



School of Curriculum

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Professional Learning 5 Multiplication and Division **Booklet 5.4: Division Computation Strategies**







This project is funded by an Australian Research Council Linkage Project grant with support from Education Queensland (EQ)

DEADLY MATH TUTORS PROGRAM

Professional Learning 5: Multiplication and Division

DIVISION COMPUTATION STRATEGIES

TRIAL PACKAGE 4

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Contributing EQ Organisations

Mount Isa EQ Regional Office
Boulia State School
Dujarra State School
Doomadgee State School
Doomadgee State School

YuMi Deadly Maths Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Island Regional Educational Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology, Australian Catholic University & Education Queensland

Deadly Maths Tutor Program

PROFESSIONAL LEARNING 5: MULTIPLICATION AND DIVISION

BOOKLET 5.4 DIVISION COMPUTATION STRATEGIES 2008

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OVERVIEW

PURPOSE

These materials were designed to be used in conjunction with a Professional Learning program for teacher aides. The objective of this Professional Learning was to empower teacher aides to enhance rural and remote Indigenous students' numeracy outcomes. This document contains the materials of the fourth of five different booklets on multiplication and division.

If your school would like to receive a YuMi Deadly Maths Professional Learning program please contact the YuMi Deadly Centre (YDC) on: 07 3138 0035 or ydc@qut.edu.au.

DIRECTIONS

(1) Interviewing the students:

Pick one or more students who appear to be having trouble understanding division. Interview these students using the interview schedule and the materials. Mark what they do and put their results on the Student Recording Sheet.

(2) Trialling the student activities:

Use the Recording Sheet to work out the activities the students need to do and trial these activities with the students (with each student one at a time or with a group of students). Keep a record of what happens and collect the students' work.

INTERVIEW SCHEDULE

Materials:

Unifix cubes, deck of playing cards, washable felt pen, pen, paper

Materials within this booklet: interview questions, cards, number tracks, number lines

Student Recording Sheet

Directions:

- 1. Photocopy and laminate attached cards.
- 2. Gather other material (unifix, paper, pencils)
- 3. Place material in front of students. Give students pen and paper to write with.
- 4. Tell the students you are trying to find out what they know. Say they are not expected to know it and you will teach what is not known.
- 5. Give the student directions slowly read problems. Do not give hints. If student cannot do a question, pass on to the next question, repeating that it is not important if they don't know how to do the question.
- 6. Allow students to use material and make drawings but only after they say they do not know how to do it with symbols alone.

INTERVIEW MATERIALS Interview Questions



Show and read Card 1

• Ask: Can you use the top answer to answer the two underneath?

Repeat this for Card 2



Show and read Card 3

- Ask: Can you answer this division operation by showing hundreds tens and ones?
- If cannot do this, **show and read Card 4** and ask: *Can you answer this one?*
- If still cannot do this, ask if they can solve it any way.



Show and read Card 5

- Ask: Can you answer this division operation by working out how many 4s in 512 looking first at 100s of 4s and then 10s of 4s?
- If cannot do this, **show and read Card 6** and ask: *Can you answer this one?*



Show and read Card 7

- Ask: Can you answer this division operation by finding an easier operation and relating this one to anything. Say: Say the problem (have student say the problem).
- If cannot do this, **show and read Card 8** and ask: Can you read this one?

Interview Cards

Tittel view cards			
Card 1	Card 2		
$6 \div 2 = 3$	$18 \div 3 = 6$		
60÷2 =	180÷3 =		
600÷2 =	1800÷3 =		
Card 3	Card 4		
576÷4	72÷4		
4)576	4) 72		
Card 5	Card 6		
Caru 5	Caru		
512÷4	76÷4		
4)512	4) 76		
Card 7	Card 8		
Caru /	Caru o		
584÷4	92÷4		
4)584	4) 92		

STUDENT RECORDING SHEET

Name:
School/Class:

Interview item	Result (✓, ≭)	Comments	Activities to be completed if incorrect
DSA: Higher decade division facts			DS1
DSB: Separation strategy for division computation			DS2
DSC: Sequencing strategy for division computation			DS3
DSD: Compensation strategy for division computation			DS4

INTRODUCTION

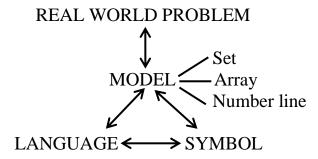
Contents

This package contains:

• four <u>tutoring activities</u> (DS1 to DS4) and their students materials (games and worksheets)

Pedagogy

The activities DS1 to DS4 are based on the Rathmell Triangle Relationship below, real world problems are related to set, array and number line models, language and symbols (and vice versa) to teach strategies for computation.



However, the focus of the activities is to develop a repertoire of strategies for computing such examples as $36 \div 6$ and $255 \div 3$. The activities are designed to allow you to tutor students who are having difficulties with division computation for 2- and 3-digit numbers \div 1-digit numbers. The activities are based on the belief that it is more important to use the algorithms to teach a variety of strategies than to get a correct answer. However, the activities also show how to get correct answers.

There are three strategies (or strategy groupings) associated with multiplication computation. These are:

(1) Separation:

This is the traditional algorithm for long division, and it is based on sharing as can be seen in the following example 732÷3. This example is considered as 7 hundreds 3 tens and 2 ones shared amongst 3 people. By separating the number into its place values, it is possible to share each place value separately. It is easily seen to be more efficient to share large place values

	244
	3 732
hundreds	6
	13
tens	_12_
	12
ones	12
	0

first. So 7 hundreds are shared between 3, 2 hundreds to each person, with 6 hundreds used and 1 hundred left over! This hundred makes 13 tens with the 3 tens and sharing continues until all ones are shared. This process works really well for money - \$100 and \$10 notes and \$1 coins. The algorithm on the right symbolically acts out this sharing process.

(2) Sequencing:

This is an alternative method/strategy. The number is not 244 separated but broken into parts which become a sequence for 3 3×244 the process of division. It is best seen in terms of arrays/area as on right. Multiplication of 244 by 3 is seen as the area of a 732÷3 3×244 rectangle. Thus, division is seen as finding the long side of a 3 high rectangle with an area of 732. The idea is to start to 3 732 think of what the long side could be. Is it over 10, over 100? Try 100. 100 multiplied by 3 is 300, leaving 432 as in figure on right. Try another 100 – yes, it works and leave 132 as on right. Then we can try 30 and 10 or

40 direct and finally 4 for the last 12. This gives the algorithm on the right. It enables the division to be worked out with a <u>sequence</u> if parts – for example, the 244 is determined by 100, 100, 30, 10 and 4.

,	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	100
	100
100 100 ? 432	
3 300 300 132 300	1000
132	
100 100 40 ? 90	30
3 300 300 120 12 42	
	10
100 100 40 4 12	
3 300 300 120 12 12	_4
	244

(3) Compensation:

This is the most efficient strategy. There is no breaking into parts by separating into place values or generating sequences. Numbers are left as they are but other examples are sought which are closer to the original division example but easy to divide. For the example $732 \div 3$, an easy example would be $750 \div 3$ which is 250, then the difference between the original and the easy example has to be <u>compensated for</u>. 750 is 18 too high, this is 6 threes, so subtract 6 from 250 = 244, which is the answer for $732 \div 3$.

To recognise easier examples requires knowledge of (a) higher-decade division (e.g., $12 \div 4 = 3$ so $120 \div 4 = 30$ and $120 \div 40 = 3$); and (b) when a division is the inverse of an easy multiplication (e.g., $\times 25$ and $\times 50$). Some examples of making division easy are as follows.

- (1) $581 \div 7$: An easy example is $560 \div 7 = 80$, 581 is 21 more than 560 so need to add $21 \div 7 = 3$ to 80, so answer is 83.
- (2) $924 \div 6$: An easy example is $900 \div 6 = 150$, 924 is 24 more than 900, so need to add $24 \div 6 = 4$ to 150, so answer is 154.

Note: There is another method for making examples easier and that is to use the fact that if, for example, $6 = 3x^2$ then $24 \div 6 = 24 \div 3 \div 2 = 8 \div 2 = 4$.

This is particularly useful for divisors based on 2. For example, $544 \div 8$ can be done by $54 \div 2 \div 2 \div 2$ as $2 \times 2 \times 2 = 8$. This is really halving, so answer half of half of half of 544 = half of half of 272 = half of 136 = 68.

It is also useful when dividing by 2 digit numbers, e.g. $888 \div 24$ as $6 \times 4 = 24$, then answer is found by $888 \div 4 = 222$ and $222 \div 6 = 37$.

All the three strategies require knowledge of basic number facts (e.g., $6 \times 8 = 48$) and also higher-decade number facts (e.g., $6 \times 80 = 480$, $600 \times 80 = 48000$).

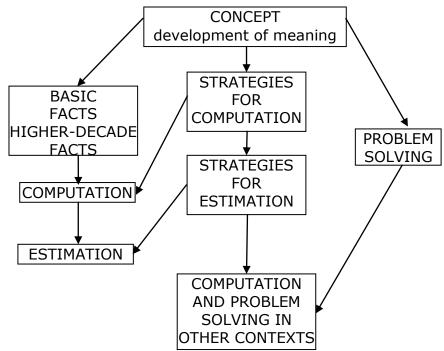
Thus, the four activities in this booklet are the higher-decade facts (Activity DS1), separation strategy (Activity DS2), sequencing strategy (Activity DS3), and compensation strategy (Activity DS4).

Approach

The four operations of addition, subtraction, multiplication and division are important in mathematics, as they reflect <u>everyday</u> important actions (of joining, separation, combining and partitioning) that happen in the lives of people and that are the basis of problems that need to be solved.

For larger numbers, procedures have been developed to achieve answers using basic facts and higher-decade facts. These are based on strategies that also can be used to teach multiplication and division for, for example, mixed numbers, measures, time, and algebra. In a world of calculators and computers, computation for answers has become less important than computation for learning strategies. However, strategies when learnt will enable answers to be acquired as well as the procedures to be understood.

