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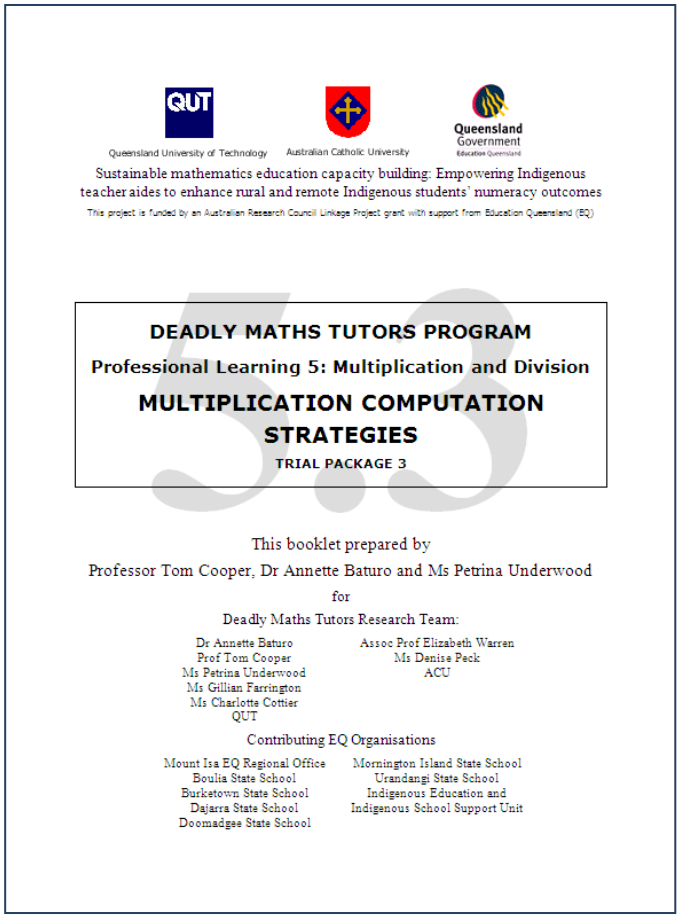
Professional Learning 5

Multiplication and Division

Booklet 5.3: Multiplication Computation Strategies



Queensland University of Technology Australian Catholic University Queensland Government

Sustainable mathematics education capacity building: Empowering Indigenous teacher aides to enhance rural and remote Indigenous students' numeracy outcomes

This project is funded by an Australian Research Council Linkage Project grant with support from Education Queensland (EQ)

DEADLY MATHS TUTORS PROGRAM

Professional Learning 5: Multiplication and Division

MULTIPLICATION COMPUTATION STRATEGIES

TRIAL PACKAGE 3

5.3

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Boulia State School
Burketown State School
Dajarra State School
Doomadgee State School
Mornington Island State School
Urandangi State School
Indigenous Education and Indigenous School Support Unit

YuMi Deadly Maths
Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Island Regional Educational Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology,
Australian Catholic University &
Education Queensland

Deadly Maths Tutor Program

PROFESSIONAL LEARNING 5: MULTIPLICATION AND DIVISION

BOOKLET 5.3

MULTIPLICATION COMPUTATION STRATEGIES

2008

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Boulia State School	Sunset State School
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Dajarra State School	Indigenous Education and Training Alliance
Doomadgee State School	(IETA)

YuMi Deadly Centre

School of Mathematics, Science and Technology Education,
Faculty of Education, QUT

CONTENTS

OVERVIEW

PURPOSE

These materials were designed to be used in conjunction with a Professional Learning program for teacher aides. The objective of this Professional Learning was to empower teacher aides to enhance rural and remote Indigenous students' numeracy outcomes. This document contains the materials of the third of five different booklets on multiplication and division.

If your school would like to receive a YuMi Deadly Maths Professional Learning program please contact the YuMi Deadly Centre (YDC) on: 07 3138 0035 or ydc@qut.edu.au.

DIRECTIONS

(1) Interviewing the students:

Pick one or more students who appear to be having trouble understanding multiplication. Interview these students using the interview schedule and the materials. Mark what they do and put their results on the Student Recording Sheet.

(2) Trialling the student activities:

Use the Recording Sheet to work out the activities the students need to do and trial these activities with the students (with each student one at a time or with a group of students). Keep a record of what happens and collect the students' work.

Interview Schedule

Materials:

Unifix cubes or counters, washable felt pens, pen, pencil, paper

Materials within this booklet: interview questions, interview cards, Student Recording Sheet

Directions:

1. Photocopy and laminate attached interview cards.
2. Gather other material (unifix or counters, paper, pens, pencils).
3. Place material in front of students. Give students pen and paper to write with.
4. Tell the students you are trying to find out what they know. Say they are not expected to know it and you will teach what is not known.
5. Give the student directions slowly – read problems. Do not give hints. If student cannot do a question, pass on to the next question, repeating that it is not important if they don't know how to do the question.
6. Allow students to use material and make drawings but only after they say they do not know how to do it with symbols alone.

INTERVIEW QUESTIONS

Strategies for Multiplication Computation

C1	**Show and read Card 1** • Ask: *What is the answer?* • If student can answer, ask: *How did you work it out?* • If cannot answer, give answer and write on card. Do the same for Card 2.
C2	**Show and read Card 3** • Ask: *What is the answer?* • If student can answer, ask: *How did you work it out?* • If cannot answer, ask: *Can you use Card 1 to assist you?* Show Card with answer. Repeat for Card 4, but refer to Card 2 and 5.
C3	**Show and read Card 6** • Say: *Calculate the answer by separating the 2-digit number into ones and tens.* • If cannot answer, say *Can you use MAB to help?*
C4	**Show and read Card 7** • Say: *Calculate the answer by leaving the 2-digit number as is and breaking the 1-digit number up somehow.* • If cannot answer, say can you use a drawing to help?
C5	**Show and read Card 8** • Say: *Calculate the answer by changing the multiplication to something that is easier to multiply and then compensating.*

INTERVIEW CARDS

Strategies for Multiplication Computation

CARD 1	**CARD 2**
$\begin{array}{r} 2 \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array}$
CARD 3	**CARD 4**
$\begin{array}{r} 20 \\ \times\ 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times\ 70 \\ \hline \end{array}$
CARD 5	**CARD 6**
$\begin{array}{r} 60 \\ \times\ 70 \\ \hline \end{array}$	$\begin{array}{r} 35 \\ \times\ 7 \\ \hline \end{array}$
CARD 7	**CARD 8**
$\begin{array}{r} 28 \\ \times\ 6 \\ \hline \end{array}$	$\begin{array}{r} 39 \\ \times\ 4 \\ \hline \end{array}$

STUDENT RECORDING SHEET

Name: _____

School/Class: _____

Interview item	Result (✓, ✗)	Comments	Activities to be completed if incorrect
C1:			Undertake practice in basic multiplication facts.
C2: Higher decade multiplication facts			MS1
C3: Separation strategy			MS2
C4: Sequencing strategy			MS3
C5: Compensation strategy			MS4

INTRODUCTION

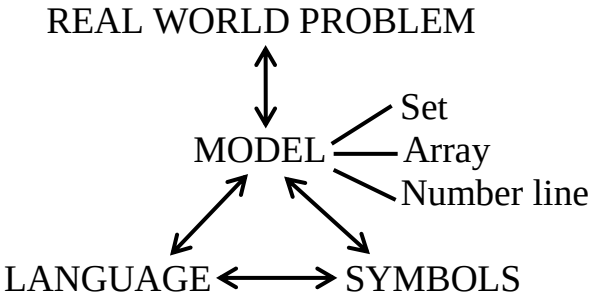
Contents

This package contains:

- four tutoring activities (MS1 to MS4) and their student materials (games and worksheets), as well as an activity feedback sheet for each activity; and

Pedagogy

The activities MS1 to MS4 are based on the Rathmell Triangle Relationship below; real world problems are related to set, array and number line models, language and symbols (and vice versa) to teach strategies for computation.



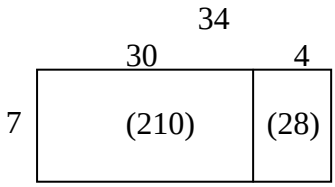
However, the focus of the activities is to develop a repertoire of strategies for computing such examples as 34×6 and 257×3. The activities are designed to allow you to tutor students who are having difficulties with multiplication computation for 2- and 3-digit numbers × 1 digit numbers. The activities are based on the belief that it is more important to use the algorithms to teach a variety of strategies than to get a correct answer. However, the activities also show how to get correct answers.

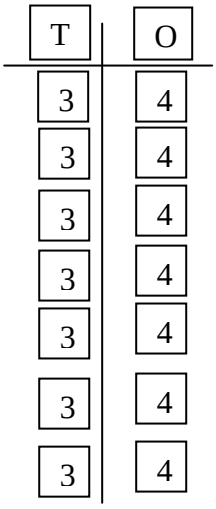
There are three strategies (or strategy groupings) associated with multiplication computation. These are:

(1) Separation:

This strategy (separate, operate, combine) is to break the 2- or 3- digit number into parts usually based on place value, multiply the numbers as separated, then recombine for the answer. It is based on the distributive law that, e.g., if 34 is 30+4 then 7×34 is 7×30 + 7×4. It is widely used in mathematics, for example, multiplying measures (m and cm), time (hrs and mins), mixed numbers (wholes and parts), and algebra (x's and y's). The strategy may involve renaming or carrying. It is based on the set or array model – materials to teach separation are place-value charts, bundling sticks, and MAB, or square tiles and graph paper. Examples of written algorithms for 7×34 below.

Example 1:

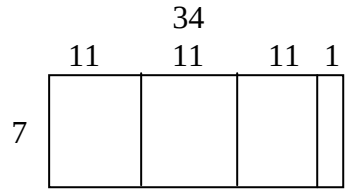




7×34
= 7×30
+ 7×4
or 7×4
+ 7×30

34	34
×7	×7
210 (7×30)	28 (7×4)
28 (7×4)	210 (7×30)
238	238

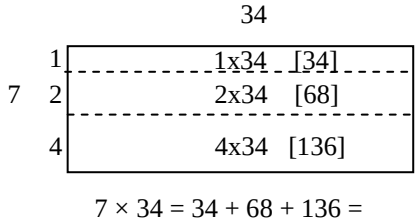
Example 2:



34
×7
77 (7×11)
77 (7×11)
77 (7×11)
7 (7×1)
238

(2) Sequencing:

This strategy is to leave the 2- or 3- digit as a whole and multiply parts of the second 1- digit number until all parts have been multiplied. It is associated with arrays and is also useful for measures, time and mixed numbers, and for variables and algebra. In some ways it is number separation because a common way to break up a number is by place value. However, as the examples below show, there are other ways to break up numbers. Again it is based on the distributive law. Materials to teach sequencing are square counters, squared paper (graph paper) and drawings. An example for 7 × 34 is below.



7 × 34 = 34 + 68 + 136 =

34	
×7	
34	(1×34)
68	(2×34)
136	(4×34)
238	

[Note: If one of the numbers has factors like 24 = 6×4, then another method is to use two stages; e.g., 7×24 can be done by 7×6 = 42 and then 42×4= 168.]

(3) Compensation:

This strategy is to leave both numbers as they are but to look for a change (to both numbers if necessary) that will make the multiplication easy, then compensate for that change if necessary. This is the basis of many of the methods used in algebra as well as being useful for multiplying measures, time and mixed numbers. Materials to teach compensation are mainly symbolic but

are based on the distributive law (as in the other two strategies) which can be developed with graph paper and geoboards. Two examples for 7×34 are below.

- An easy problem related to 7×34 would be 10×34. This equals 340 but the 10 is 3 larger than the 7, so have to compensate by removing 3×34 or three thirty-fours. Thus 7×34 is 340-34 = 306-34 = 272-34 = 238.
- Another easy problem would be 7×35. This equals: 3½ 70s = 210 + ½ 70 = 210+35 = 245 but 35 is 1 more than 34, so have to compensate by removing 7, so answer is:245−7 = 238. Another way is to look at 7×34 as 10×34=340, halve this to get 5×34=170 and then add 2×34 or 68.

