0 \end{array}$ | $\begin{array}{r} 100 \\ 100 \\ 9 \\ \hline \hline 209 \end{array}$ | Is there 10 or 1004 s in 836? Try 100 how much left? [436] Ok there's another 400 so remove another 100 - how much left? [36] Finish off the $1 s$ - how many $4 s$ in 36? [9] |
| :---: | :---: | :---: | :---: |

6. Complete Worksheet 3.2.
7. Play games: "Sequencing Division Snap, Rummy or Concentration", "Sequencing Division Cover-the-Board", "Sequencing Division Mix-and-Match", and "Sequencing Division Bingo". The rules for these games are in DS2.

## DS3 - Dot Paper

## DS3 - Worksheet 3.1

Do the following using the diagram! (Check with calculators)
(1) 8
$8 \longdiv { 9 7 6 }$
S

(2)

$6 \longdiv { 8 7 0 }$
C
(4)

(6)


## We want what the song says!

$$
\begin{array}{lllllll}
\overline{492} & \overline{124} & \overline{122} & \overline{126} & \overline{124} & \overline{145} & \overline{307}
\end{array}
$$

## DS3 - Worksheet 3.2

Do the following thinking of the diagram! (Check with calculators)
(1)
(2)
$6 \longdiv { 8 8 2 }$
(4)
$3 \longdiv { 7 1 1 } \quad G$
(5)
$4 \longdiv { 9 6 4 }$
L
(6)
$2 \longdiv { 6 0 2 }$
(3)
$2 \longdiv { 7 8 6 }$
(7)
(8)
$4 \longdiv { 8 2 8 } \quad \mathrm { N }$

D
(9)
$8 \longdiv { 8 1 6 } \quad \mathrm { R }$
(10)
$7 \longdiv { 7 5 6 }$
H

| We want on which we walk! |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\overline{241}$ | $\overline{199}$ | $\overline{207}$ | $\overline{301}$ | $\overline{202}$ | $\overline{147}$ | 237 | 108 | $\overline{306}$ | $\overline{393}$ |

## DS3 - Sequencing Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board

| $215 \div 5=43$ | $124 \div 4=31$ | $432 \div 6=72$ | $196 \div 7=28$ |
| :--- | :--- | :--- | :--- |
| $201 \div 3=67$ | $752 \div 8=94$ | $85 \div 5=17$ | $128 \div 2=64$ |
| $768 \div 8=96$ | $441 \div 7=63$ | $464 \div 8=58$ | $693 \div 9=77$ |
|  |  |  |  |


| $5 \longdiv { 2 1 5 }$ $\begin{array}{r}200 \\ 15 \\ 15 \\ \hline 0\end{array} \begin{gathered}40 \\ \frac{3}{43}\end{gathered}{ }^{\text {a }}$ ( | $\begin{array}{r}4 \longdiv { 1 2 4 } \\ \begin{array}{r}120 \\ 4 \\ 4 \\ 0\end{array} \\ \hline 30\end{array}$ | $\begin{array}{r}6 \longdiv { 4 3 2 } \\ \frac{420}{12} \\ 12 \\ \hline 0\end{array} \begin{aligned} & \\ & 70 \\ & \\ & 72\end{aligned}$ | $7 \longdiv { 1 9 6 }$ <br> $\begin{array}{r}140 \\ 56 \\ 56 \\ \hline 0\end{array}$ <br> 20 <br> $\frac{8}{28}$ |
| :---: | :---: | :---: | :---: |
| $\begin{array}{rr}3 \\ \begin{array}{r}201 \\ 180 \\ \hline 21 \\ 21 \\ \hline 0\end{array} & 60 \\ & \frac{7}{67}\end{array}$ | 8752  <br> 720  <br> 32 90 <br> 32  <br> 0 $\frac{4}{94}$ | $5 \longdiv { 8 5 }$  <br> $\begin{array}{r}50 \\ \hline 35 \\ 35 \\ \hline 0\end{array}$ 10 <br> 17  | $\begin{array}{r}2 \longdiv { 1 2 8 } \\ \hline 120 \\ \hline 8 \\ 8 \\ \hline 0\end{array} \begin{array}{r}60 \\ \\ \hline 64\end{array}$ |
| $\begin{array}{r}8 \longdiv { 7 6 8 } \\ \begin{array}{r}720 \\ 48 \\ 48 \\ \hline 0\end{array} \\ \hline 96\end{array} \begin{array}{r}9 \\ 96\end{array}$ | $7 \longdiv { 4 4 1 }$ <br> $\begin{array}{r}420 \\ \hline 21 \\ 21 \\ \hline 0\end{array}$ <br> 60 <br> $\frac{3}{63}$ | $\begin{array}{r}\text { 8} \begin{array}{r}464 \\ 400 \\ 64 \\ 64 \\ 0\end{array} \\ \hline\end{array}$ | $\begin{array}{r}9 \lcm{693} \\ \frac{630}{63} \\ \hline 63 \\ \hline 0\end{array} \begin{aligned} & \\ & 70 \\ & \\ & 77\end{aligned}$ |