Thus, the sequence for the development of any of the four operations with whole number is:



Therefore, in this booklet we will focus on strategies (leaving problem solving for booklet 5.5) so as to develop mathematics ideas that will pay off in later years in other contexts (e.g. decimals). For this reason, much of the activity in this booklet is learning a repertoire of strategies (not just one strategy to get answers).

ACTIVITIES

ACTIVITY DS1

[Higher-decade division facts]

Materials: MAB, unifix cubes, pens, paper, calculators, attachments

Directions:

- 1. Consider 8÷4. Act this out by sharing 8 MAB units amongst 4 people. Ask: *How many does each person get?* [2] Now consider 80÷4. Act this out by sharing 8 tens amongst 4 people. Ask: *How many does each person get?* [2 tens or twenty] Repeat this for 800÷4.
- 2. Consider 15÷5. Act this out with unifix by making rows of 5. Ask: *How many rows?* [3] Consider 150÷5. Act this out by making rows of 5. Do it in your mind. Will 10 rows of 5 give 150, 20 rows of 5, 30 rows of 5 and so on. Ask: *How many rows?* [30] Repeat this thinking for 1500÷5.
- 3. Write down the results of 2 and 3. Use a calculator to work out the results of the examples under them.

$8 \div 4 = 2$	$80 \div 4 = 20$	$800 \div 4 = 200$
$15 \div 5 = 3$	$150 \div 5 = 30$	$1500 \div 5 = 300$
24÷6 =	240÷6 =	2400÷6 =
12÷3 =	120÷3 =	1200÷3 =

Ask: Can you write down a rule for dividing with tens and hundreds? If difficulty, ask students to look at relationship of other columns to left hand column.

4. Use a calculator to work out the answers for these examples:

9÷3 =	90÷30 =	900÷300 =
8÷2 =	80÷2 =	800÷20 =
14÷7 =	140÷70 =	1400÷70 =
18÷6 =	180÷6 =	1800÷600 =
32÷8 =	3200÷80 =	3200÷800 =

Ask: Can anyone think of a rule for dividing these numbers? If difficulty, ask: Can you see how these [point to 2 right hand columns] relate to this [point to left hand column]?

- 5. Complete Worksheet 1.1 and 1.2.
- 6. Complete game "Ride them equal".

DS1 - Worksheet 1.1

1. Complete the following using a calculator:

6÷2 =	49÷7 =	28÷4 =	
60÷2 =	490÷7 =	280÷4 =	
600÷2 =	4900÷7 =	2800÷4 =	
6000÷2 =	49 000÷7 =	28 000÷4 =	
60÷2 =	490÷70 =	280÷40 =	
600÷20 =	4900÷70 =	2800÷40 =	
6000÷20 =	49 000÷70 =	28 000÷40 =	
600÷200 =	4900÷70 =	2800÷400 =	
6000÷200 =	49 000÷70 =	28 000÷400 =	
6000÷2000 =	4900÷700 =	28 000÷4000 =	

2. Complete these without a calculator:

48÷6 =	8	30÷5 =	6	72÷4 =	18
480÷6 =		300÷5 =		720÷4 =	
4800÷6 =		3000÷5 =		7200÷4 =	
48 000÷6 =		30 000÷5 =		72 000÷4 =	
480÷60 =		300÷50 =		720÷40 =	
4800÷60 =		3000÷50 =		7200÷40 =	
48 000÷600 =		30 000÷50 =		72 000÷40 =	
4800÷600 =		3000÷500 =		7200÷400 =	
48 000÷600 =		30 000÷500 =		72 000÷400 =	
48 000÷6000 =		30 000÷5000 =		72 000÷4000 =	

3. Write down a rule that these examples follows:

4. Show how your rule can explain why if $65 \div 5 = 13$ then $6500 \div 50 = 130$:

DS1 – Worksheet 1.2

$$\underline{6 \div 3 = 2}$$

How many zeroes in the answers?

For each question, determine the number of zeros and colour one area with the colour given by the number of zeroes - 0 - green; 1 - brown; 2 - blue; 3 - yellow. When you are finished, colour all other areas. What have you got?

- (1) $600 \div 20$
- (2) 6000 \div 2
- (3) 60 000 \div 200

 $6 \div 2$ (4)

- (5) $6\,000\,000 \div 200\,000$ (6) $600 \div 200$

- (7) $60\ 000\ 000 \div 2000$
- (8) $6\,000\,000 \div 20\,000$
- (9) $6000\ 000 \div 200\ 000$

(10) $600 \div 2$

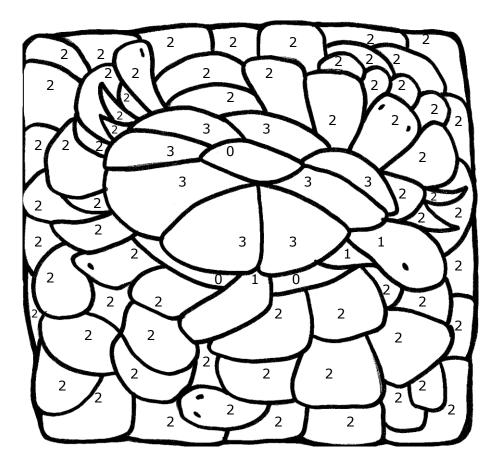
- $(11) 60 000 \div 2000$
- $(12)\ 600\ 000\ 000 \div 200\ 000$

- (13) $600\ 000 \div 20\ 000$
- $(14)\ 60\ 000 \div 20$
- $(15) 60 \div 20$
- (16) $60\ 000\ 000 \div 200\ 000 \quad (17)\ 600\ 000 \div 2000$
- $(18) 6000 \div 2000$

(19) $60 \div 2$

- $(20) 6 000 000 \div 2000$
- $(21) 6000 \div 20$

- (22) $6000 \div 200$
- $(23)\ 600\ 000 \div 200$
- $(24) 60 000 \div 20 000$



It keeps on paddling?

DS1 - Game: "Ride Them Equal"

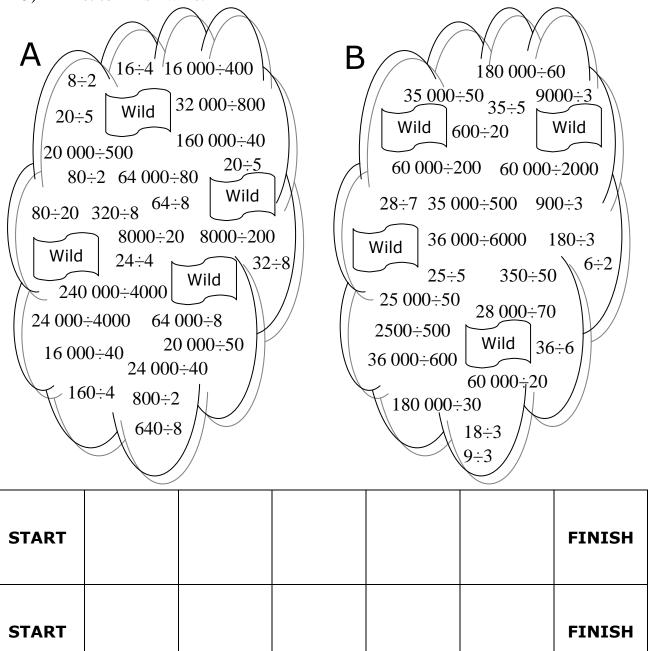
Materials: A marker (coloured unifix) for each player (start at "START"),

calculator, pen.

Number of players: 2

Directions:

- 1) Pick on exercise from each cloud. Cross out the examples chosen.
- 2) Use a calculator to work out the answers.
- 3) If the number of zeros are the same, move marker one step towards finish line.
- 4) Wild can be used to make your own problem.
- 5) First to finish wins.



DS1 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME		STUDENTS' REACTIONS	
		Boring —	•
		Difficult —	•
		Not learning —	Learning
		Boring —	_
		Difficult —	Easy
		Not learning —	Learning
2.	How did you feel about trialling the Mark the line with an X: Unconfi	•	nfident
3.	Do you think the student was engage	ed in the activity? Explain.	
4.	What do you think the student learnt from the activity?		
5.	Do you think the student has gained an understanding of the concept being taught? Explain.		
6.	What do you think of the activity?		
7.	What are your suggestions for improving the activity?		
8.	What else do you suggest could be dethis activity?	lone to help students who have tro	ouble with

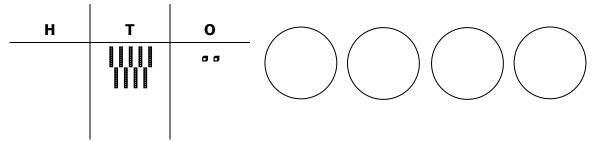
ACTIVITY DS2

[Separation strategy for division computation]

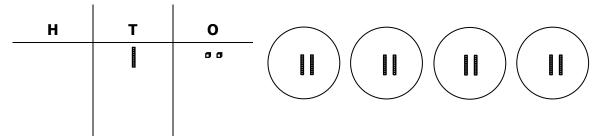
Materials: MAB (for 100, for 10, o for 1), plates, place value chart (100, 10, 1), pen, paper, attachments

Directions:

- 1. Consider the division example 92÷4. From booklet 5.2, this can be considered as "92 shared amongst 4" or "how many groups of 4 in 92". Discuss this with students. Ask: *How can we think about 92÷4? What does 92÷4 mean?* Also ask: *Is there another way to write it?* Lead students to remember that 92÷4 can also be written as 4) 92. State: *For computation we use this second way!* [point to 4) 92]
- 2. State: The first strategy is based on separating the number being divided [quotient] into place values and sharing! Ask: How many tens in 92? [9] How many ones left over? [2] State: Show this with MAB. [put out 9 tens and 2 ones on place value chart] Show the 4 to share amongst. [put out 4 plates]



Ask: What shall we share first? [let students discuss but encourage them to see that in real life it is easier to share the larger notes first – the tens] State: Share the tens. ["a ten to you, a ten to you" and so on] Ask: How many tens did each plate get? [2] How many tens used all together? [8] How many tens left over? [1]

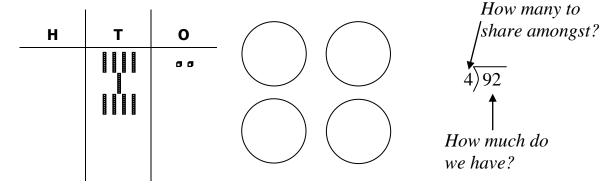


Ask: What can we do with the left over 10 to keep sharing? [discuss but encourage students to see need to break/trade it for ones] Ask: How many ones does this give us? [12] State: Share these ones.