All the three strategies require knowledge of basic number facts (e.g., 6×8=48) and also higher-decade number facts (e.g., 6×80 = 480, 60×80 = 4800).

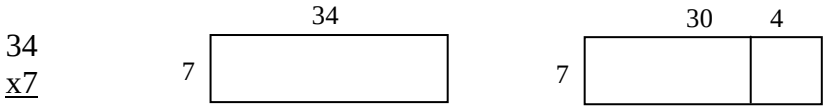
Thus, the four activities in this booklet are the higher decade facts (Activity MS1), separation strategy (Activity MS2), sequencing strategy (Activity MS3), and compensation strategy (Activity MS 4).

Approach

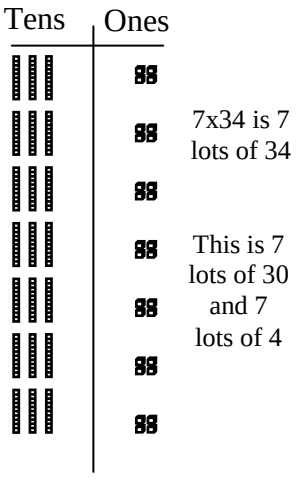
Meaning, not answers

The first imperative is to get across understanding – to focus on meaning and not on answers. Thus time is spent on

(a) Relating different representations of multiplication, for example, 7×34 is "7 thirty-fours", "34 multiplied by 7", 34+34+34+34+34+34+34, problems such as "I had seven bags of lollies; there were 34 lollies in each bag, how many lollies overall?", and vertical settings out and arrays such as:



(b) Sequencing from basic facts to algorithms, for example, for 7×34 need to develop basic facts (7×3, 7×4) higher decade multiplication facts (7×3=21, 7×30=210), distributive law (see below) and finally 7×34 is 210+28.



(c) Variety, not one algorithm, for example, for 7×34 will develop a variety of ways to teach it (sequencing, compensation, etc). This allows students to choose the best ways for numbers they are given. This requires metacognition (being in control of own thinking).

ACTIVITIES

ACTIVITY MS1

[Higher decade multiplication facts]

Materials: Square tiles, graph paper (2mm), calculators, pen, paper, resources attached (worksheets, games)

Directions:

1. Use tiles to form a 3 by 7 rectangle. Ask: *Is this similar to an array?* [Yes] *What is the answer?* [21] *What is the operation?* 3×7, "three sevens", "seven multiplied by three".

   Use graph paper to show 3×2 and 3×20. Ask: *Calculate the number of squares in each. What do you notice?* [6 and 60]. Repeat this for 7×4 and 7×40. See if students can propose a rule or pattern.

2. Use graph paper to show 2×3 and 20×30. Ask: *Calculate the number of squares in each. What do you notice?* [6 and 600]. *Why is the answer 600 and not 60?* [2 zeros in 20 and 30]. Repeat this for 4×6 and 40×60. See if students can propose a rule or pattern.

3. Use calculators to answer:

   3×4 ____ 30×4 ____ 3×40 ____ 30×40 ____

   6×7 ____ 60×7 ____ 6×70 ____ 60×70 ____

   5×8 ____ 50×8 ____ 5×80 ____ 50×80 ____

   Ask: *Can anyone see a pattern? Can you use this pattern to answer 4×80 if 4×8=32?*

4. Complete Worksheets 1.1 and 1.2.

5. Play the game; "Higher Decade Multiplication Tic-Tac-Toe".

MS1 – Worksheet 1.1

1. Complete these <u>with</u> a calculator:

   (a) 5×3 = ____ 50×3 = ____ 5×30 = ____ 50×30 = _____

   (b) 2×7 = ____ 20×7 = ____ 2×70 = ____ 20×70 = _____

   (c) 4×9 = ____ 40×9 = ____ 4×90 = ____ 40×90 = _____

   (d) 6×4 = ____ 60×4 = ____ 6×40 = ____ 60×40 = _____

   (e) 8×5 = ____ 80×5 = ____ 8×50 = ____ 80×50 = _____

2. Can you see a pattern? Write it down:

3. Complete these <u>without</u> a calculator:

   (a) 4×2 = 8 40×2 = ____ 4×20 = ____ 40×20 = _____

   (b) 8×9 = 72 80×9 = ____ 8×90 = ____ 80×90 = _____

   (c) 5×4 = 20 50×4 = ____ 5×40 = ____ 50×40 = _____

4. Can you extend the idea – complete these without a calculator:

   (a) 6×8 = 48 60×80 = _____ 600×80 = _____

   (b) 4×7 = 28 40×700 = _____ 400×700 = _____

   (c) 5×9 = 45 500×900 = _____ 5000×9000 = _____

MS1 – Worksheet 1.2

WHAT IS AN IG?

1.	3×6 = 18	30×60 =	S
2.	3×6 = 18	30×600 =	K
3.	5×7 = 35	500×7 =	U
4.	5×7 = 35	50×700 =	E
5.	5×6 = 30	50×600 =	O
6.	5×6 = 30	50×6000 =	T
7.	6×7 = 42	60×70 =	M
8.	6×7 = 42	6000×7 =	W
9.	5×8 = 40	50×800 =	H
10.	5×8 = 40	50×8000 =	L
11.	5×8 = 40	500×8 =	I

35000	1800	18000	4000	4200	30000	
40000	30000	3500	1800	35000		
42000	4000	300000	40000	30000	3500	300000
300000	30000	4000	400000	35000	300000	

MS1 – Game:
Higher Decade Multiplication Tic-Tac-Toe

Materials: Boards below, Unifix (one colour per page), calculators

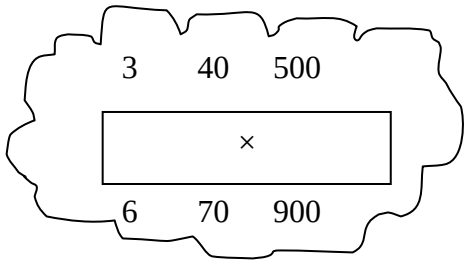
Number of players: 2

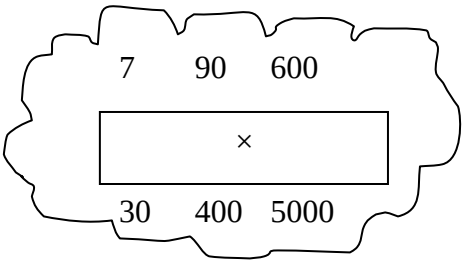
Directions:

1) Players in turn select a number from the top and a number from the bottom. Calculate the product of these (use calculator to check).
2) Cover vacant square with unifix (if square already has unifix, miss a turn).
3) First player with 3-in-a-row wins (across, down or diagonal).

Use the following:

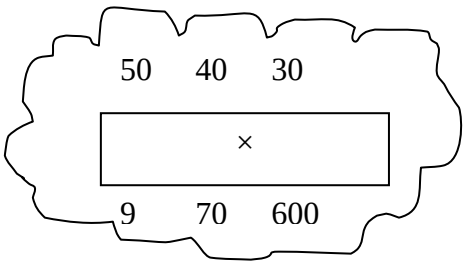
3×6=18, 3×7=21, 3×9=27, 4×6=24, 4×7=28, 4×9=36, 5×6=30, 5×7=35, 5×9=45

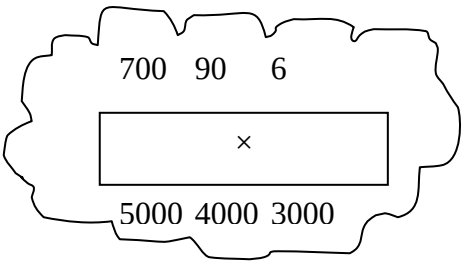




240	18	210
35.000	2,800	450.000
36,000	2.700	3.000

450,000	2,800	18,000
210	240,000	35.000
3,000,000	2.700	36,000





2,100	24 000	450
270	3,500	30.000
360	18,000	2.800

360,000	30,000	18,000
2.800.000	450,000	2.100.000
270.000	24.000	3.500.000

MS1 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning

2. How did you feel about trialling the activity?

   Mark the line with an X: Unconfident ——— Very confident

3. Do you think the student was engaged in the activity? Explain.

   _____

4. What do you think the student learnt from the activity?

   _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

   _____

6. What do you think of the activity?

   _____

7. What are your suggestions for improving the activity?

   _____

   _____

8. What else do you suggest could be done to help students who have trouble with this activity?

   _____

   _____

ACTIVITY MS2

[Sequencing strategy for Multiplication Computation]

Materials: MAB blocks, place value chart (PVC), pen, paper, calculator, attached resources.

Directions:

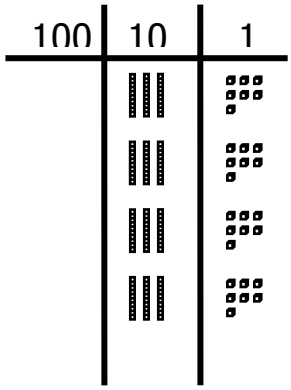
1. *Meaning.* Ask students: *What does 4×37 mean?* Focus on set model ["4 lots of 37"]. Ask: *Can you think up a story?*

2. *Separation.* Ask students: *How we can show 37 with MAB.* Ask: *How many tens?* [3]. *How many ones left over?* [7]. Direct, *put out 3 tens in tens place. Put out 7 ones in ones place.*

3. *Distributive Law.* Ask, *what does 4×37 mean?* [4 lots of 37]. State, *we have one 37 on PVC with MAB. How can we show 4×37?* [4 lots of 37]. *Put this on PVC with MAB.* State, *look at tens. How many 30s? Look at ones. How many7's?* Get students to finish 4 lots of 37 = _____ of 30s and _____ of ones.

4. *Computation.* Put out 4 lots of 37 on PVC and record as you go. Ask, *how many in each lot?* [37].*How many lots?* [4]. *How do we write this?* [37 on top, × 4 under and a line to show equals].

Step 1

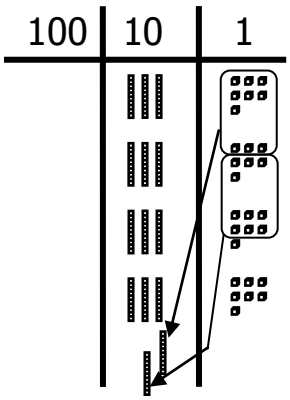


4 lots of 37

37
×4

Step 2

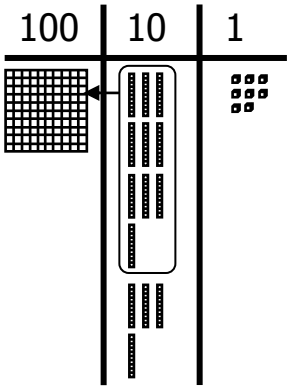
Discuss that we have to find 4 lots of 7 and regroup. Separate 37 into tens and ones and look at the ones. Ask, *what is 4×7 (use calculator)?* [28]. Ask, *have we enough ones to make a ten?* [Yes]. *Have we enough ones to make 2 tens?* [Yes]. *Have we enough ones to make 3 tens?* [No]. Direct, *make the tens. Move to tens position. How many ones are left over?* [8]. *How many extra tens?* [2]. *Write 28 under the 4.*



37
×4
28 (4×7)

Step 3

Direct, now look at tens. How many tens in each lot? How many lots? Discuss that we have to find 4 lots of 30 and regroup. Ask, *what is 4×30 (use calculator)?* [120]. *How many tens?* Remind that 4×3=12 means 4×30=120 or 12 tens. Ask, *do we have enough tens to form a hundred, two hundred, …?* [one 100]. *Make the one hundred and move to hundreds position. How many tens left over?* [2]. *How many extra hundreds?* [1]. Write 120 under the 28.