## DS3 - Sequencing Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

| ? | $\begin{gathered} ? \\ \hline \quad 124 \\ \hline \end{gathered}$ |  |  |  | 7 | ? 196 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $5 \square 215$ |  |  | 6 | ? 432 |  |  |
| ? |  | ? |  | ? |  | ? |
| $3 \bigcirc$ | 8 | 752 | 5 | 85 | 2 | 128 |
| ? |  | ? |  | ? |  | ? |
| $8 \bigcirc 768$ | 7 | 441 | 8 | 464 | 9 | 693 |


| 540 <br> $200 \quad 15$ | 430 <br> 4 <br> $120 \quad 4$ | 670 <br> $420 \quad 12$ | 720 <br> 140 |
| :---: | :---: | :---: | :---: |
|  | 8 $8 \begin{array}{r}90 \\ \hline 720 \quad 32 \\ \hline\end{array}$ | 5c <br> 5 <br> $10 \quad 30$ | 220 <br> 20 <br> 120 |
| 8 $8 \begin{array}{r}90 \\ \hline 720 \quad 48 \\ \hline\end{array}$ | $7 \begin{array}{r}60 \\ 420 \\ \hline\end{array}$ | 8850 $?$ <br> $400 \quad 64$  | 70 <br> 9 <br> 630 |

DS3 - Sequencing Division Mix-and-Match Cards


## DS3 - Sequencing Division Mix-and-Match Cards Continued



## DS3 - Sequencing Division Mix-and-Match Cards Continued



DS3 - Sequencing Division Mix-and-Match Cards Continued


DS3 - Sequencing Division Mix-and-Match Cards Continued


## DS3 - Sequencing Division Mix-and-Match Cards

 Continued

## DS3－Sequencing Division Bingo Flashcards

| $\stackrel{m}{\square}$ | $\cdots$ | $N$ | $\stackrel{\infty}{N}$ |
| :---: | :---: | :---: | :---: |
| II | II | II | II |
| $\square$ | ナ | $\bigcirc$ | N |
| $\cdot 1 \cdot$ | $\cdots$ | $\cdot 1 \cdot$ | － |
| $\stackrel{\llcorner }{\underset{N}{N}}$ | $\underset{\underset{\sim}{\underset{\sim}{N}}}{\substack{2}}$ | $\underset{\sim}{\underset{\sim}{N}}$ | 0 $\cdots$ $\cdots$ |
| $\hat{\omega}$ | ণ | $\xrightarrow{\text { N }}$ | $\pm$ |
|  | 11 | II | II |
| $m$ | $\infty$ | ค | N |
| $\cdot 1 \cdot$ | $\cdots$ | $\cdot 1 \cdot$ | $\cdots$ |
| $\stackrel{-1}{\mathrm{~N}}$ | $\begin{gathered} N \\ N \end{gathered}$ | $\infty$ | $\stackrel{\infty}{N}$ |
| ¢ | $\cdots$ | $\cdots$ | $N$ |
| II | II | 11 | II |
| $\infty$ | N | $\infty$ | 0 |
| $\cdots$ | $\cdots \cdot$ | $\cdots$ | $\cdots$ |
| $\begin{aligned} & \infty \\ & \underset{\sim}{\circ} \end{aligned}$ | $\underset{寸}{\underset{寸}{7}}$ | $\begin{aligned} & \underset{\sim}{*} \\ & \underset{y}{*} \end{aligned}$ | $\begin{aligned} & \text { m } \\ & \hat{0} \end{aligned}$ |