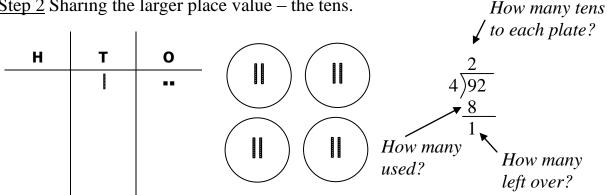
Ask: How many ones does each plate get? [3] How many ones are used? [12] How many ones are left over? [0] What is the answer? [2 tens and 3 ones – 23]

3. Repeat direction 2 but record with a pen and paper as go

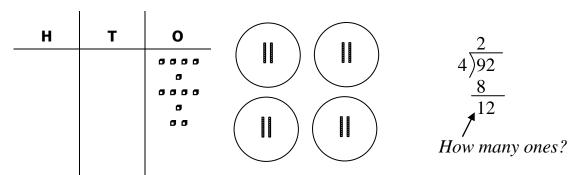
Step 1 Setting up the sharing.



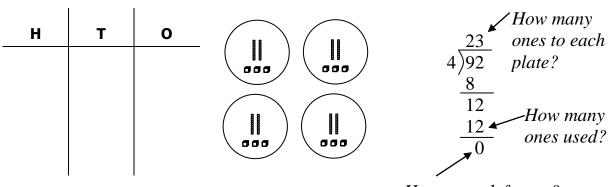
<u>Step 2</u> Sharing the larger place value – the tens.



Step 3 Regrouping.



Step 4 Sharing the ones.

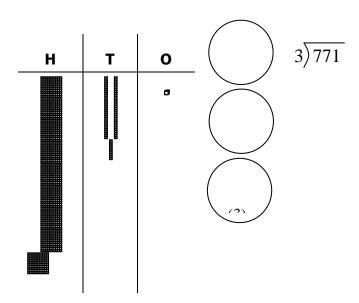


Step 5 State the answer. [23]

How many left over?

4. Repeat for 771÷3. State: *Act this out with MAB and 3 plates.* State: *Point at numbers and material as you ask the questions.*

Step 1 Set up.



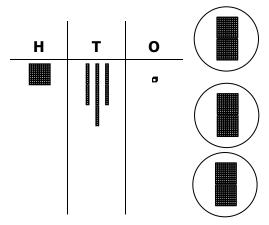
How many hundreds?

How many tens left over?

How many ones left over?

How many to share amongst?

Step 2 Sharing the hundreds.

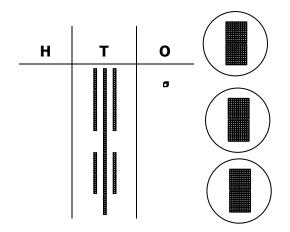


 $\begin{array}{c}
2\\3{771}\\
\underline{6}\\
1
\end{array}$

How many hundreds to each plate?

How many hundreds used? How many hundreds left over?

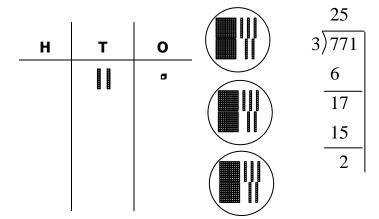
Step 3 Regrouping $100s \rightarrow 10s$.



 $\begin{array}{c|c}
2\\
3{\overline{\smash{\big)}\,771}}\\
\underline{6}\\
17
\end{array}$

How many tens now?

Step 4 Sharing the tens.



How many tens to each plate?

How many tens used?

How many tens left over?

Step 5 Regrouping $10s \rightarrow 1s$.

н	т	0	$\begin{array}{c} 25 \\ 3 \overline{)771} \end{array}$
		0000	6 17 15 21

How many ones now?

Step 6 Sharing the ones.

н	т	0	$\begin{array}{c} 257 \\ 3 \overline{)771} \end{array}$
			$\frac{6}{17}$ 15
			$ \begin{array}{c c} \hline 21 \\ \hline 21 \\ \hline 0 \end{array} $

How many to each plate?
How many used?
How many ones left over?

Step 7 State the answer. [257]

5. Complete worksheet 2.1.

6. Go through separation strategy without material for example 936÷4. Point to numbers as ask questions.

Step 1 Set up.

Step 2 Divide the 100s.

How many 100s to divide? [9]
$$\frac{8}{1}$$
How many 100s to each group? [9÷4 gives 2 to each group]
How many 100s left? [1]

Step 3 Regroup and divide the 10s.

$$\begin{array}{c}
23 \\
4) \overline{)936} \\
\underline{8} \\
13 \\
\underline{12} \\
1
\end{array}$$
Change the 100s to 10s.

How many 10s to divide? [13]

How many 10s to each group? [13÷4 gives 3 to each group and 1 left over]

How many 10s left? [1]

Step 4 Regroup, divide the 1s and state answer.

234	Change the 10s to 1s
4)936	How many 1s to divide? [16]
<u>8</u> 13	How many 1s to each group? [4]
12	How many 1s left? [0]
16	What is the answer? [234]
<u> 16</u>	With is the diswer. [234]
0	

- 7. Repeat direction 6 for 4)828. Discuss the zero for tens.
- 8. Complete Worksheet 2.2.
- 9. Play games: "Separation Division Snap", "Separation Division Rummy", "Separation Division Concentration", "Separation Division Cover-the-board", "Separation Division Mix-and-Match" and "Separation Division Bingo".

DS2 - Place Value Chart

HUNDREDS
TENS
ONES

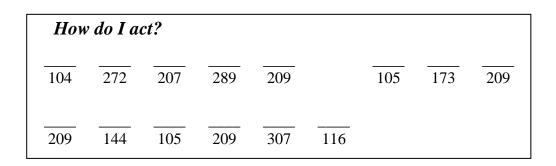
DS2 - Worksheet 2.1

Use MAB and Place Value Chart to complete the following. (Put letters on top of answers in the box below).

- (1) $3\sqrt{81}$ G (2) $4\sqrt{96}$ N (3) $6\sqrt{78}$ D
- (4) $5\sqrt{85}$ M (5) $7\sqrt{98}$ O (6) $3\sqrt{96}$ L
- (7) $3\sqrt{627}$ I (8) $4\sqrt{784}$ T (9) $6\sqrt{804}$ A
- (10) 5)940 R (11) 4)912 S

Complete without MAB. (Put letters on top of answers in the box below).

- (1) $6 \ 864$ E (2) $7 \ 994$ R (3) $3 \ 921$ L
- (4) $3\sqrt{867}$ U (5) $5\sqrt{865}$ N (6) $7\sqrt{812}$ Y
- (7) $6\sqrt{624}$ P (8) $3\sqrt{621}$ O (9) $7\sqrt{735}$ A
- (10) 4 > 836 D



DS2 Games

Snap, Rummy and Concentration

Materials: Four pages of pictures following these instructions (equations,

algorithms, array, partitioned array).

Number of players: 2-4

Directions:

- 1. Print the four pages in 4 different colours, laminate and cut each page into 12 cards (making 48 cards of 4 different colours). Shuffle the cards. Follow instructions for these games:
- 2. **Snap**: Two players, cards divided equally between players. Together, the two players play the top card of their deck face up in front of them. First to call snap when two cards who the same number wins a point. The player with the most points wins when all cards played.
- 3. **Rummy**: Two to four players, deal out 7 cards to each player. Remaining cards face down in middle with one card face-up beside deck. Players put out any doubles or triples. Players in turn pick up a card (either the top face-up card or face-down card) and place a card face-up on the face-up pile. Doubles and triples are put out as they are formed. The winner is the first player to put out all their cards as doubles and triples.
- 4. **Concentration**: Place all cards face down on table. Players take turns selecting 2 cards. If they are the same, keep the pair and take another turn. The winner is the player with the most pairs when all cards used.

Cover-the-Board Game

Materials: Same materials as for card games except the symbols page is

kept as a base board, while the other 3 pages are cut into 12

picture cards.

Number of players: 2-4

Directions:

- 1. Print the four pages from the card games, each on different colour paper or light card (laminate if possible).
- 2. Use the symbols as a base board.
- 3. Cut the other pages into 12 cards each.
- 4. Each player gets a set of cards.
- 5. In turn, each player places a card correctly on base board (card and board have to display same number) or on top of another card already placed.
- 6. At the end, the player with most cards on top wins.

DS2 Games Continued

Mix-and-Match Cards

Materials: 12 mix and match cards (two per page).

Number of players: 1 (though can be a group)

Directions:

- 1. Print all cards in same colour.
- 2. Cut cards along lines into pieces.
- 3. Mix pieces together.
- 4. Students put pieces back together to form cards.

Bingo

Materials: 7 pages of material – one set of symbol flash cards and six bingo

base boards, unifix cubes.