37
× 4
28 (4×7)
120 (4×30)

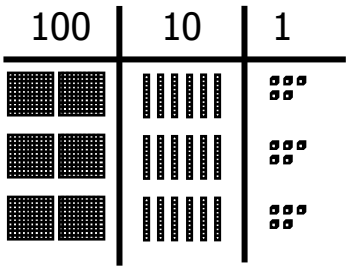
Step 4

Add the 28 and 120 to give 148, the result of 4×37. Note: can do the tens first:

37
× 4
120 (4×30)
28 (4×7)

5. Repeat direction 4 for example 3×265:

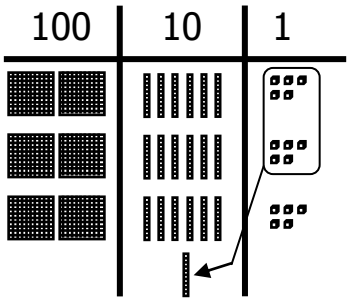
Step 1



3 lots of 265

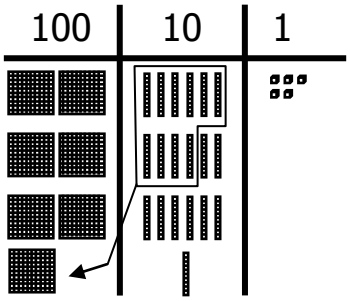
265
× 3

Step 2



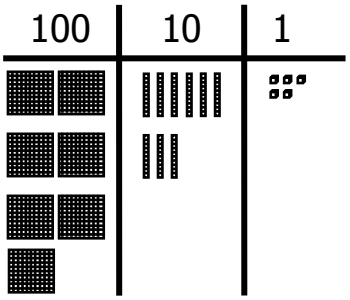
265
× 3
15 (3×5)

Step 3



265
× 3
15 (3×5)
180 (3×60)

Step 4



265
× 3
15 (3×5)
180 (3×60)
600 (3×200)

Step 5

Add all the parts. Note: could have done larger place values first as on right:

265
× 3
15 (3×5)
180 (3×60)
600 (3×200)
795

265
× 3
600 (3×200)
180 (3×60)
15 (3×5)
795

6. Repeat directions 4 and 5 but without materials.

   (a) 7×48: separate 48 into 40 and 8 and multiply (note: 7×8=56, 7×4=28 so 7×40=280)

   48
   × 7
   56 (7 × 8)
   280 (7 × 40)
   336

   (b) 6×149: separate 149 into 100, 40 and 9 and multiply (note: 6×9=54, 6×4=24, 6×1=6, so 6×40=240 and 6×100=600)

   149
   × 6
   600 (6 × 100)
   240 (6 × 40)
   54 (6 × 9)
   894

7. Complete Worksheets 2.1 and 2.2.

8. Complete the games "Multiplication Separation Tic-Tac-Toe" and "Multiplication Computation Mix and Match".

MS2 – Worksheet 2.1

1. Use your calculator to complete the following:

(a)
40	5	45		
× 7	× 7	× 7	A+B=	_____
___A	___B			

(b)
60	8	68		
× 3	× 3	× 3	A+B=	_____
___A	___B			

(c)
200	30	7	237		
× 4	× 4	× 4	× 4	A+B+C=	_____
___A	___B	___C			

(d)
400	80	5	485		
× 9	× 9	× 9	× 9	A+B+C=	_____
___A	___B	___C			

2. Look at (a), (b), (c) and (d) what pattern do you see? Write it down:

_____

_____

3. Complete these without your calculator

(a)
70	6		
× 5	×5	76×5=	_____
350	30		

(b)
200	40	6		
× 7	× 7	× 7	246×7=	_____
1400	280	42		

(c)
300	50	9		
× 6	× 6	× 6	359×6=	_____
1800	300	54		

(d)
800	70	2		
× 8	× 8	× 8	872×8=	_____
6400	560	16		

MS2 – Worksheet 2.2

Why did the chicken cross the road?

1. 24 (E)

× 6

24 (6×4)

120 (6×20)

144

2. 56 (W)

× 7

(7×6)

(7×50)

3. 38 (H)

×8

(8×8)

(8×30)

4. 53 (D)

× 5

5. 49 (R)

× 2

6. 44 (T)

×9

7. 236 (L)

× 4

24 (4×6)

120 (4×30)

800 (4×200)

8. 347 (O)

× 5

(5×7)

(5×40)

(5×300)

9. 683 (I)

× 3

(3×3)

(3×80)

(3×600)

10. 284 (S)

× 4

11. 629 (A)

× 8

12. 844 (K)

× 9

396	1735		392	5032	944	7596		396	1735		396	304	144
1735	396	304	144	98				1126	2049	265	144		

MS2 – Games

Separation Multiplication Tic-Tac-Toe

Materials: Playing boards, unifix of one colour for each player

Number of Players: 2

Directions:

1) Players in turn choose a number from top row and a number from bottom row.
2) Players multiply the numbers and cover the answer with unifix.
3) First player to cover 3 in a row, column or diagonal wins.

64	37	48
	×	
6	8	7

222	336	296
288	384	512
448	259	384

Separation Mix and Match

Materials: Mix and Match cards (attached)

Number of Players: 1 (or a group)

Directions:

1) Print all cards in same colour.
2) Cut cards along lines into pieces.
3) Mix pieces together.
4) Students put pieces back together to form cards.

MS2 – Separation Multiplication Tic-Tac-Toe games

Game 1

$64	$37	$48
	×	
6	8	7

$222	$336	$296
$288	$384	$512
$448	$259	$384

Game 2

$78	$55	$63
	×	
7	9	5

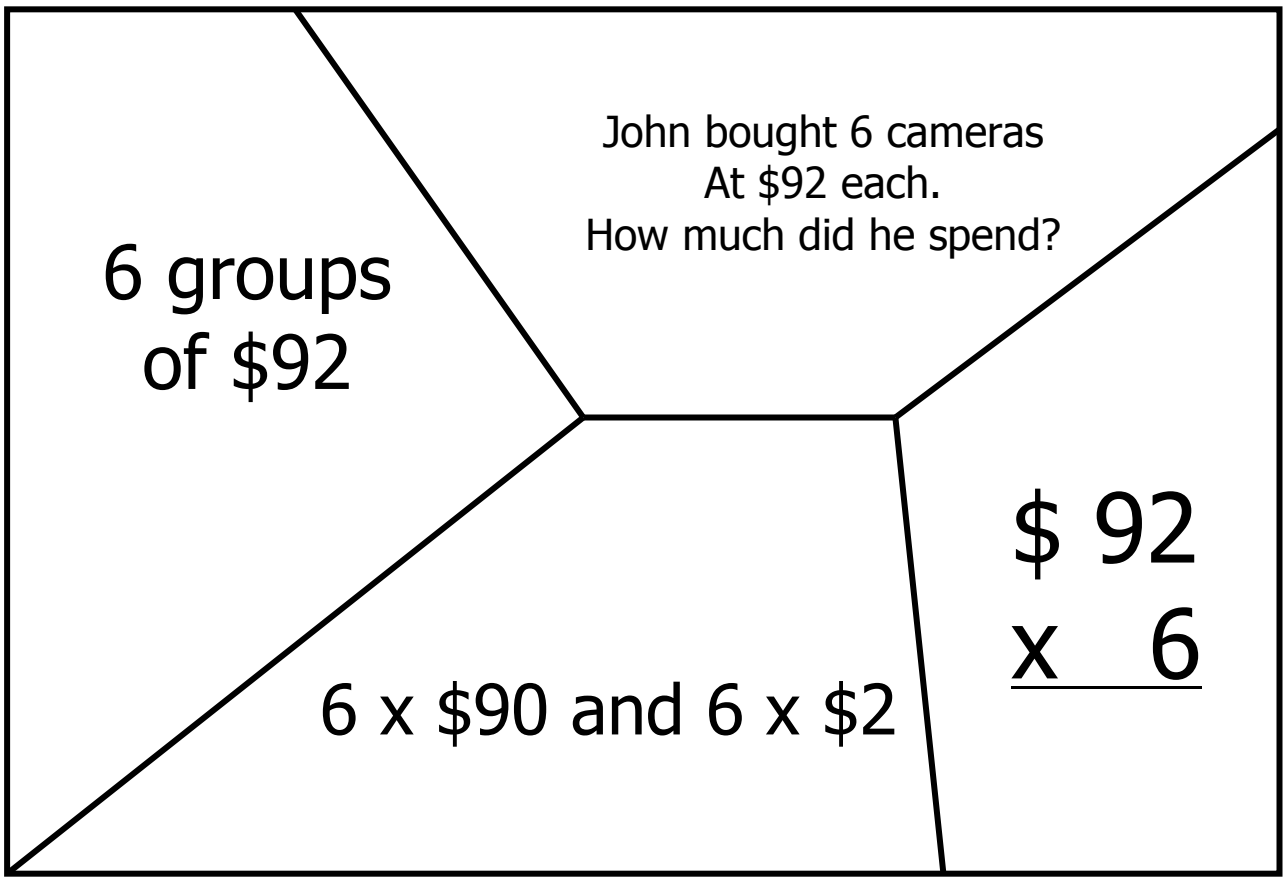
$441	$275	$702
$546	$385	$495
$315	$390	$567