DS3 - Sequencing Division Bingo Boards

| $5 \begin{gathered} ? \\ \\ \hline \end{gathered}$ | $\begin{gathered} \\ \\ \\ \hline \end{gathered}$ | $6 \begin{gathered} ? \\ \hline \end{gathered}$ | $7 \begin{gathered} ? \\ \hline 196 \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| $\begin{array}{r}3 \% \\ \hline 180 \\ \hline 21 \\ \hline 21 \\ \hline 00 \\ \\ \hline 67\end{array}$ | 852  <br> 720  <br> 32 90 <br> 32  <br> 0 $\frac{4}{94}$ | $5 \longdiv { 8 5 }$ $\left.\begin{array}{r}50 \\ \hline 35 \\ 35 \\ \hline 0\end{array} \right\rvert\, \begin{array}{r}10 \\ \\ \hline 17\end{array}{ }^{7}$ | $\begin{array}{r}2 \longdiv { 1 2 8 } \\ \hline 120 \\ \hline 8 \\ \hline 8 \\ \hline 0\end{array}$60 <br> 64 |
| $\begin{array}{r}\text { \% } \\ \hline 80 \\ \hline 720 \\ \hline\end{array}$ | 760 | cr <br> 80 <br> 400 | 70 <br> 930 <br>  |


| $7 \begin{gathered} ? \\ \hline 196 \end{gathered}$ | $\begin{gathered} ? \\ \\ \hline 124 \\ \hline \end{gathered}$ | $\begin{array}{r}3 \longdiv { 2 0 1 } \\ \begin{array}{r}180 \\ 21 \\ 21 \\ \hline 0\end{array} \\ \hline\end{array} \quad \frac{7}{67}$ | 8850 <br> $400 \quad 64$ |
| :---: | :---: | :---: | :---: |
| $6 \longdiv { ? }$ |  | 90   <br> 720   <br> 788   | $\begin{array}{r}2 \begin{array}{r}128 \\ 120 \\ 8 \\ 8 \\ \hline 0\end{array} \\ \hline 60 \\ \\ \hline 64\end{array}$ |
| $\begin{array}{r}8 \lcm{752} \\ \frac{720}{32} \\ 32 \\ \hline 0\end{array} \frac{4}{94}$ | 760 <br> 7 <br> 420 | $5 \longdiv { ? }$ | 970 <br> 1030 <br> 630 |

## DS3 - Sequencing Division Bingo Boards Continued

| $\begin{array}{cc}  & 40 \\ \cline { 3 - 3 } & 200 \\ \hline \end{array}$ | $8$ | $\begin{array}{r} 8 \longdiv { 4 6 4 } \\ \frac{400}{64} \\ 64 \\ \hline 0 \end{array} \begin{aligned} & \\ & 50 \\ & 58 \end{aligned}$ | $3$ |
| :---: | :---: | :---: | :---: |
| $2 \begin{gathered} ? \\ \hline 128 \\ \hline \end{gathered}$ |  | 670 <br> 420 | 720 <br> 7 <br> $140 \quad 56$ |
| $\begin{array}{r} 8 \longdiv { 7 6 8 } \\ \frac{720}{48} \\ 48 \\ \hline 0 \end{array} \begin{gathered} \\ 90 \\ 96 \end{gathered}$ | 30 $?$ <br> $120 \quad 4$  |  | $\left.\begin{array}{r} 7 \longdiv { 4 4 1 } \\ \frac{420}{21} \\ 21 \\ \hline 0 \end{array} \right\rvert\, \begin{aligned} & \\ & 60 \\ & \hline 63 \end{aligned}$ |


| $2 \begin{gathered} ? \\ \\ \hline \end{gathered}$ | $9 \begin{array}{r}693 \\ \frac{630}{63} \\ 63 \\ \hline 0\end{array} \frac{7}{77}$ | 670 $?$ <br> $420 \quad 12$  | 20  $?$ <br> 140   |
| :---: | :---: | :---: | :---: |
|  | $\begin{array}{r\|} 8 \longdiv { 4 6 4 } \\ \frac{400}{64} \\ 64 \\ \hline 0 \end{array} \begin{aligned} & \\ & 50 \\ & 58 \end{aligned}$ | $3$ |  |
| 868  <br> 720  <br> 48  <br> 48  <br> 0 90 <br> 96  | 30 $?$ <br> 120 4 | 8 ? | $\begin{array}{r} 7 \longdiv { 4 4 1 } \\ \frac{420}{21} \\ \frac{21}{0} \end{array} \begin{aligned} & \\ & 60 \\ & \\ & \hline 63 \end{aligned}$ |