Number of players: 2-6

Directions:

- 1. Print the flash cards (symbols) on white paper or card, cut out and laminate if possible.
- 2. Print the six bingo base boards, each on different coloured paper or card (laminate if possible).
- 3. One player (caller) takes the flash cards and shuffles them, the rest take a base board and unifix cubes.
- 4. Caller shows cards one at a time.
- 5. Players cover same operation on their board with unifix cube.
- 6. First player to get 3 in a row (across, down or diagonal) is the winner (calls "bingo") and becomes caller in next game.

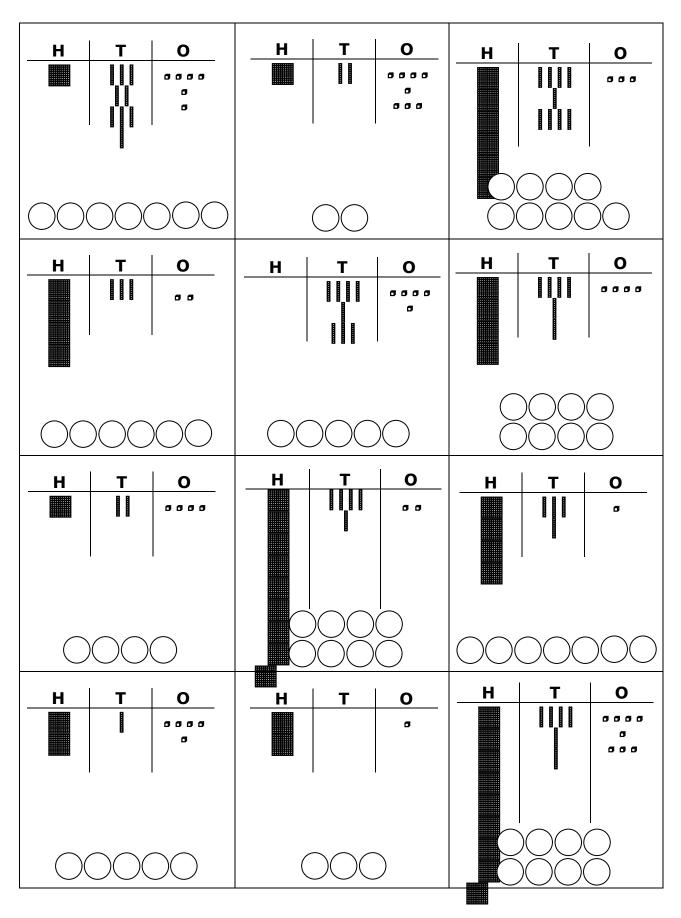
DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board

196 ÷ 7 = 28	128 ÷ 2 = 64	693 ÷ 9 = 77
432 ÷ 6 = 72	85 ÷ 5 = 17	464 ÷ 8 = 58
124 ÷ 4 = 31	752 ÷ 8 = 94	441 ÷ 7 = 63
215 ÷ 5 = 43	201 ÷ 3 = 67	768 ÷ 8 = 96

DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

28	64	77
7)196	2)128	9)693
14	12	63
56	08	63
56	8	63
0	0	0
72	17	58
6)432	5)85	8)464
42	5	<u>40</u>
12	35	64
12	35	<u>64</u>
0	0	0
31	94	63
4)124	8)752	7)441
12	<u>72</u>	<u>42</u>
04	32	21
4	32	<u>21</u>
0	0	0
43	67	96
5)215	3)201	8)768
20	18	<u>72</u>
15	21	48
15	21	<u>48</u>
0	0	0

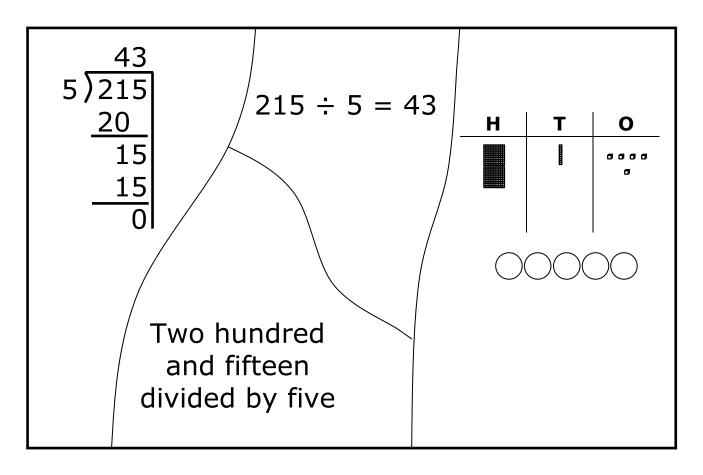
DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

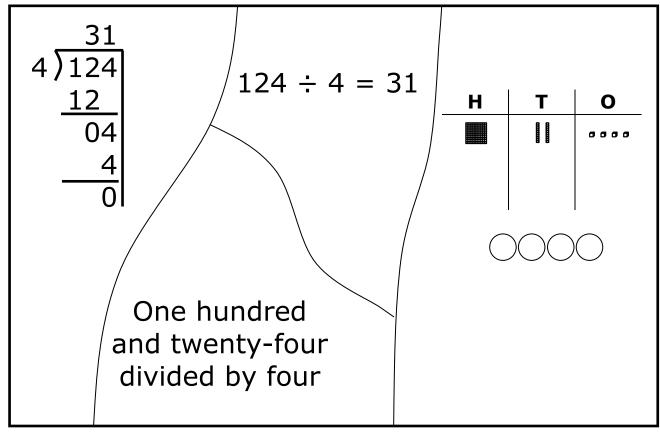


DS2 - Separation Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

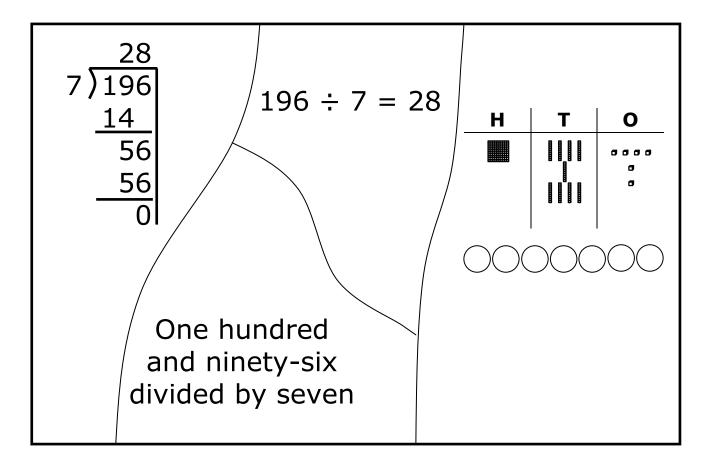
One hundred	One hundred	Six hundred
and ninety-six	and twenty-	and ninety
divided by	eight divided	three divided
seven	by two	by nine
Four hundred and thirty-two divided by six	Eighty-five divided by five	Four hundred and sixty-four divided by eight
One hundred	Seven hundred	Four hundred
and twenty-	and fifty-two	and forty-one
four divided by	divided by	divided by
four	eight	seven
Two hundred and fifteen divided by five	Two hundred and one divided by three	Seven hundred and sixty-eight divided by eight

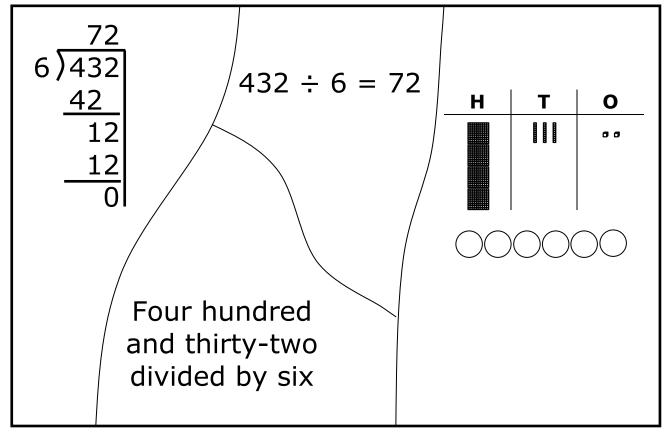
DS2 - Separation Division Mix-and-Match Cards



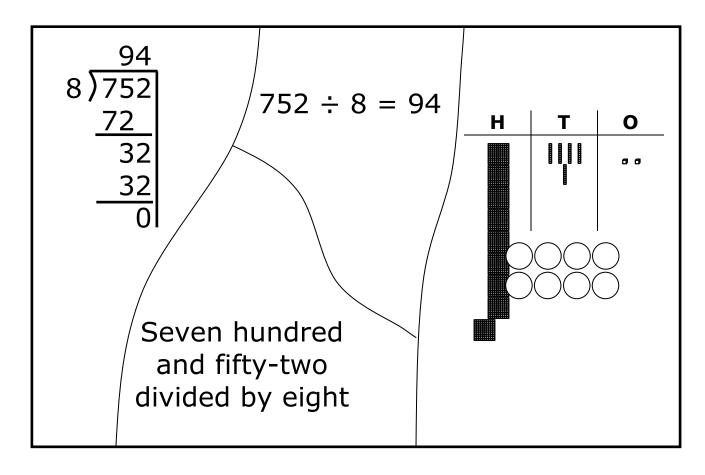


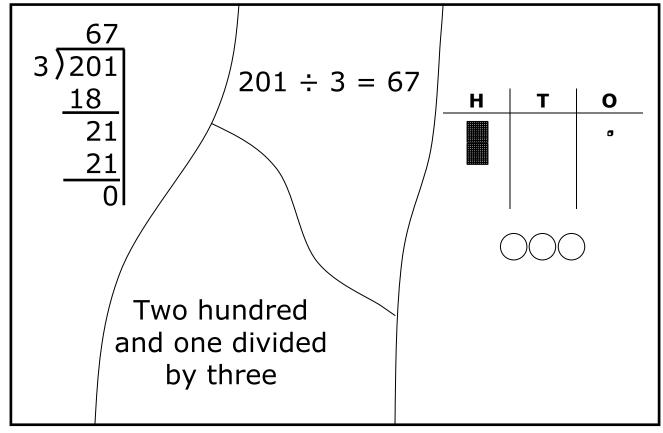
DS2 – Separation Division Mix-and-Match Cards Continued