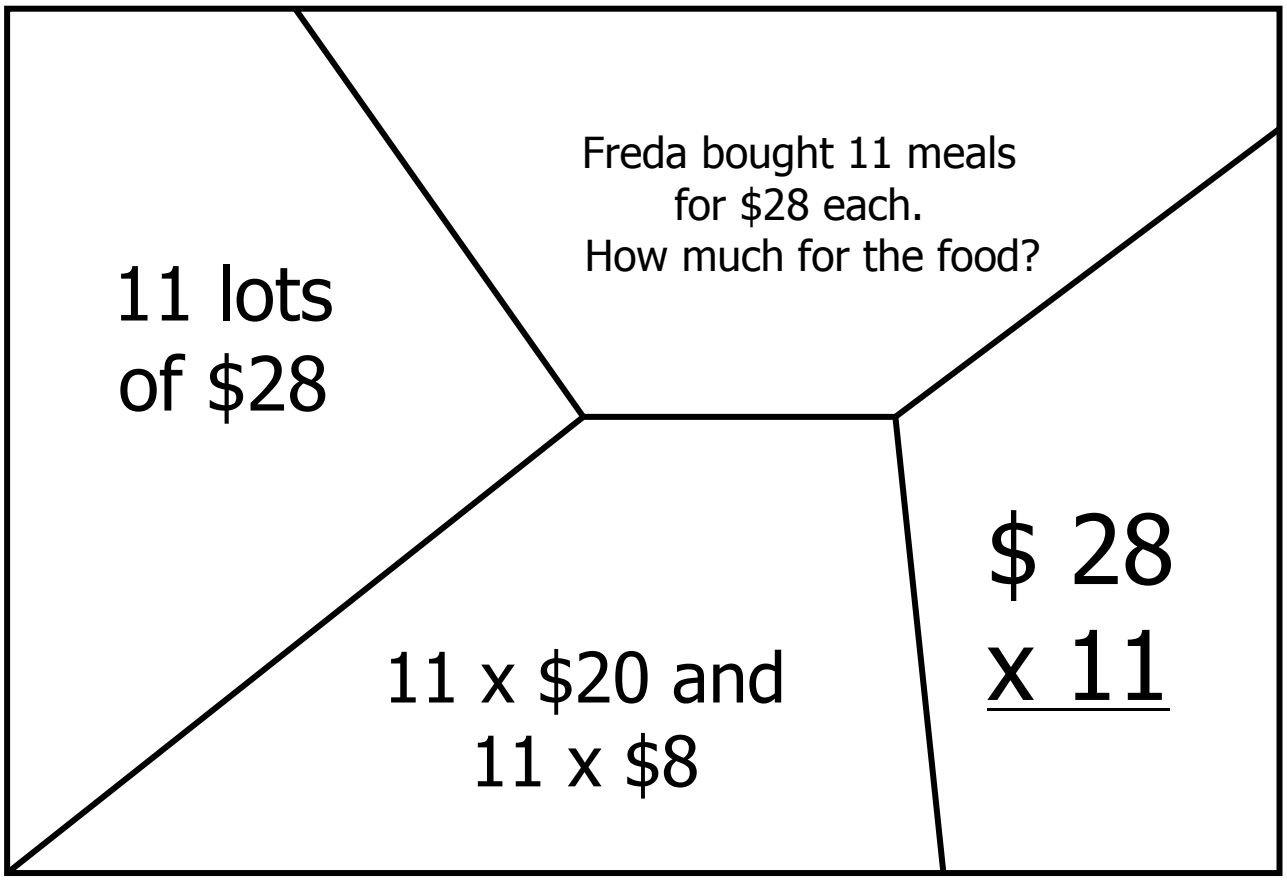
Game 3

$49	$81	$68
	×	
8	7	9

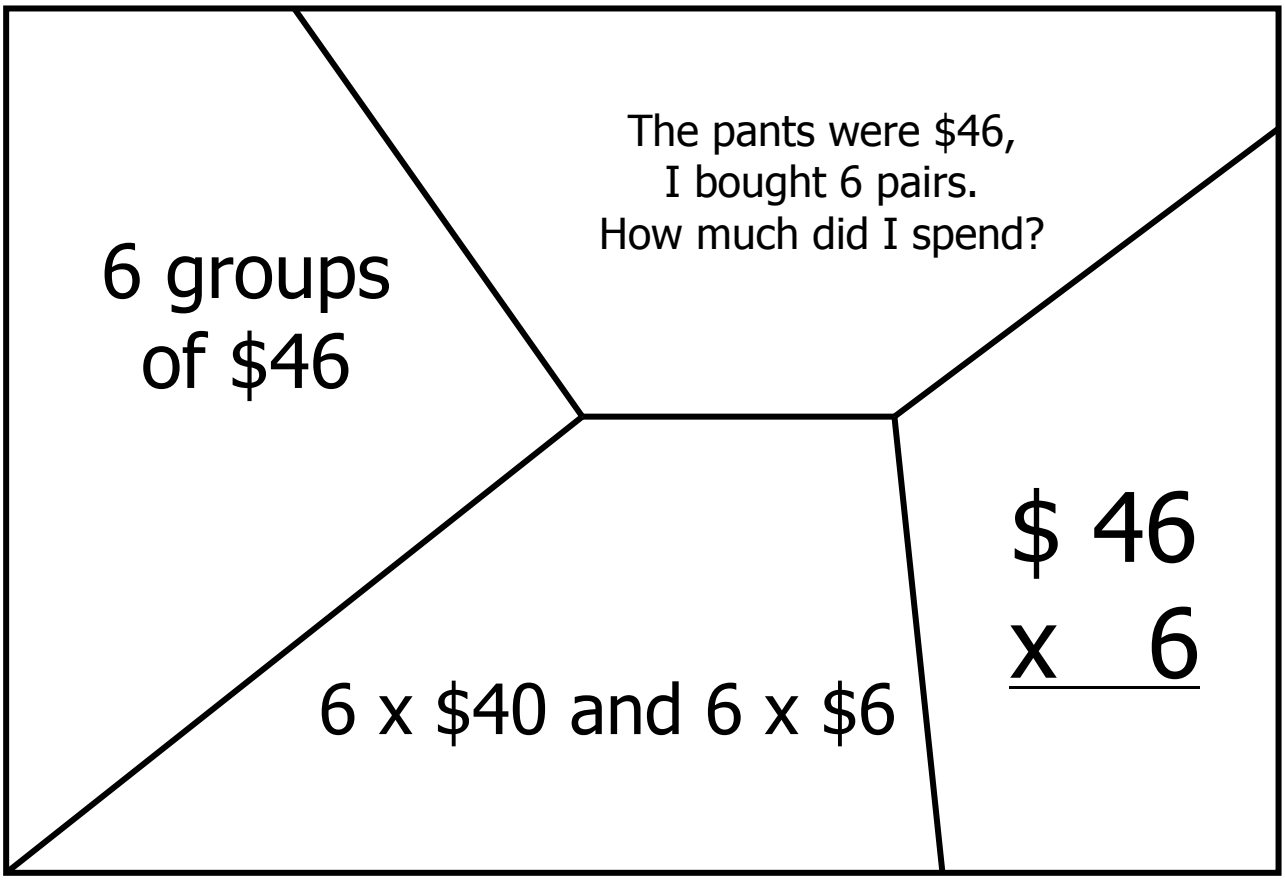
$648	$544	$343
$441	$392	$729
$476	$567	$612

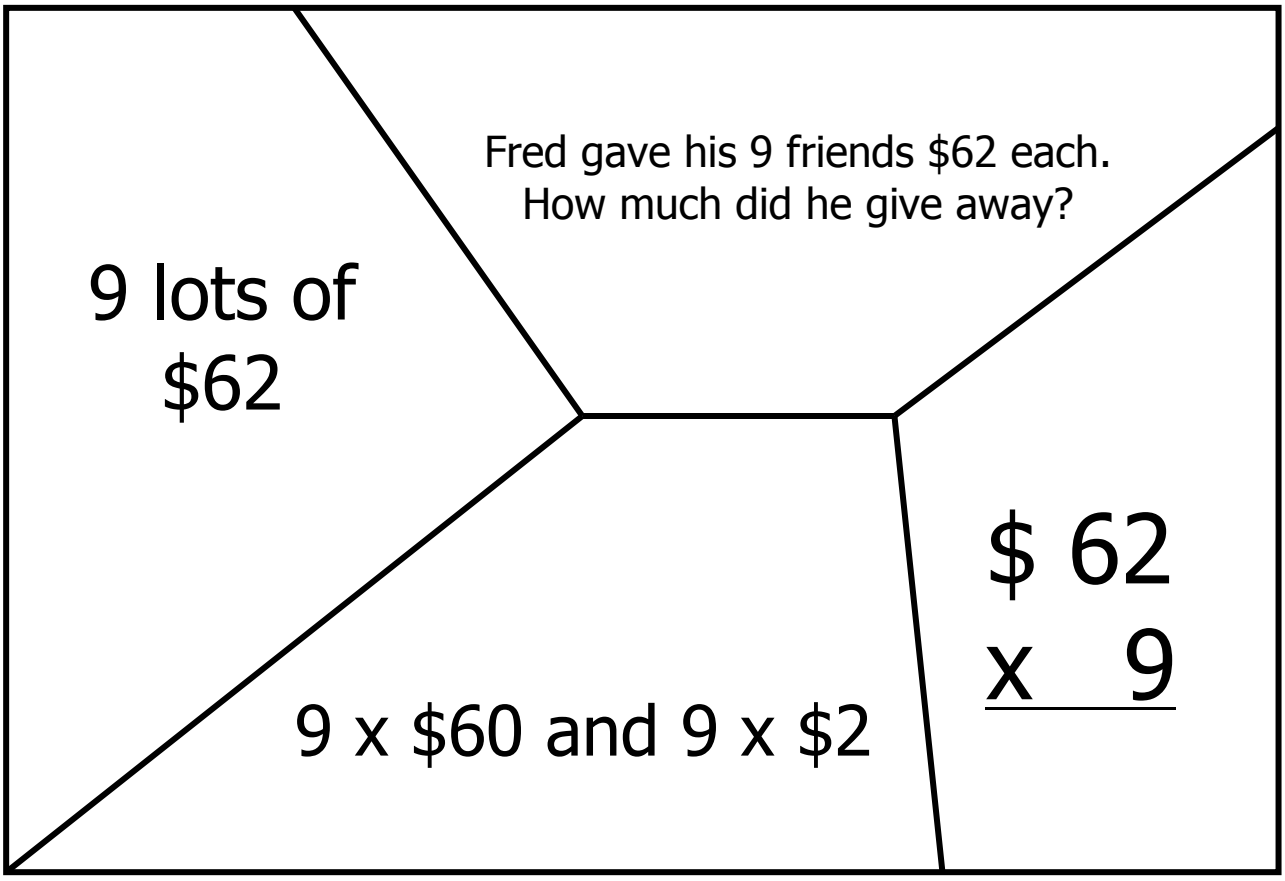
MS2 – Separation Multiplication Mix & Match Cards