DS3 - Sequencing Division Bingo Boards Continued

| $2 \begin{array}{rr} 60 & ? \\ & 8 \\ \hline \end{array}$ | $7 \begin{gathered} ? \\ \hline 441 \end{gathered}$ | 88 | $\begin{array}{r\|} \hline 7 \longdiv { 1 9 6 } \\ \frac{140}{56} \\ 56 \\ \hline 56 \end{array} \begin{aligned} & \\ & 20 \\ & \hline 28 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $9 \begin{gathered} ? \\ \\ \hline \end{gathered}$ | $6 \lcm{432}$  <br> $\frac{420}{12}$ 70 <br> 12  <br> 0 $\frac{2}{72}$ | $5 \%$  <br> 215  <br> 200  <br> 15 40 <br> 15  <br> 0 $\frac{3}{43}$ | $\begin{array}{r\|r} 4 \lcm{124} \\ 120 & 30 \\ 4 & \\ \hline 0 & \frac{1}{31} \end{array}$ |
| $\begin{gathered} ? \\ \hline \quad 768 \\ \hline \end{gathered}$ | 5cr | 360 $?$ <br>  180 | $\begin{array}{r}\text { ¢ } \\ \hline 80 \\ \hline 720 \quad 32 \\ \hline\end{array}$ |


| $\begin{array}{r}4 \begin{array}{r}124 \\ 120 \\ 4 \\ 4 \\ 4\end{array} \\ \hline 0 \\ \hline\end{array} \begin{aligned} & \\ & 31\end{aligned}$ |  | $5 \begin{array}{r}215 \\ 200 \\ \hline 15 \\ 15 \\ \hline 0\end{array}$ <br> 40 <br> 43 | 5(10   |
| :---: | :---: | :---: | :---: |
| $8 \longdiv { ? }$ | $\begin{array}{r\|} 7 \longdiv { 1 9 6 } \\ \frac{140}{56} \\ 56 \\ \hline 0 \end{array} \begin{aligned} & \\ & 20 \\ & \frac{8}{28} \end{aligned}$ | 60 $?$ <br> 120 8 | 7$?$ <br> 44 |
| 90 $?$ <br>  720 | $8 \quad \begin{gathered} ? \\ \\ \hline \end{gathered}$ | 9 ? ${ }^{\text {? }}$ | $\begin{array}{r}6 \longdiv { 4 3 2 } \\ \frac{420}{12} \\ 12 \\ \hline 0\end{array} \begin{aligned} & \\ & \\ & 70 \\ & 72\end{aligned}$ |

## DS3 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

| NAME | STUDENTS' REACTIONS |  |
| :--- | :--- | :--- |
|  | Boring $\quad$ | Interesting |
|  | Difficult | Easy |
|  | Not learning | Learning |
|  | Boring $\quad$ | Interesting |
|  | Difficult | Easy |
|  | Not learning | Learning |

2. How did you feel about trialling the activity?

Mark the line with an X: Unconfident - Very confident
3. Do you think the student was engaged in the activity? Explain.
4. What do you think the student learnt from the activity?
5. Do you think the student has gained an understanding of the concept being taught? Explain.
$\qquad$
6. What do you think of the activity?
$\qquad$
7. What are your suggestions for improving the activity?
$\qquad$
$\qquad$
8. What else do you suggest could be done to help students who have trouble with this activity?

## ACTIVITY DS4

[Compensation strategy for division computation]
Materials: Pen, paper, attachments

## Directions:

1. State: We are going to look at very quick ways to do division if you can spot the shortcut. We have to start by reminding ourselves of easy multiplications.
Organise students to remember the following:
(1) multiplying by $10-8 \times 10=80$
(2) multiplying by $5-\frac{1}{2} \times 10: 8 \times 5=\frac{1}{2}(8 \times 10)=\frac{1}{2} 80$
(3) multiplying by $100-7 \times 100=700$
(4) multiplying by $50-\frac{1}{2} \times 100: 7 \times 50=\frac{1}{2}(7 \times 100)=350$
(5) multiplying by $25-\frac{1}{4} \times 100: 7 \times 25=\frac{1}{4}(7 \times 100)=175$
(6) higher decade $\quad-2 \times 4=8: 20 \times 4=80 \quad 20 \times 40=800 \quad 200 \times 4=800$
(7) multiplying by $2-$ doubling : $2 \times 62=124$
(8) multiplying by $4-$ double double : $4 \times 62=2 \times 124=248$
(9) multiplying by $8-$ double double double : $8 \times 62=4 \times 124=2 \times 248=496$