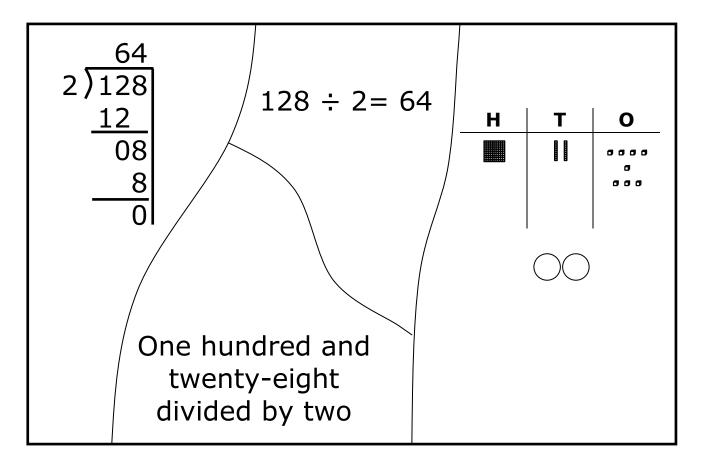


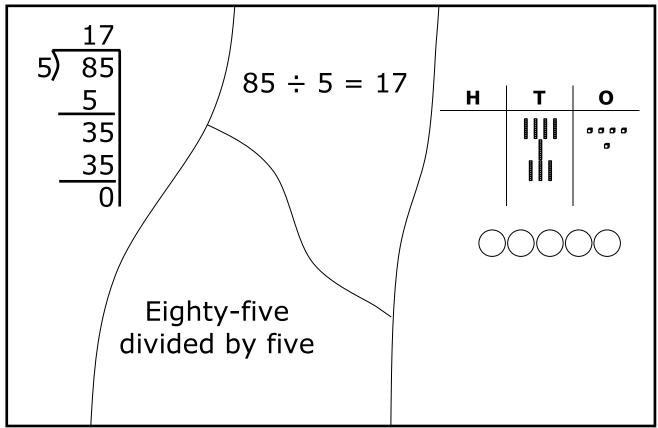
DS2 - Separation Division Mix-and-Match Cards Continued



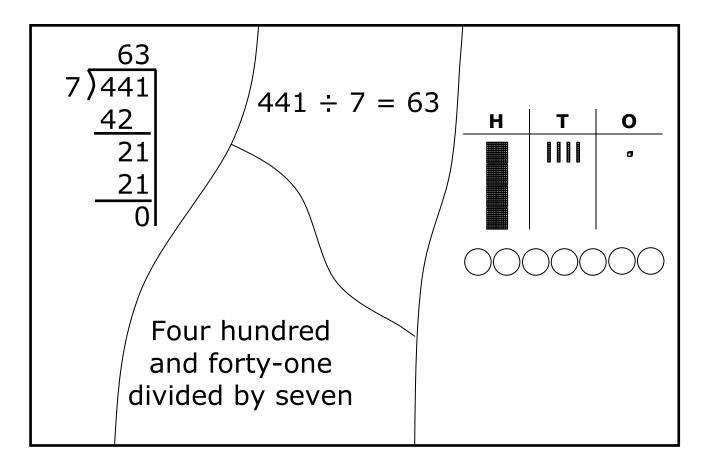


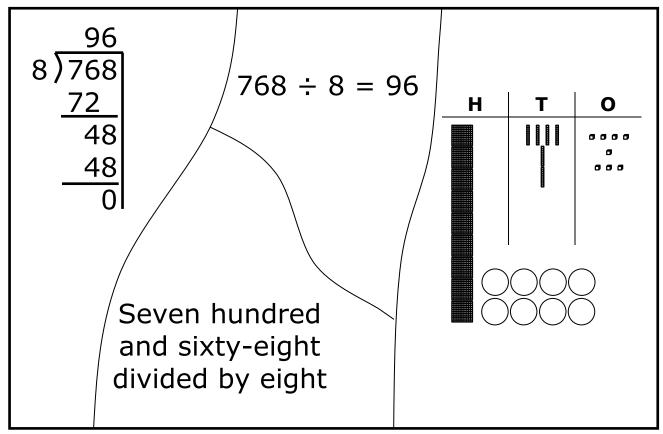
DS2 – Separation Division Mix-and-Match Cards Continued



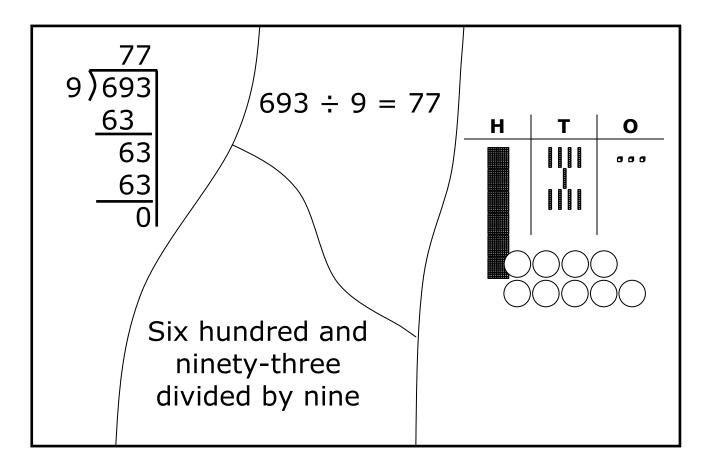


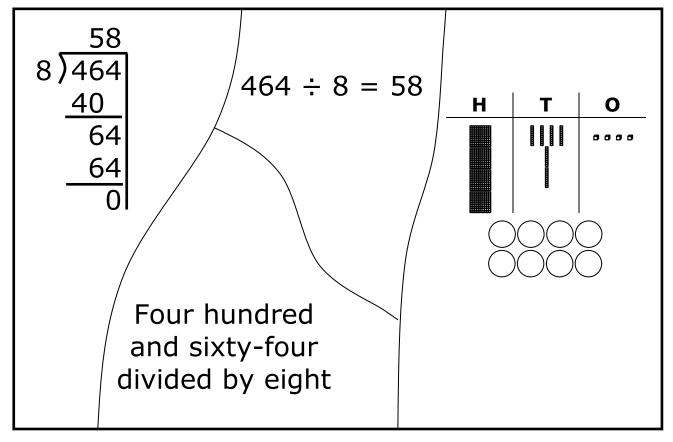
DS2 - Separation Division Mix-and-Match Cards Continued





DS2 – Separation Division Mix-and-Match Cards Continued





DS2 - Separation Division Bingo Flashcards

196 ÷ 7 = 28	128 ÷ 2 = 64	693 ÷ 9 = 77
432 ÷ 6 = 72	85 ÷ 5 = 17	464 ÷ 8 = 58
124 ÷ 4 = 31	752 ÷ 8 = 94	441 ÷ 7 = 63
215 ÷ 5 = 43	201 ÷ 3 = 67	768 ÷ 8 = 96

DS2 - Separation Division Bingo Boards

DSZ Schara	icion Division				
One hundred and ninety-six divided by seven	Four hundred and thirty- two divided by six	One hundred and twenty- four divided by four	Two hundred and fifteen divided by five		
2)128 12 08 8 0	17 5) 85 5 35 35 0	94 8) 752 72 32 32 0	67 3)201 18 21 21 0		
н т о	н т о	н т о	н т о		
One hundred and ninety-six divided by seven	One hundred and twenty-four divided by four	67 3)201 180 21 21 0	H T O		
Four hundred and thirty- two divided by six	17 5) 85 5 35 35 0	H T O	64 2)128 12 08 8 0		
94 8) 752 72 32 32 0	H T O	Two hundred and fifteen divided by five	H T O		

DS2 - Separation Division Bingo Boards Continued

H	T 	0	hun fi	Sevendred fty-two ided eight	and vo by	58 8)464 40 64 64 0			Two hundred and one divided by 3			
and eigh	hund twe t div by tw	nty- ided		77 9) 693 630 63	3 <u>)</u> 3 3	H	T 	0	H	T 	0	
96 8) 768 72 48 48 0		H	T IIIII IIII	0	_	ihty- ided five	by	7	63)441 42 21 21 0			

and eigh	One hundred 9) 693 and twenty-eight divided by two 63 0				H	T IIII	0	H T O				
H	T I	0	8	58 8) 464 40 64 64 0			Two hundred and one divided by 3			Eighty-five divided by five		
96 8) 768 72 48 48 0			H	II	0	hun fit div	Sever dred fty-tw rided eight	and vo by	7	63)441 42 21 21 0		

DS2 - Separation Division Bingo Boards Continued

1					•						
H T	0	and div	hund forty ided sever	one by	an fou	hun d six r divi y eig	ided	28 7)196 14 56 56 0			
Six hun and nin three di by ni	ety- vided	6	6)432 42 12 12 12 15 0					31 4)124 12 04 4 0			
Seve hundred sixty-e divided eigh	Н	T 	0	H	T	O	H	T IIIII	O		
4)124 12 04	<u> </u>	H	T	O	5	43) 215 20 15 15		Н	T	0	
Four hundred and sixty- four divided by eight		28 7) 196 14 56 56			H T O			Four hundred and forty-one divided by seven			
H T	0	hun six div	Sever dred ty-eig ided eight	and ght by	Six hundred and ninety-three divided by nine			72 6)432 42 12 12 0			

DS2 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

	NAME	STUDENTS' REACTIONS								
		Boring —	Interesting							
		Difficult —	Easy							
		Not learning —	Learning							
		Boring —	Interesting							
		Difficult —	——— Easy							
		Not learning —	Learning							
2.	How did you feel about trialling to Mark the line with an X: Unco	he activity?	— Very confident							
3.	Do you think the student was eng	aged in the activity? Expl	ain.							
4.	What do you think the student learnt from the activity?									
5.	Do you think the student has gain taught? Explain.	ned an understanding of th	e concept being							
6.	What do you think of the activity	?								
7.	What are your suggestions for im	proving the activity?								
8.	What else do you suggest could be this activity?	e done to help students w	ho have trouble with							

4

72

?