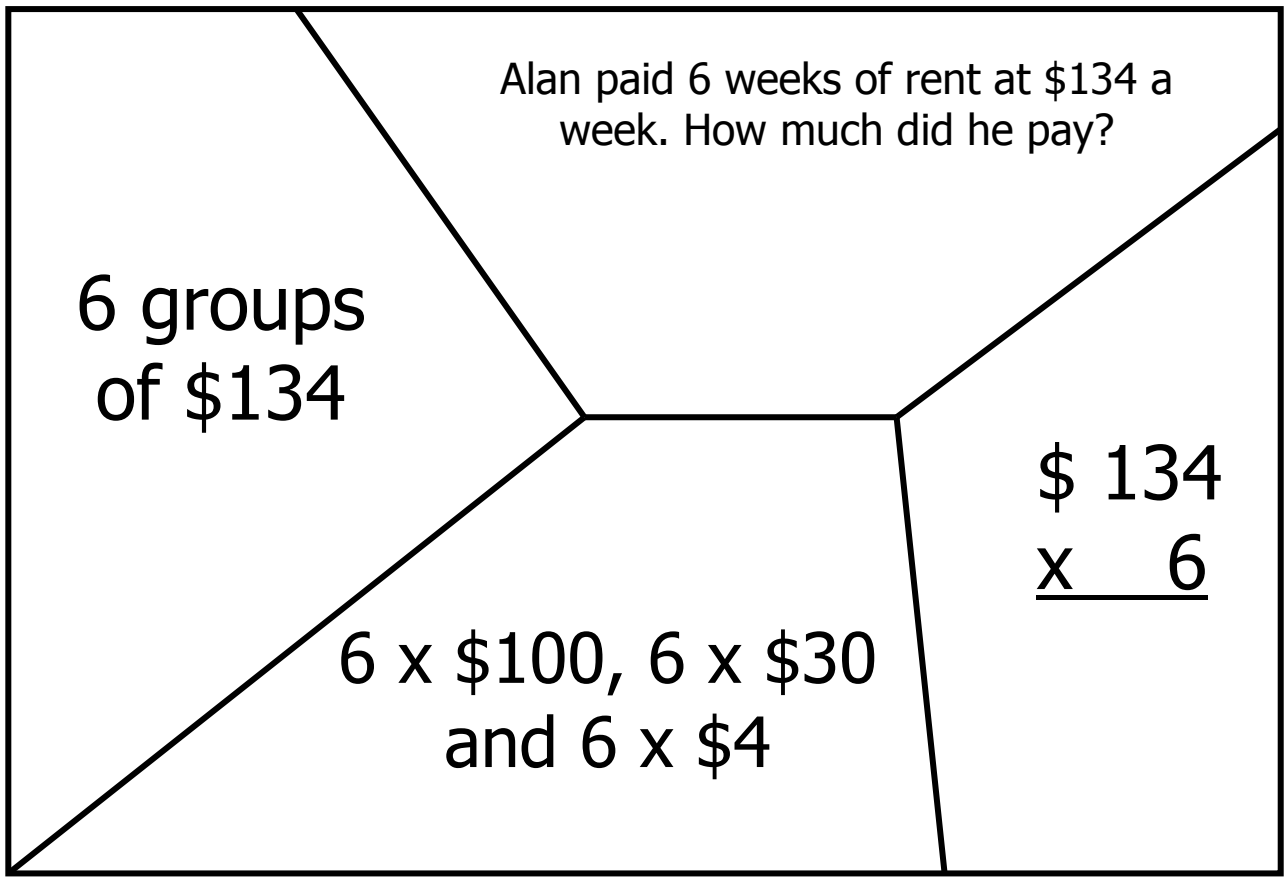


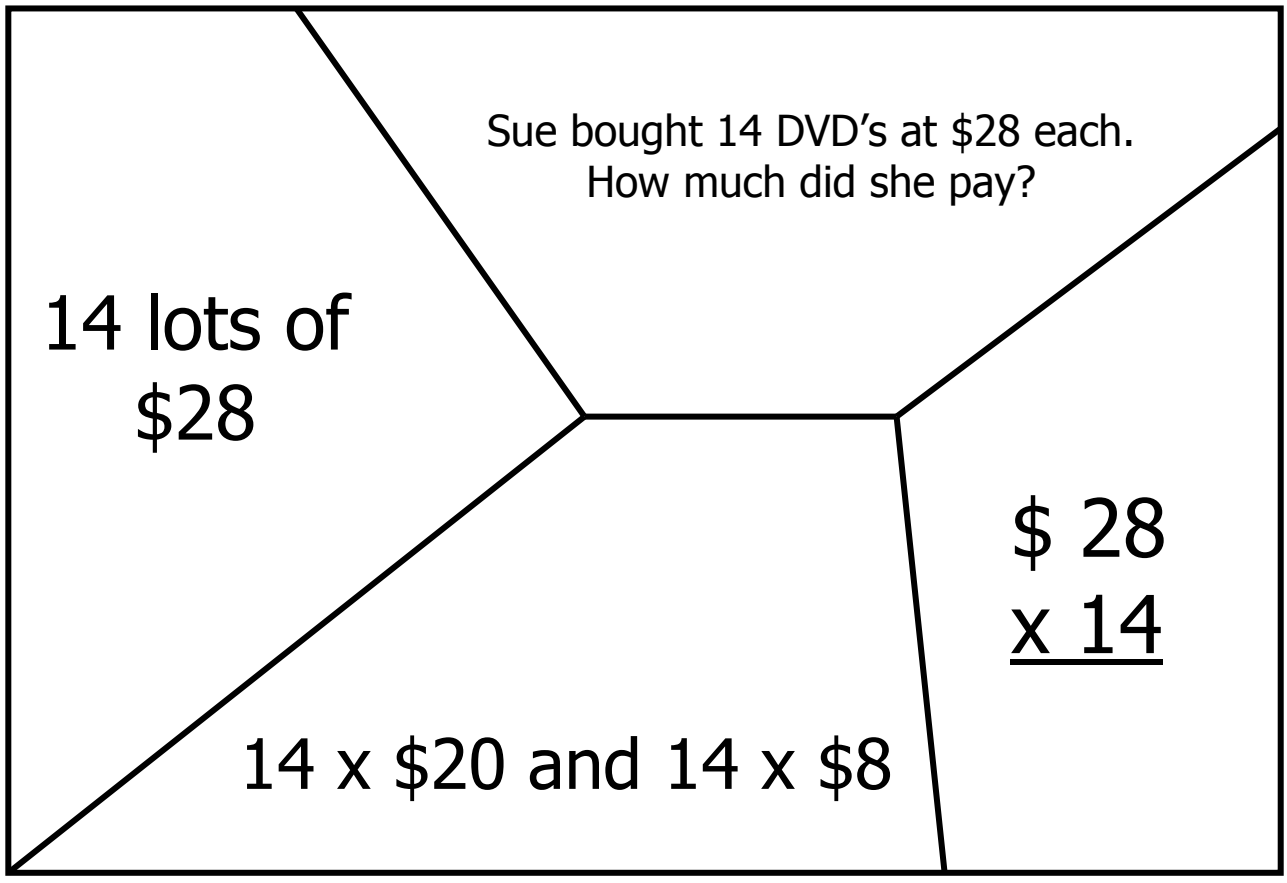
MS2 – Separation Multiplication Mix & Match Cards (continued)



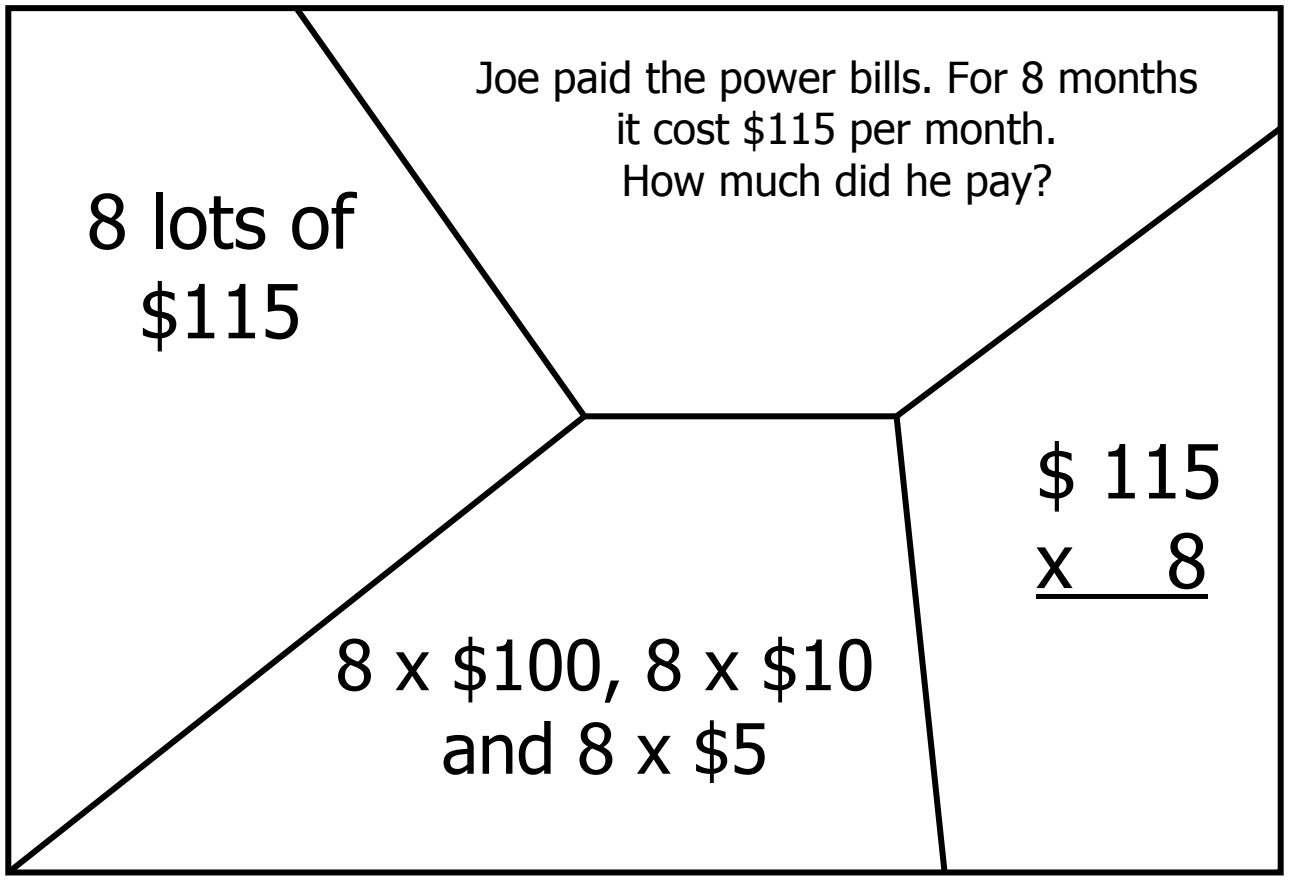


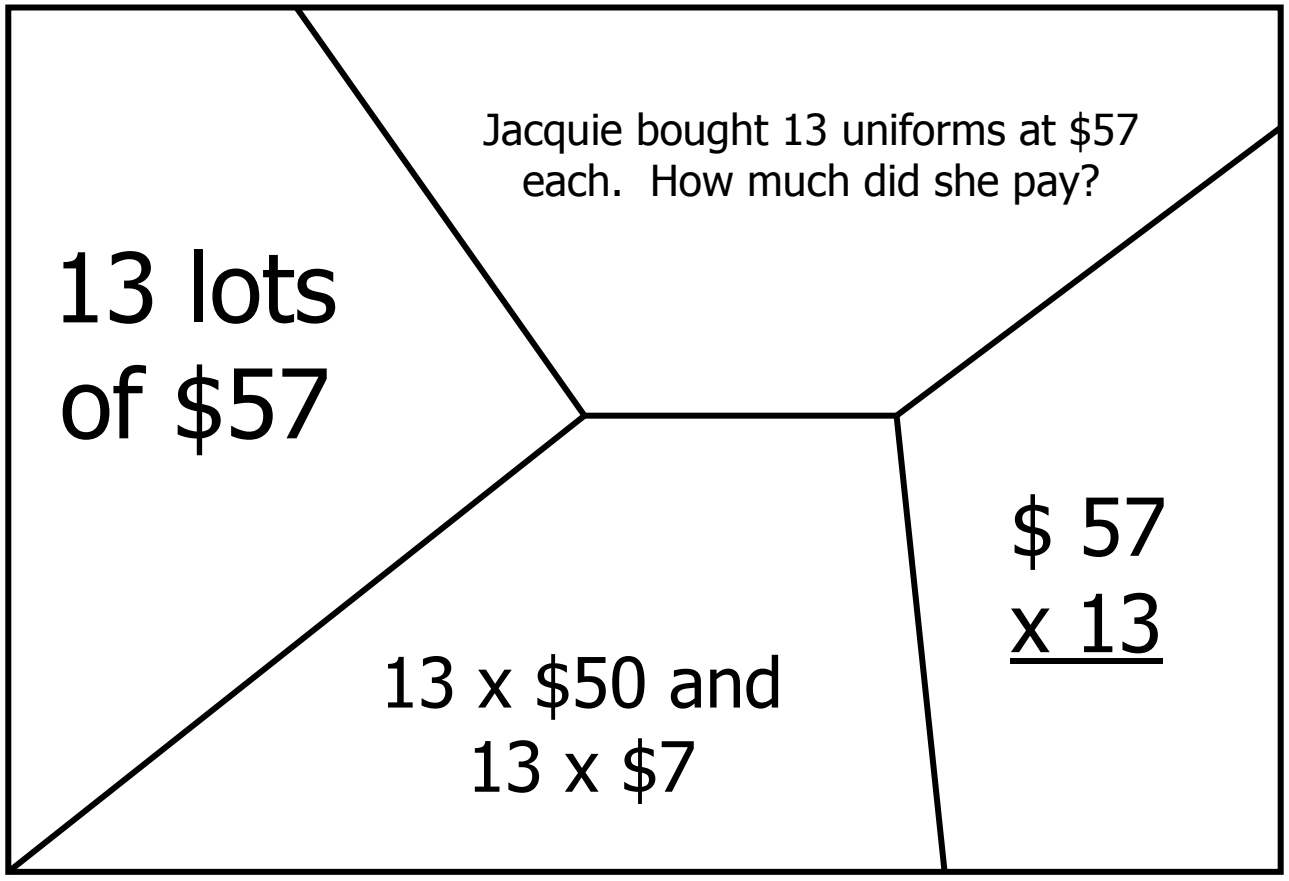
MS2 – Separation Multiplication Mix & Match Cards (continued)





MS2 – Separation Multiplication Mix & Match Cards (continued)





MS2 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning

2. How did you feel about trialling the activity?

   Mark the line with an X: Unconfident ——— Very confident

3. Do you think the student was engaged in the activity? Explain.

   _____

4. What do you think the student learnt from the activity?

   _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

   _____

6. What do you think of the activity?

   _____

7. What are your suggestions for improving the activity?

   _____

   _____

8. What else do you suggest could be done to help students who have trouble with this activity?

   _____

   _____

ACTIVITY MS3

[Sequencing Strategy for Multiplication Computation]

Materials: Graph paper (2mm), pen, paper, calculator, attached resources

Directions:

1. Multiplication as rectangular arrays

   Hand out 2mm graph paper. Ask the students to draw rectangles to enclose the following squares:

   (a) 4×3 (b) 8×7 (c) 3×24 (d) 6×58

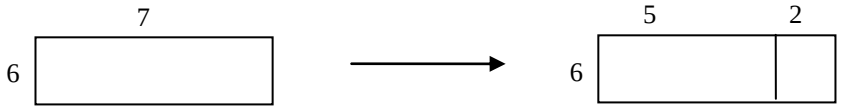
   Draw diagrams to show larger numbers:

   (a) 8×87 (b) 9×234 (c) 23×642

   Discuss how multiplication can be represented by rectangles.