State: We have to remember these as easy ones.
2. State: In this new way to divide, we change the example to an easy one and then compensate for this (that's why it's called compensation). Let's look at a few examples $72 \div 4,438 \div 3$, and $924 \div 6$. Check with calculators!

$\left.\begin{array}{lllr}438 \div 3 & 3 \times 100=300 & \begin{array}{l}438 \text { is } 12 \text { less than } 450 \\ \text { so } 150 \text { has to be }\end{array} & \begin{array}{r}450 \div 3=150 \\ \text { reduced by } 12 \div 3\end{array} \\ & 3 \times 50=\frac{1}{2} 300=150 \\ 3 \times 150=450\end{array}\right)$
3. State: We can also manipulate the number we divide by. Consider $576 \div 4,936 \div 8$ and 786 $\div 6$. (Check with calculators!)

| Example | Thinking | Answer <br> $576 \div 4$ |
| :--- | :--- | ---: |
| $\div 4$ is halving and halving | half of 576 $=288$ <br> half of 288 $=\underline{144}$ |  |
| $936 \div 8$ | $\div 8$ is halving, halving and |  |
| halving |  |  |$\quad$| half of $936=468$ |  |
| ---: | :--- |
| half of 468 | $=234$ |
| half of 234 | $=\underline{\underline{117}}$ |

4. Complete Worksheets 4.1 and 4.2.

## DS4 - Worksheet 4.1

Do the following by thinking of easy ways and compensating. Tell us what you did. Put letters where answers are at bottom of sheet.

| (1) | $815 \div 2=$ | W | (2) | $915 \div 3=$ | R |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (3) | $985 \div 5=$ | D | (4) | $716 \div 2=$ | E |
| (5) | $968 \div 4=$ | V | (6) | $477 \div 3=$ | S |
| (7) | $864 \div 6=$ | P | (8) | $924 \div 7=$ | N |
| (9) | $840 \div 8=$ | O | (10) | $645 \div 5=$ | I |

What do we want?

| $\overline{193}$ | $\overline{129}$ | $\overline{242}$ | $\overline{129}$ | $\overline{159}$ | $\overline{129}$ | $\overline{105}$ | $\overline{132}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\overline{144}$ | $\overline{105}$ | - | $\overline{358}$ | $\overline{305}$ |  |  |  |

## DS4 - Worksheet 4.2

## The Human Cannonball!

$($ Power $\times$ Angle $=$ Distance $)$


Complete the chart - check your answer with a calculator.
The first has been done with you.

| Distance | Power | Angle | Power x Angle <br> (use calculator) |
| :---: | :---: | :---: | :---: |
| 456 | 4 | Think: <br> $440 \div 4=110$ <br> 456 is 16 more so <br> $456 \div 4=114$ |  |
| (1) 875 | 5 |  | $114 \times 4=456$ |
| (2) 510 | 6 |  |  |
| (3) 742 | 7 |  |  |
| (4) 464 | 8 |  |  |
| (5) 531 | 9 |  |  |
| (6) 658 | 8 |  |  |
| (7) 736 | 6 |  |  |
| (8) 774 | 4 |  |  |
| (9) 996 | $(10)$ | 987 |  |

## DS4 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

| NAME | STUDENTS' REACTIONS |  |
| :--- | :--- | :--- |
|  | Boring $\quad$ | Interesting |
|  | Difficult $\quad$ | Easy |
|  | Not learning $\quad$ | Learning |
|  | Boring $\quad$ | Interesting |
|  | Difficult $\quad$ | Easy |
|  | Not learning | Learning |

2. How did you feel about trialling the activity?

Mark the line with an X: Unconfident Very confident
3. Do you think the student was engaged in the activity? Explain.
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$\qquad$
6. What do you think of the activity?
$\qquad$
7. What are your suggestions for improving the activity?
$\qquad$
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8. What else do you suggest could be done to help students who have trouble with this activity?