?

ACTIVITY DS3

[Sequencing strategy for division computation]

Materials: Unifix, dot paper, calculator, pen, paper, attachments

Directions:

- 1. Use unifix and dot paper to make arrays. Ask: *How many rows for the following?*
 - (a) 18 cubes, 3 per row (b) 32 cubes, 4 per row (c) 54 cubes, 9 per row

Ask: What do you notice? Elicit that, for example (a), 6 rows, 6x3 = 8 and $18 \div 3 = 6$. State: By finding how many in each row, we are really finding division.

2. State: We are going to learn a new method for division based on sequencing. It uses rows and columns. It thinks about division like $18 \div 3 = 6$ as "how many threes in 18?" Ask: Look at $72 \div 4$. State: We think 72 cubes, 4 per row, how many rows? Ask: Look at drawing of this [on right]. Think! How many 4s in 72? Is there 10? 20? For each answer, work out how many would be used.

4 10x4 = 40; OK 20x4 = 80; too many 40 10 State: Need to be less than 72. State: Redraw the diagram to remove 32

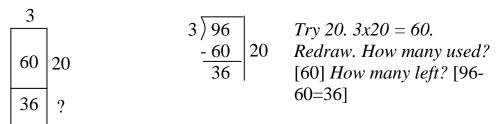
the 10 rows of 4. Ask: How many used? [40] How many left? [72-40] = 32

4 Look at 32. Ask: *How many 4s in 32?* [8] State: *Recheck the* diagram again. Redraw to include 8x4 = 32. Ask: What is the 40 10 *answer?* [18] 8

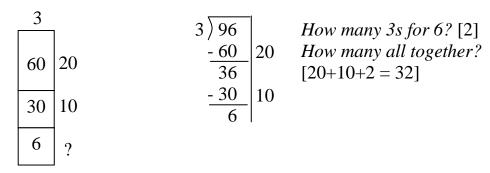
3. State: This new method has a special algorithm which makes it really easy. Let's look at it for 96÷3 and 972÷4. Check answers with calculators.

<u>Problem</u>	<u>Drawing</u>	<u>Algorithm</u>	Questioning / Directions
Step 1 Set up.			
96÷3	3 96 ?	3 \ 96	How many 3s in 96? Is there 10, 20?

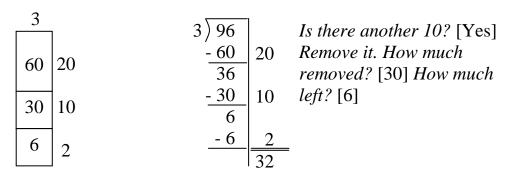
Step 2 Try tens that are less than needed.



Step 3 Keep going.

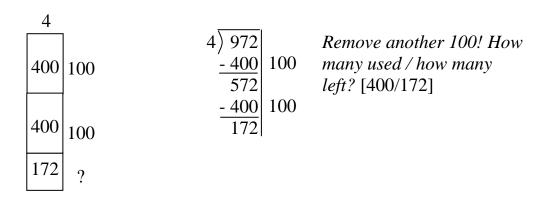


Step 4 Finish off the ones and work out answer.



Problem	<u>Drawing</u>	<u>Algorithm</u>	Questioning / Directions
Step 1 Setting	up.		
972÷4	972 ?	4) 972	Start thinking! How many 4s in 972? Is there 10? Is there 100? Many 10s? Many 10os?

Step 2 Start removing and keep going.

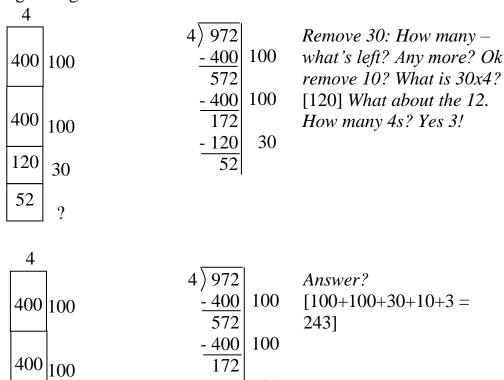


Step 3 Keep going through tens and ones.

 $120|_{30}$

40 10

 $12 \mid ? = 3$



- 120

52 - 40

12

30

10

Note: State: The example could be completed by working out the 100s, 10s and 1s more accurately. e.g.

4	4	4	4		
972 ?	800 200	800 200	800 200		
	172 ?	160 40	160 40		
		12 ?	12 3		

4) 972	4) 972	$4\overline{)972}$	$4\overline{)972}$
	<u>- 800</u> 200	<u>- 800</u> 200	<u>- 800</u> 200
	172	172	172
		- 160 40	- 160 40
		12	12
			- 12 3
			$\overline{0}$ $\overline{243}$

- 4. Complete Worksheet 3.1.
- 5. State: *Now we are going to do this method without a drawing (though we can think of a drawing).* (Check answers with a calculator.)

Look at examples 657÷9, 715÷5 and 836÷4 on the next page.

Example	<u>Algorithm</u>	<u>Thinking</u>
657÷9	9) 657	How many 9s in 657? 10, 100? Not 100, so try a fair number of 10s! What about 5 10s or 50? [50x9 = 450]
	$ \begin{array}{c c} 9 & 657 \\ -450 & 50 \\ \hline 207 & \\ -180 & 27 \end{array} $	Not there yet? Try another 2 10s or 20! [20x9 = 180]
	$ \begin{array}{c c} 9 & 657 \\ -450 & 50 \\ \hline 207 & 207 \\ -180 & 20 \\ \hline 27 & 3 \\ \hline 0 & 73 \end{array} $	Finish the 1s! How many 9s in 27? [3] How much left? [0] So answer is? [50+20+3 = 73]
715÷5	$ \begin{array}{c c} 5 & 715 \\ -500 & 100 \\ \hline 215 & 40 \\ \hline -200 & 40 \\ \hline -15 & 3 \\ \hline 0 & 143 \\ \end{array} $	How many 5s in 715? Is there 1, 10, 100? How many of these? [one 100] Try a 100 – how much left? [215] How many 10 lots of 5? [4 10s or 40]. How much do we remove? [40x5 = 200] Finish off the 1s.

Is there 10 or 100 4s in 836? Try 100 – how much left? [436] Ok there's another 400 so remove another 100 – how much left? [36] Finish off the 1s – how many 4s in 36? [9]

- 6. Complete Worksheet 3.2.
- 7. Play games: "Sequencing Division Snap, Rummy or Concentration", "Sequencing Division Cover-the-Board", "Sequencing Division Mix-and-Match", and "Sequencing Division Bingo". The rules for these games are in DS2.

DS3 – **Dot Paper**

•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
											•	•				
•		•								•				•		•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	٠	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	٠	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
•	•	•	٠	•	•	•	٠	٠	•	•	•	•	•	•	•	•

DS3 - Worksheet 3.1

Do the following using the diagram! (Check with calculators)

(1) 8

(2) 6

(3) 4

 $4)\overline{904}$

(4)

3

3) 921

T

(5) 2

2) 984

(6)

7) 868

E

We want what the song says!

492

124

122

126

124

145

307

DS3 - Worksheet 3.2

Do the following thinking of the diagram! (Check with calculators)

$$\begin{array}{c} (3) \\ 2 \overline{)786} \end{array}$$

G

L

$$\begin{array}{c|c} (6) & \\ 2 & 602 \end{array}$$

(7) 3 918

,

$$\begin{array}{c|c}
(8) \\
4 \overline{\smash{\big)}\,828}
\end{array}$$

N

R

$$7 \overline{)756}$$

Н

241 199 207

202

301

147

237

108

306

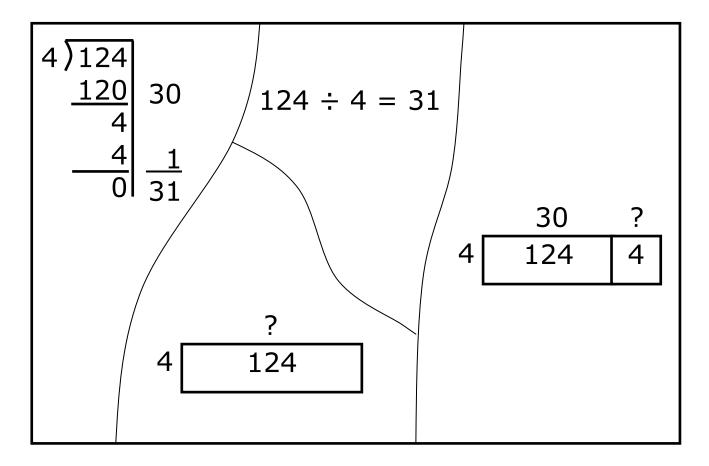
393

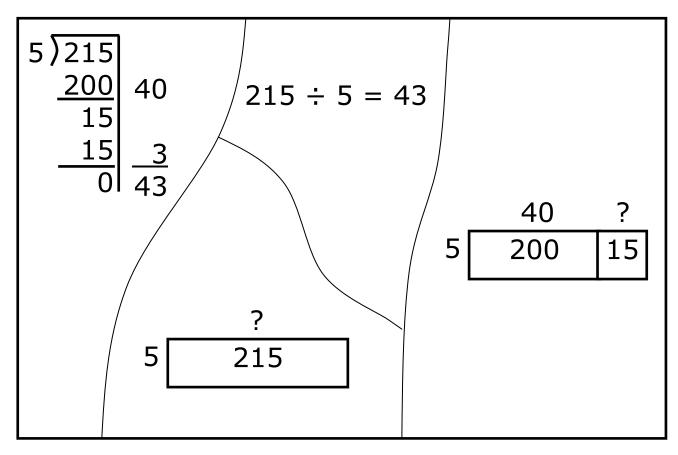
DS3 - Sequencing Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board