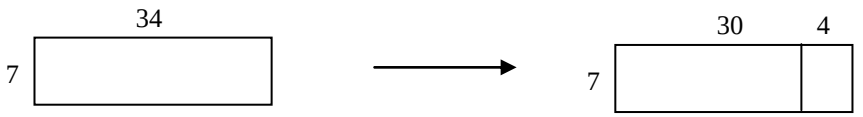
2. Distributive Law

   Discuss how rectangles can be broken up. Draw 6×7 on rectangle. Look at what happens when break 7 into 5 and 2. Discuss how it breaks multiplication 6×7 into 6×5 and 6×2.



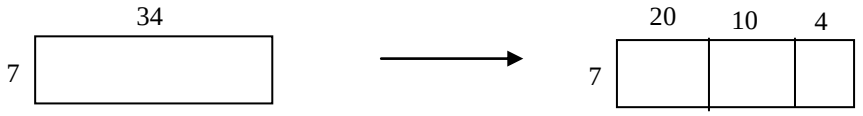
   Look at 7×34 and discuss ways each number could be broken up for example:

   (a)



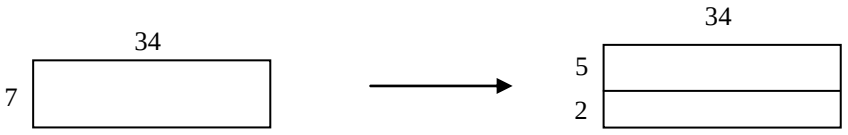
7×34
= 7×30
+ 7×4

   (b)



7×34
= 7×20
+ 7×10
+ 7×4

   (c)



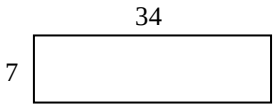
7×34
= 5×34
+ 2×34

   Obviously when the 34 side of the rectangle is broken up by place value (as in example (a) or example (b)), we are looking at separation. For sequencing, we look at breaking up the 1-digit number (as in example (c)). We will focus on sequencing in this activity (but will include separations).

3. Applying Distributive Law to computation

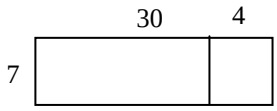
   Ask: *We are going to solve multiplications by thinking arrays and breaking up one number. Look at example 7×34.*

Step 1 *Make a drawing*

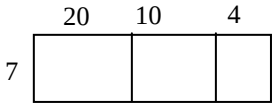


Step 2 *Break up one of the numbers, e.g.:*

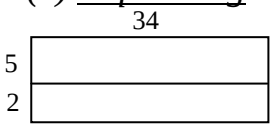
(a) separation



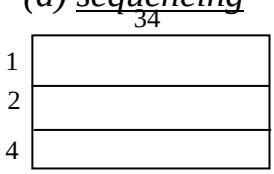
(b) separation



(c) sequencing



(d) sequencing



Step 3 *Consider the multiplication as a number of parts and add them:*

(a) separation

34
×7
210 (7×30)
28 (7×4)
238

(b) separation

34
× 7
140 (7×20)
70 (7×10)
28 (7×4)
238

(c) sequencing

34
× 7
170 (5×34)
68 (2×34)
238

(d) sequencing

34
× 7
34 (1×34)
68 (2×34)
136 (4×34)
238

4. Good/easy multiplications

Ask students: *We could have broken 34 into 17 and 17, and 7 into 3 and 4. Why did we choose 30 and 4 for 34 and 5 and 2 for 7?* State: *Answer is that these are easy.* Ask: *Why are they easy and what other numbers are easy?*

After discussion, elicit the following:

Answer (a) Multiples of 10 are easy

7×30 = (7×3) tens = 21 tens = 210

(b) 1, 2, 4 and 8 are easy

1×34 =34

2×34 = double 34 = 68

4×34 = double double 34 = double 68 =136

8×34 = double double double 34 = double double 68 = double 136 = 272

(c) 10 and 5 are easy

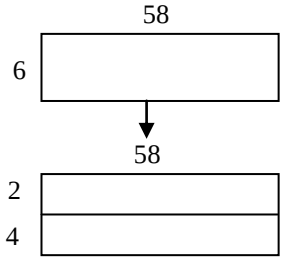
10×34 = 340

5×34 = ½ (10×34) = 120

5. Sequencing strategy for multiplication

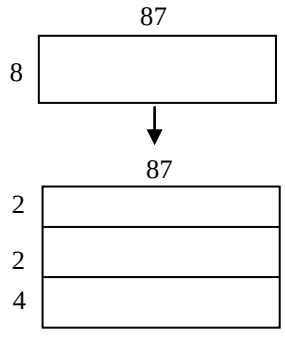
Ask: *In 7×34, the separation strategy involves breaking up the 34 and the sequencing strategy involves breaking up the 7. Let's use arrays to see what happens for some examples.* Repeat step 3 for a variety of numbers, discussing different ways of breaking things up by sequencing

(a) 6×58



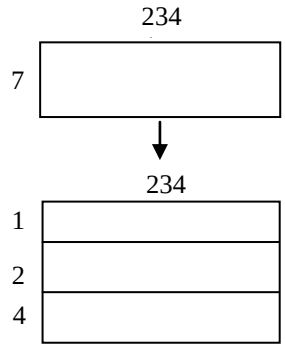
× 6
116 (2×58)
232 (4×58)
348

(b) 8×87



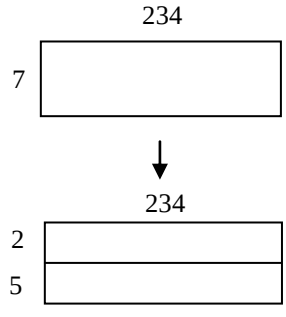
87
× 8
174 (2×87)
174 (2×87)
348 (4×87)
696

(c) 7×234



234
× 7
234 (1×234)
468 (2×234)
936 (4×234)
1638

OR

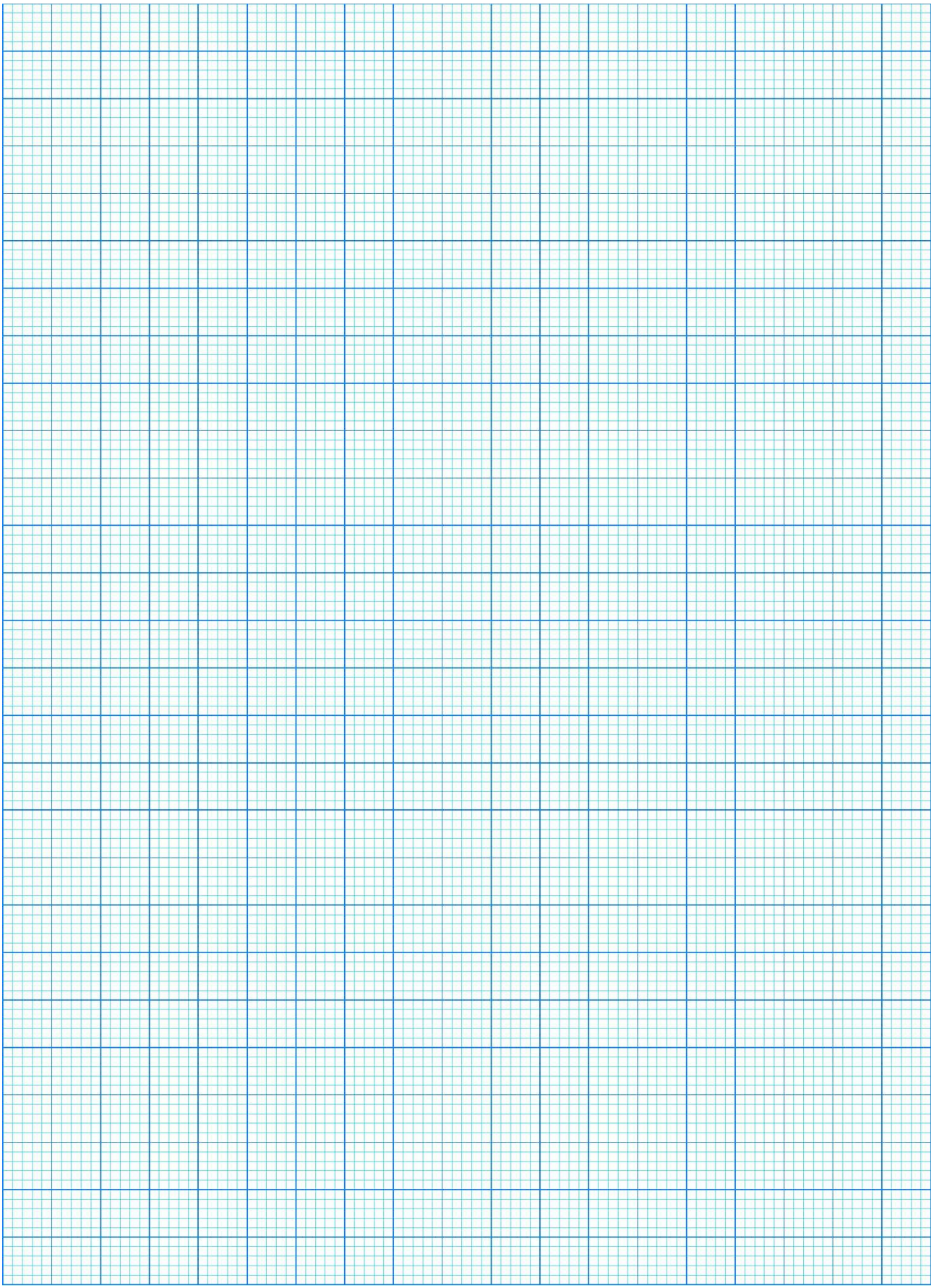


234
× 7
468 (1×234)
1170 (2×234)
1638 *(4×234)*

6. Complete Worksheets 3.1 and 3.2.

7. Play games: "Sequence Multiplication Snap", "Sequence Multiplication Rummy", "Sequence Multiplication Concentration", "Sequence Multiplication Cover the Board", "Sequence Multiplication Mix-and Match", and "Sequence Multiplication Mix-and Match Bingo".

MS3 – 2mm Graph Paper



MS3 – Worksheet 3.1

[Sequencing and separation use of breaking up rectangles]

Complete the drawings. The first one has been done for you.

1. PROBLEM	*SEPARATION WAYS*	*SEQUENCING WAYS*
53 5 ☐	50 3 5 ☐ ☐ 20 20 10 3 5 ☐ ☐ ☐ ☐	53 1 ☐ 2 ☐ 2 ☐ 53 1 ☐ 4 ☐
2. 28 6 ☐		
3. 86 3 ☐		
4. 32 9 ☐		
5. 48 7 ☐		
6. 347 8 ☐		

MS3 – Worksheet 3.2

Complete the following by sequencing. The first one has been done for you.

PROBLEM	*ARRAY*	*WRITTEN ALGORITHM*
1. *6×48*	48 1 | 5 |	48 × 6 48 (1×48) 240 (5×48) (½10×48) 288
2. *3×74*		
3. *9×93*		
4. *5×128*		
5. *6×329*		
6. *7×807*		

MS3 - Multiplication Computation Card Games

Snap, Rummy and Concentration

Materials: Five pages of pictures following these instructions (equation, array, algorithm, partitioned array, extended algorithm).

Number of players: 2-4

Directions:

- Print the five pages in 5 different colours, cut each page into 12 cards (making 60 cards of 5 different colours) for Snap, Rummy and Concentration. Shuffle the cards. Follow instructions for these games:
- **Snap**. Two players, cards divided equally between players. Together, the two players play the top card of their deck face up in front of them. First to call snap when two cards show the same number wins a point. The player with the most points wins when all cards played.
- **Rummy**. Two to four players, deal out 7 cards to each player. Remaining cards face down in middle with one card face-up beside deck. Players put out any doubles or triples. Players in turn pick up a card (either the top face-up card or face-down card) and place a card face-up on the face-up pile. Doubles, triples, quadruples, and quintuples are put out as they are formed. The winner is the first player to put out all their cards as doubles, triples, quadruples or quintuples.
- **Concentration**. Two to three players or two groups of players. Place all cards face down on table. Players take turns selecting 2 cards. If they are the same, keep the pair and take another turn. The winner is the player with the most pairs when all cards used.