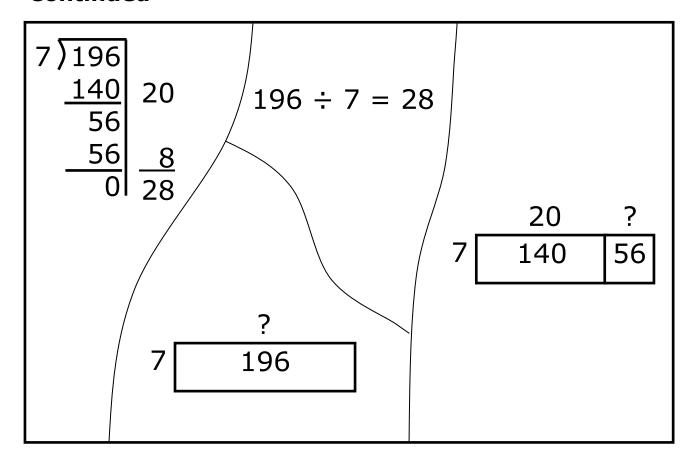
			•
215 ÷ 5 = 43	124 ÷ 4 = 31	432 ÷ 6 = 72	196 ÷ 7 = 28
201 ÷ 3 = 67	752 ÷ 8 = 94	85 ÷ 5 = 17	128 ÷ 2 = 64
768 ÷ 8 = 96	441 ÷ 7 = 63	464 ÷ 8 = 58	693 ÷ 9 = 77
5) 215 200 15 15 3 0 43	4) 124 120 4 -4 0	$ \begin{array}{c c} 6)432 \\ \underline{420} \\ 12 \\ \underline{12} \\ 0 \end{array} $	7)196 140 56 56 8 0 28
3) 201 180 21 21 7 0 67	$ \begin{array}{c c} 8)752 \\ 720 \\ 32 \\ 32 \\ \hline 0 \\ \hline 94 \end{array} $	5) 85 50 35 35 7 0 17	2) 128 120 8 8 8 4 64
8)768 720 48 48 6 96	$ \begin{array}{c c} 7) & 441 \\ \underline{420} & 60 \\ \hline 21 & 3 \\ \hline 0 & 63 \end{array} $	8) 464 400 64 64 64 0 58	9) 693 630 70 63 7 63 7 77

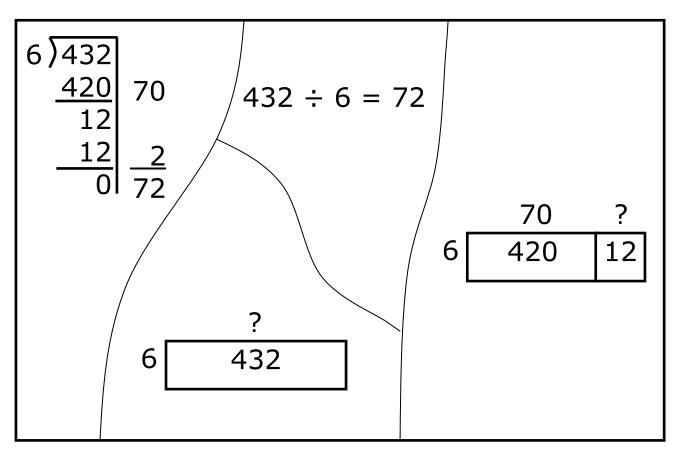
DS3 - Sequencing Division Snap, Rummy, Concentration and Cover-the-Board Cards/Board Continued

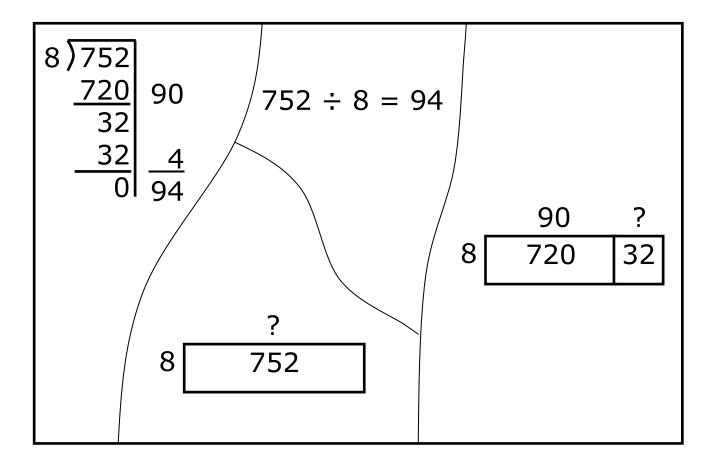
DS3 - Sequencing Division Mix-and-Match Cards

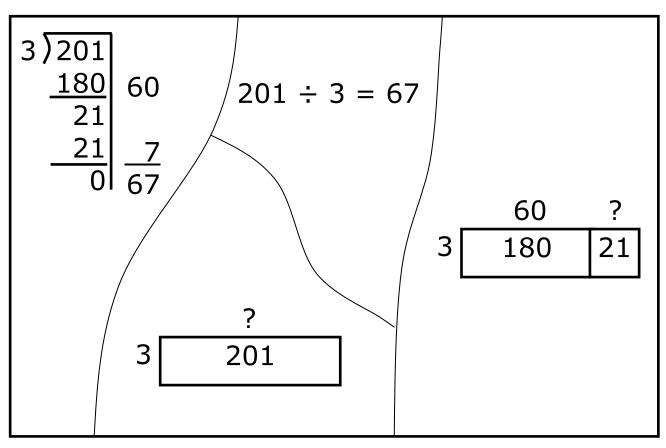


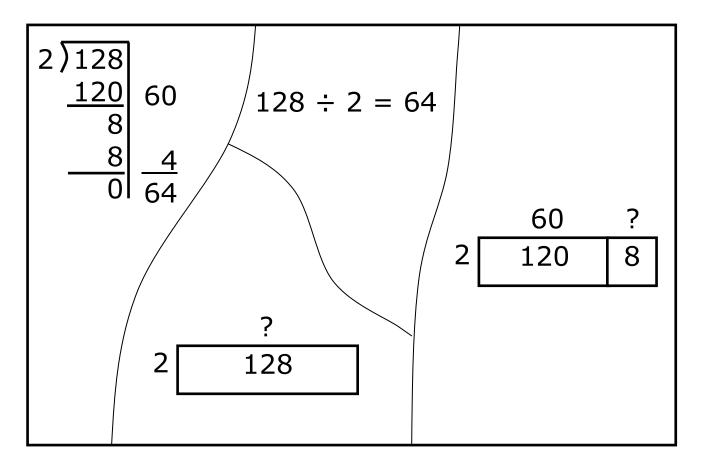


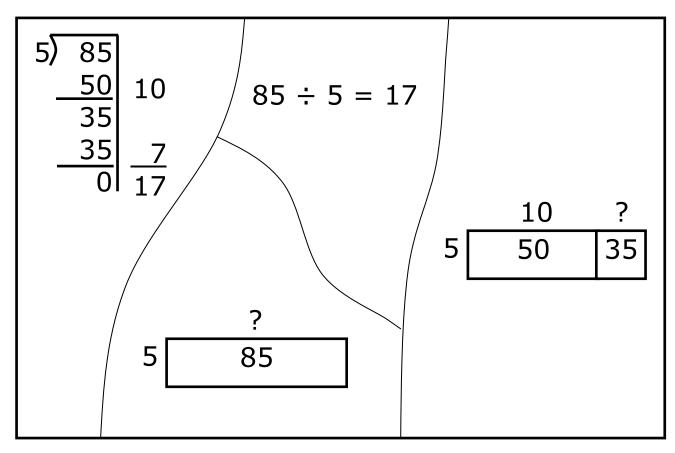


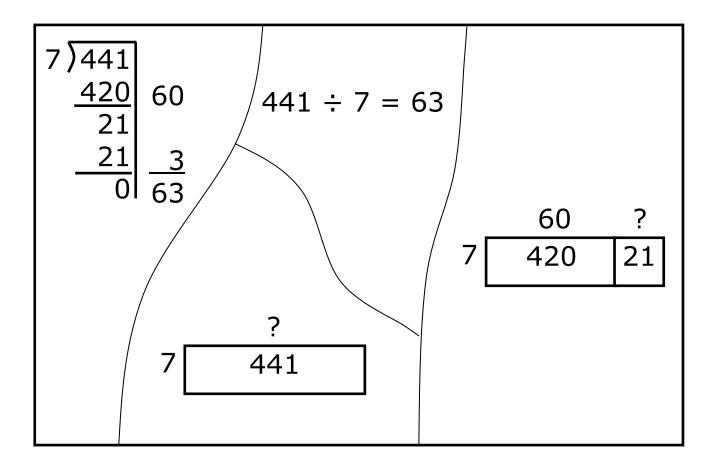


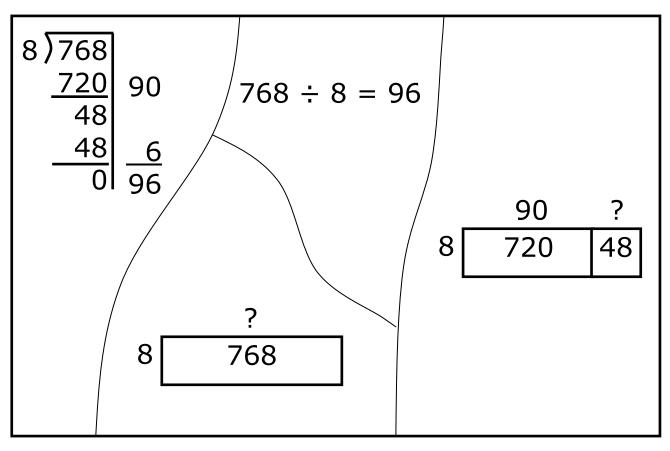


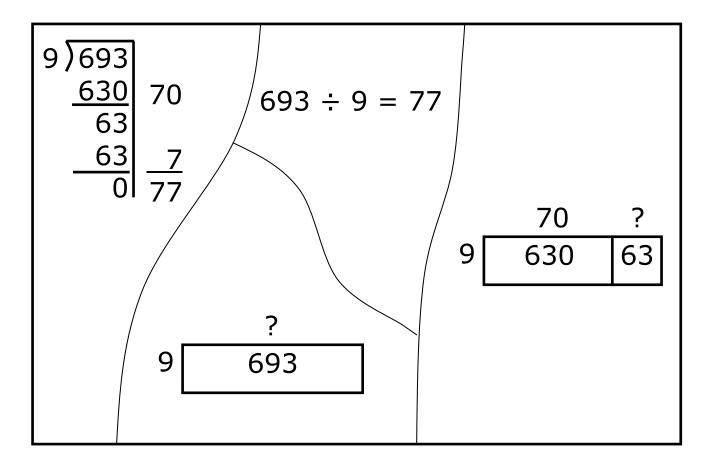


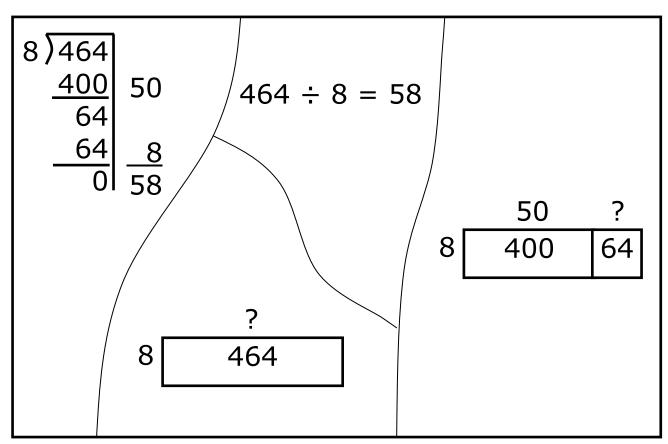












DS3 – Sequencing Division Bingo Flashcards

215 ÷ 5 = 43	124 ÷ 4 = 31	432 ÷ 6 = 72	196 ÷ 7 = 28
201 ÷ 3 = 67	752 ÷ 8 = 94	85 ÷ 5 = 17	128 ÷ 2 = 64
768 ÷ 8 = 96	441 ÷ 7 = 63	464 ÷ 8 = 58	693 ÷ 9 = 77

DS3 - Sequencing Division Bingo Boards

? 5 215	? 4 124	? 6 432	? 7 <u>196</u>
3) 201 180 21 21 7 67	8) 752 720 32 32 32 4 94	5) 85 50 35 35 0 7 17	2) 128 120 8 8 8 0 4 64
90 ? 8 720 48	60 ? 7 420 21	50 ? 8 400 64	70 ? 9 630 63
? 7 196	? 4 124	3) 201 180 21 21 7 0 67	50 ? 8 400 64
?	5) 85 50 35	90 ? 8 720 48	2) 128 120 8 60

DS3 - Sequencing Division Bingo Boards Continued

40 ? 5 200 15	? 8 752	8) 464 <u>400</u> 50 64 <u>64</u> <u>8</u> 0 58	? 3 <u>201</u>
? 2 128	9) 693 630 70 63 63 7 77	70 ? 6 420 12	20 ? 7 140 56
8) 768 720 48 48 0 96	30 ? 4 120 4	? 5 85	7) 441 420 21 21 3 63
? 2 128	9) 693 630 63 63 77	70 ? 6 420 12	20 ? 7 140 56

12

DS3 - Sequencing Division Bingo Boards Continued

DS3 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

	NAME	STUDENTS' REACTION	NS
		Boring —	Interesting
		Difficult —	Easy
		Not learning —	Learning
		Boring —	Interesting
		Difficult —	Easy
		Not learning —	Learning
2.	How did you feel about trialling the Mark the line with an X: Uncon	•	onfident
3.	Do you think the student was engage	ged in the activity? Explain.	
4.	What do you think the student learn	nt from the activity?	
5.	Do you think the student has gained taught? Explain.	an understanding of the concept	being
6.	What do you think of the activity?		
7.	What are your suggestions for impr	roving the activity?	
8.	What else do you suggest could be this activity?	done to help students who have tro	ouble with

ACTIVITY DS4

[Compensation strategy for division computation]

Materials: Pen, paper, attachments

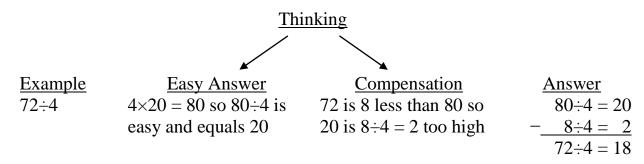
Directions:

- 1. State: We are going to look at very quick ways to do division if you can spot the shortcut. We have to start by reminding ourselves of easy multiplications.

 Organise students to remember the following:
 - (1) multiplying by 10 $-8 \times 10 = 80$
 - (2) multiplying by 5 $-\frac{1}{2} \times 10 : 8 \times 5 = \frac{1}{2} (8 \times 10) = \frac{1}{2} 80$
 - (3) multiplying by $100 7 \times 100 = 700$
 - (4) multiplying by 50 $-\frac{1}{2} \times 100 : 7 \times 50 = \frac{1}{2} (7 \times 100) = 350$
 - (5) multiplying by 25 $-\frac{1}{4} \times 100 : 7 \times 25 = \frac{1}{4} (7 \times 100) = 175$
 - (6) higher decade $-2\times 4 = 8:20\times 4 = 80$ $20\times 40 = 800$ $200\times 4 = 800$
 - (7) multiplying by 2 doubling : $2 \times 62 = 124$
 - (8) multiplying by 4 double double : $4\times62 = 2\times124 = 248$
 - (9) multiplying by 8 double double : $8 \times 62 = 4 \times 124 = 2 \times 248 = 496$

State: We have to remember these as easy ones.

2. State: In this new way to divide, we change the example to an easy one and then compensate for this (that's why it's called compensation). Let's look at a few examples 72÷4, 438÷3, and 924÷6. Check with calculators!



438÷3	$3 \times 100 = 300$	438 is 12 less than 450	$450 \div 3 = 150$
	$3 \times 50 = \frac{1}{2}300 = 150$	so 150 has to be	$-12 \div 3 = 4$
	$3 \times 150 = 450$	reduced by 12÷3	$438 \div 3 = 146$
	so $450 \div 3 = 150$		
924÷6	$6 \times 100 = 600$	924 is 24 more than	$900 \div 6 = 150$
	$6 \times 50 = 300$	900 so 150 has to be	$+ 24 \div 6 = 4$
	$6 \times 150 = 900$	increased by 24÷6	$924 \div 6 = 154$
	so $900 \div 6 = 150$		

3. State: We can also manipulate the number we divide by. Consider 576÷4, 936÷8 and 786÷6. (Check with calculators!)

<u>Example</u>	<u>Thinking</u>	<u>Answer</u>
576÷4	÷4 is halving and halving	half of $576 = 288$ half of $288 = \underline{144}$
936÷8	÷8 is halving, halving and halving	half of $936 = 468$ half of $468 = 234$ half of $234 = \underline{117}$
786÷6	6 is 3×2 so $\div6$ is $\div2$ (halving) and then $\div3$	half of $786 = 393$ dividing by $3 = 131$

4. Complete Worksheets 4.1 and 4.2.

DS4 - Worksheet 4.1

Do the following by thinking of easy ways and compensating. Tell us what you did. Put letters where answers are at bottom of sheet.

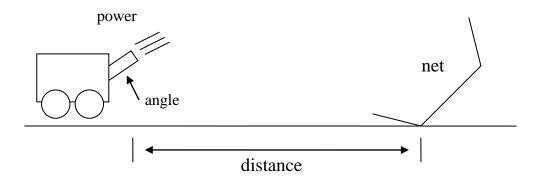
(1)	815÷2 =	W	(2)	915÷3 =	R
(3)	985÷5 =	D	(4)	716÷2 =	Е
(5)	968÷4 =	V	(6)	477÷3 =	S
(7)	864÷6 =	P	(8)	924÷7 =	N
(9)	840÷8 =	O	(10)	645÷5 =	I

		Wh	at do	we wa	int?		
193	129	242	129	159	129	105	132
144	105		358	305			

DS4 - Worksheet 4.2

The Human Cannonball!

 $(Power \times Angle = Distance)$



Complete the chart – check your answer with a calculator.

The first has been done with you.

Distance	Power	Angle	Power x Angle (use calculator)
456	4	456÷4 Think: 440÷4 = 110 456 is 16 more so 456÷4 = 114	114×4 = 456
(1) 875	5		
(2) 510	6		
(3) 742	7		
(4) 464	8		
(5) 531	9		
(6) 658	7		
(7) 736	8		
(8) 774	6		
(9) 996	4		
(10) 987	3	_	

DS4 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' RI	EACTIONS
	Boring —	——— Interesting
	Difficult —	——— Easy
	Not learning —	Learning
	Boring —	Interesting
	Difficult —	——— Easy
	Not learning —	Learning

•	_	•

Mark the line with an X: Unconfident — Very confident

- 3. Do you think the student was engaged in the activity? Explain.
- 4. What do you think the student learnt from the activity?
- 5. Do you think the student has gained an understanding of the concept being taught? Explain.
- 6. What do you think of the activity?
- 7. What are your suggestions for improving the activity?
- 8. What else do you suggest could be done to help students who have trouble with this activity?