2-D × 1-D Multiplication Cover-the-Board

Materials: Same materials as for card games except the symbols page is kept as a base board, while the other 4 pages are each cut into 12 picture cards.

Number of players: 2-5

Directions:

1) Print the five pages from the above card games, each on different coloured paper or light card. Laminate the equation page to use as a base board.
2) Cut the other 4 pages into 12 cards each.
3) Each player gets a set of cards.
4) In turn, each player places a card correctly on the base board (card and board have to display same number) or on top of another card already placed.
5) At the end when all cards played, the player with most cards on top wins.

Multiplication Computation Mix & Match Cards

Materials: 12 mix and match cards (two per page for the next 6 pages)

Number of players: 1 (though can be a group)

Directions:

1) Print all cards in same colour.
2) Cut cards along lines into pieces.
3) Mix pieces together.
4) Students put pieces back together to form cards.

Multiplication Computation Bingo Game

Materials: 7 pages of material following these instructions – one set of equation flash cards and six bingo base boards, unifix cubes.

Number of players: 2-6

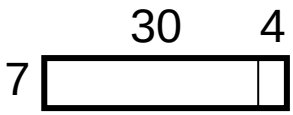
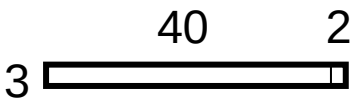
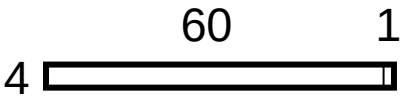
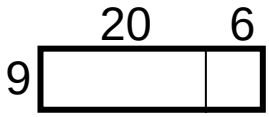
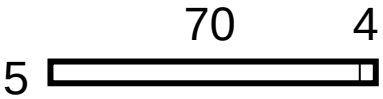
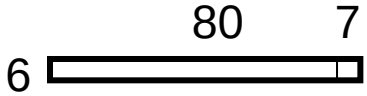
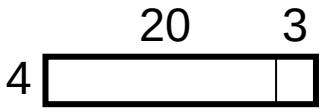
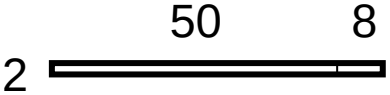
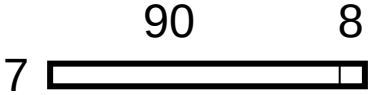
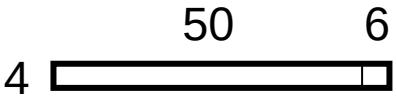
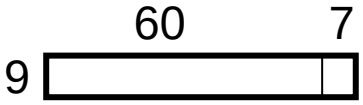
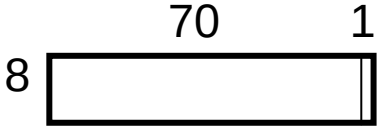
Directions:

1) Print the flash cards (equations) on white paper or card, laminate and cut out.
2) Print the six bingo base boards, each on different coloured paper or card (laminate if possible).
3) One player (caller) takes the flash cards and shuffles them, other players take a base board and unifix cubes.
4) Caller shows cards one at a time.
5) Players cover same operation on their board with unifix cube.
6) First player to get 3 in a row (across, down or diagonal) is the winner (calls "bingo") and becomes caller in next game.

MS3 – Multiplication Computation Cards (equation)

$7 \times 34 = 238$	$3 \times 42 = 126$	$4 \times 61 = 244$
$9 \times 26 = 234$	$5 \times 74 = 370$	$6 \times 87 = 522$
$4 \times 23 = 92$	$2 \times 58 = 116$	$7 \times 98 = 686$
$4 \times 56 = 224$	$9 \times 67 = 603$	$8 \times 71 = 568$

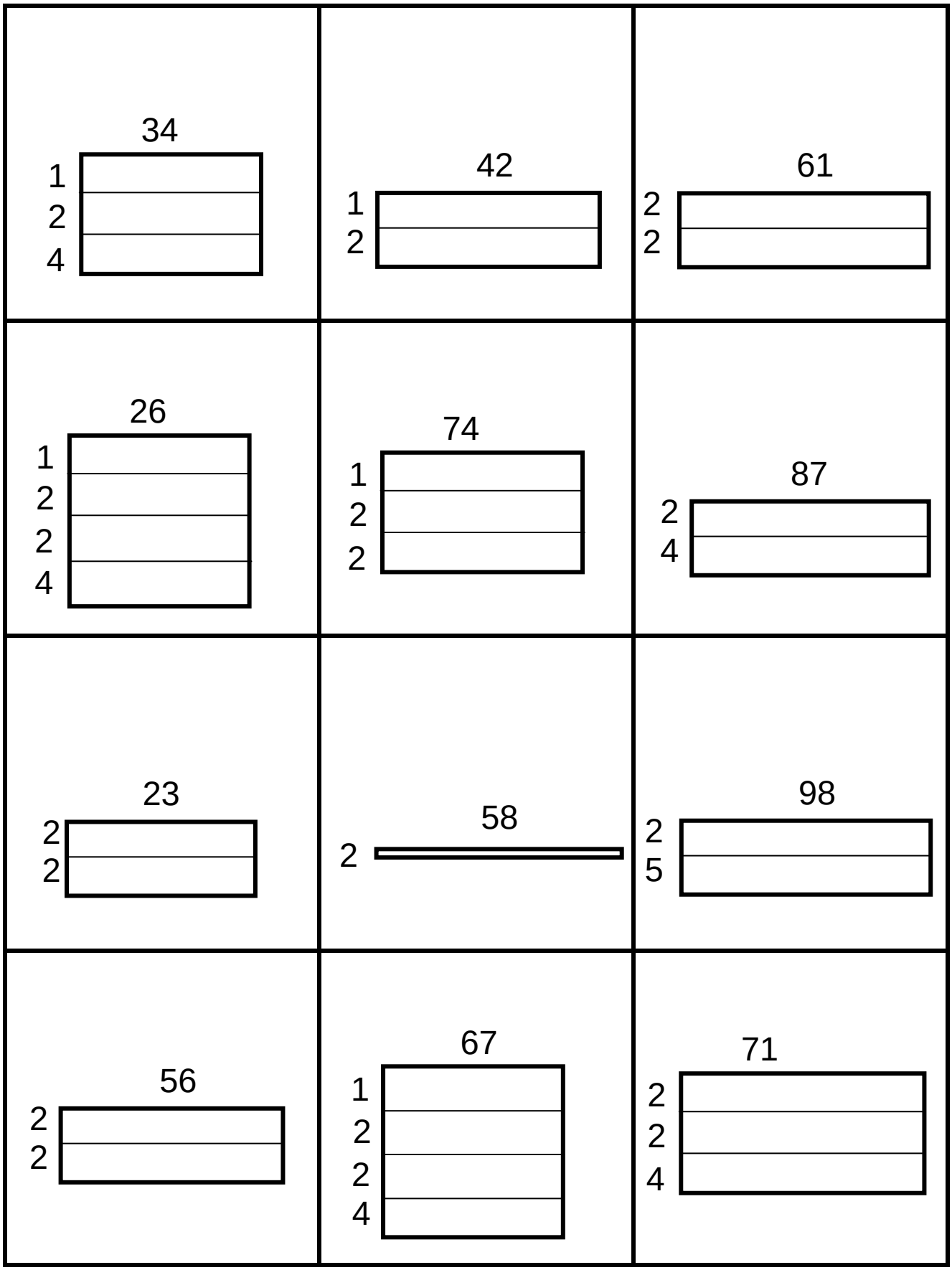
MS3 – Multiplication Computation Cards (array)

MS3 – Multiplication Computation Cards (algorithm)

34 x 7 28 210 238	42 x 3 6 120 126	61 x 4 4 240 244
26 x 9 54 180 234	74 x 5 20 350 370	87 x 6 42 480 522
23 x 4 12 80 92	58 x 2 16 100 116	98 x 7 56 630 686
56 x 4 24 200 224	67 x 9 63 540 603	71 x 8 8 560 568

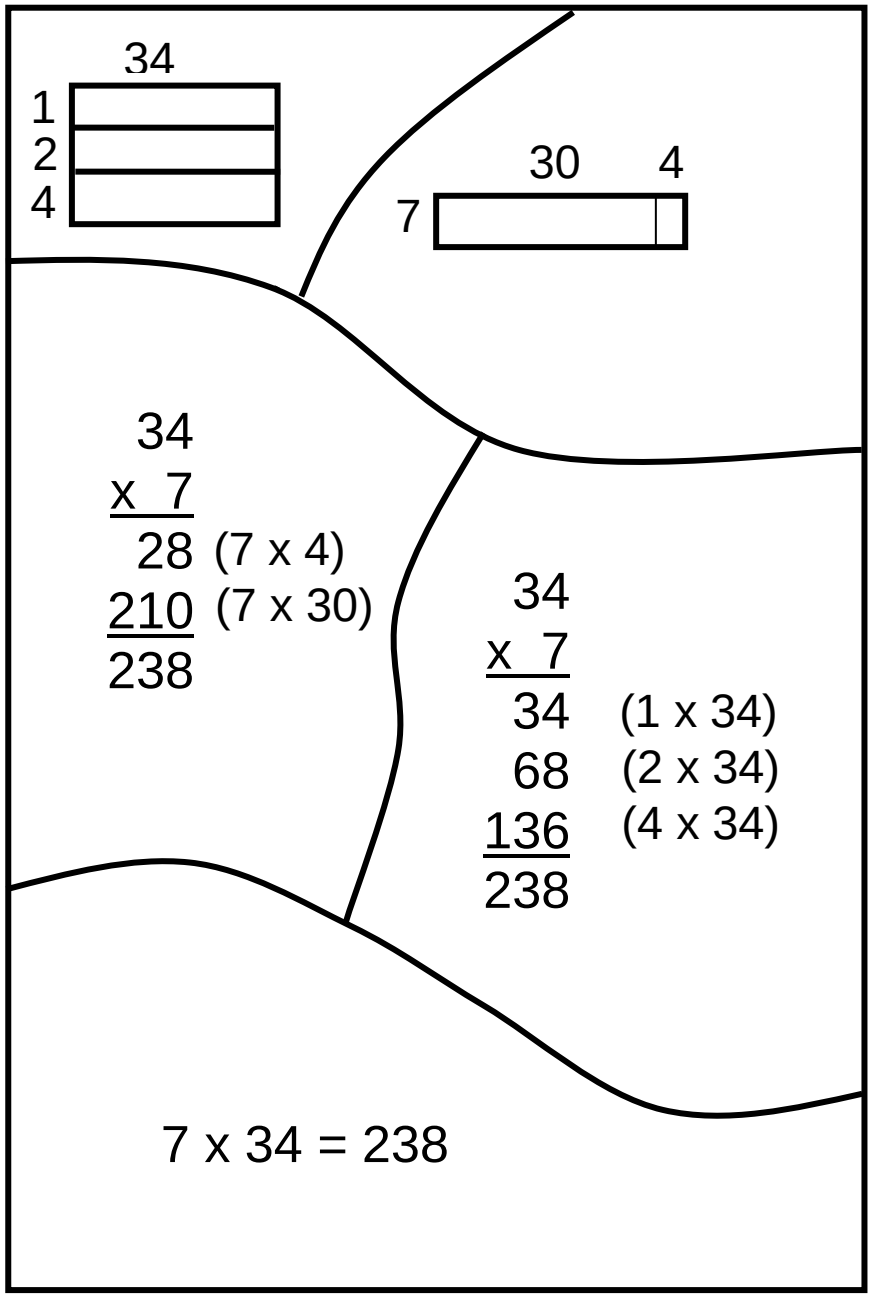
MS3 – Multiplication Computation Cards (partitioned array)

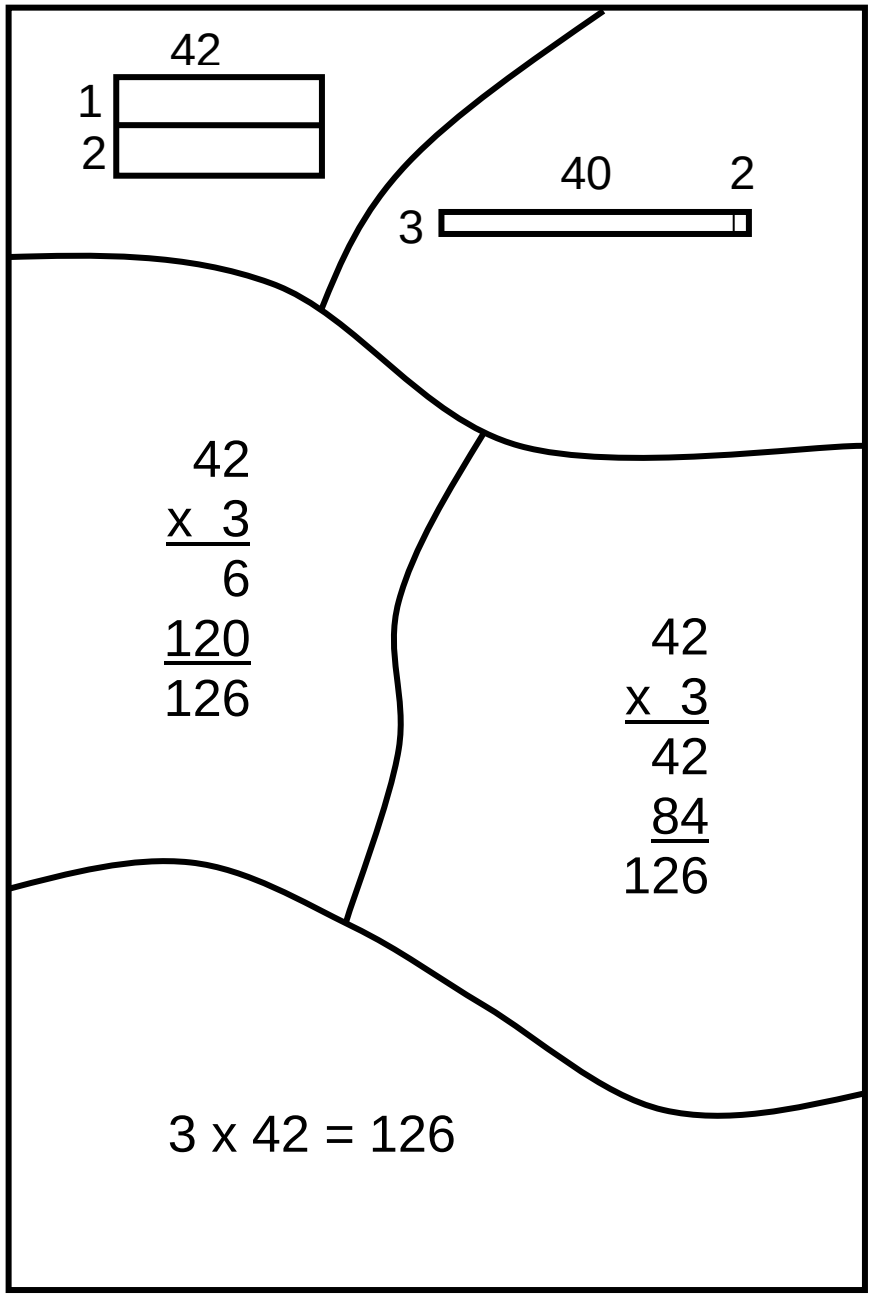


MS3 – Multiplication Computation Cards (extended algorithm)

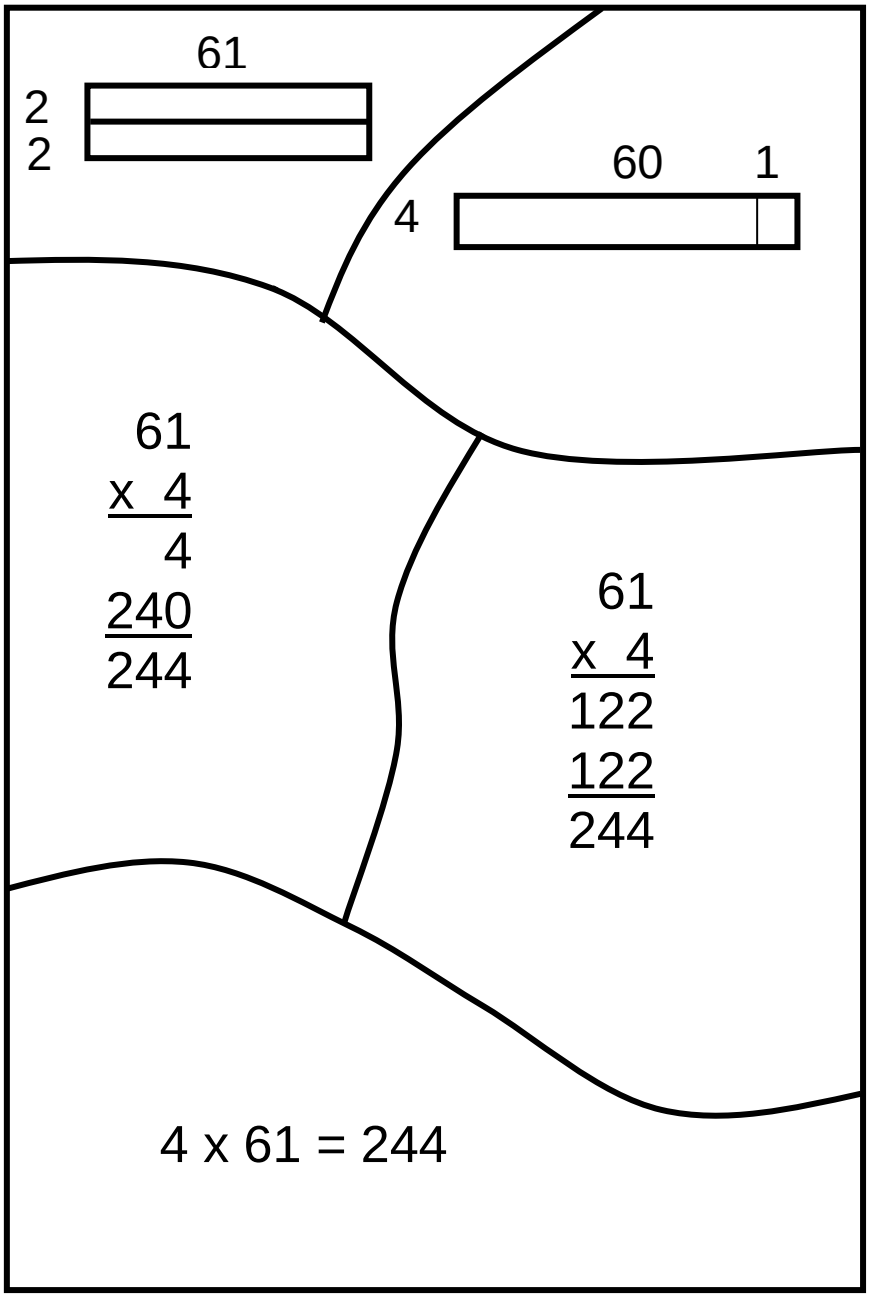
34 x 7 34 (1x34) 68 (2x34) 136(4x34) 238	42 x 3 42 (1x42) 84 (2x42) 126	61 x 4 122 (2x61) 122 (2x61) 244
26 x 9 26 (1x26) 52 (2x26) 52 (2x26) 104 (4x26) 234	74 x 5 74 (1x74) 148 (2x74) 148 (2x74) 370	87 X 6 174 (2x87) 480 (4x87) 522
23 x 4 46 (2x23) 46 (2x23) 92	58 x 2 116 (2x58)	98 x 7 196 (2x98) 490 (5x98) 686
56 x 4 112 (2x56) 112 (2x56) 224	67 x 9 67 (1x67) 134 (2x67) 134 (2x67) 268 (4x67) 603	71 x 8 142 (2x71) 142 (2x71) 284 568

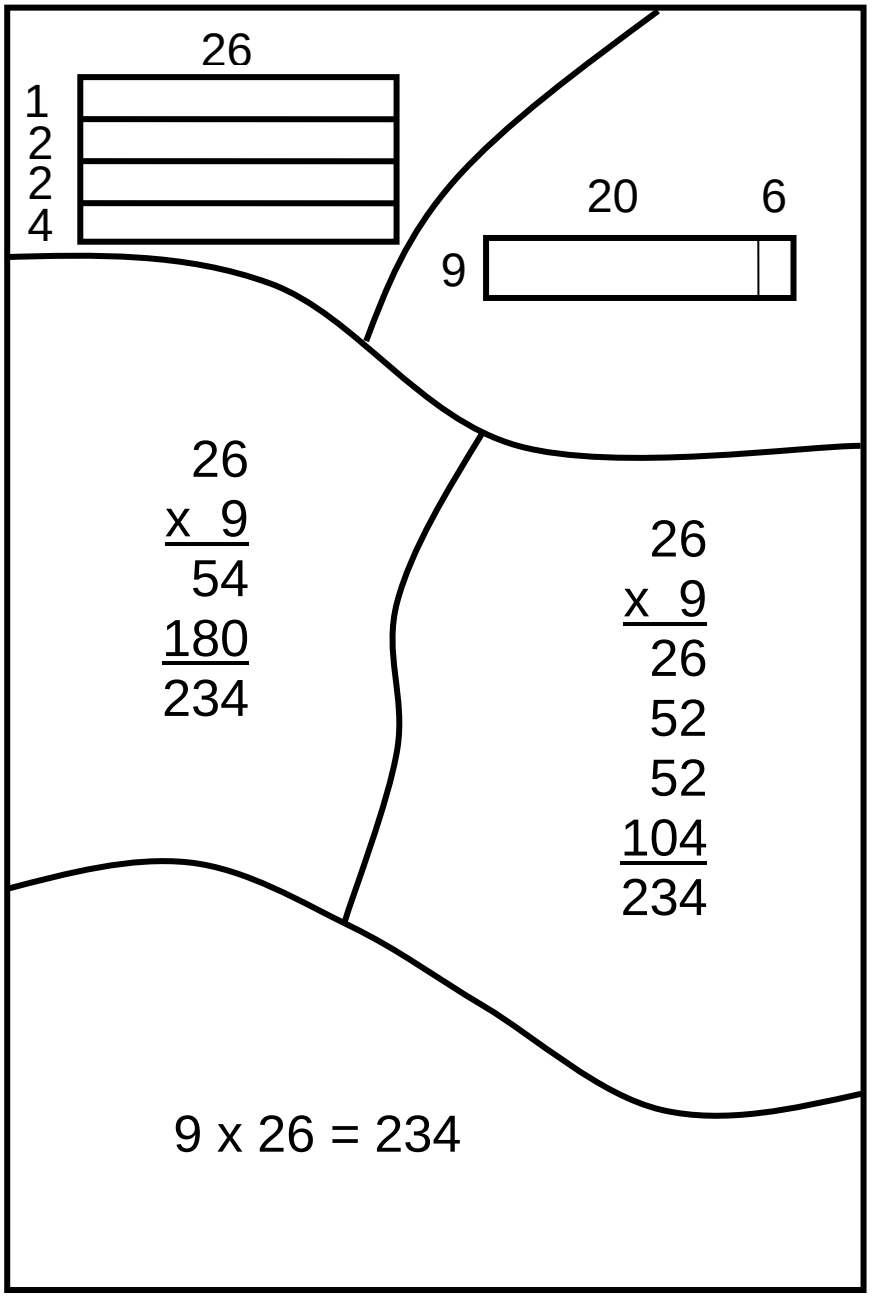
MS3 - Multiplication Computation Mix and Match Cards



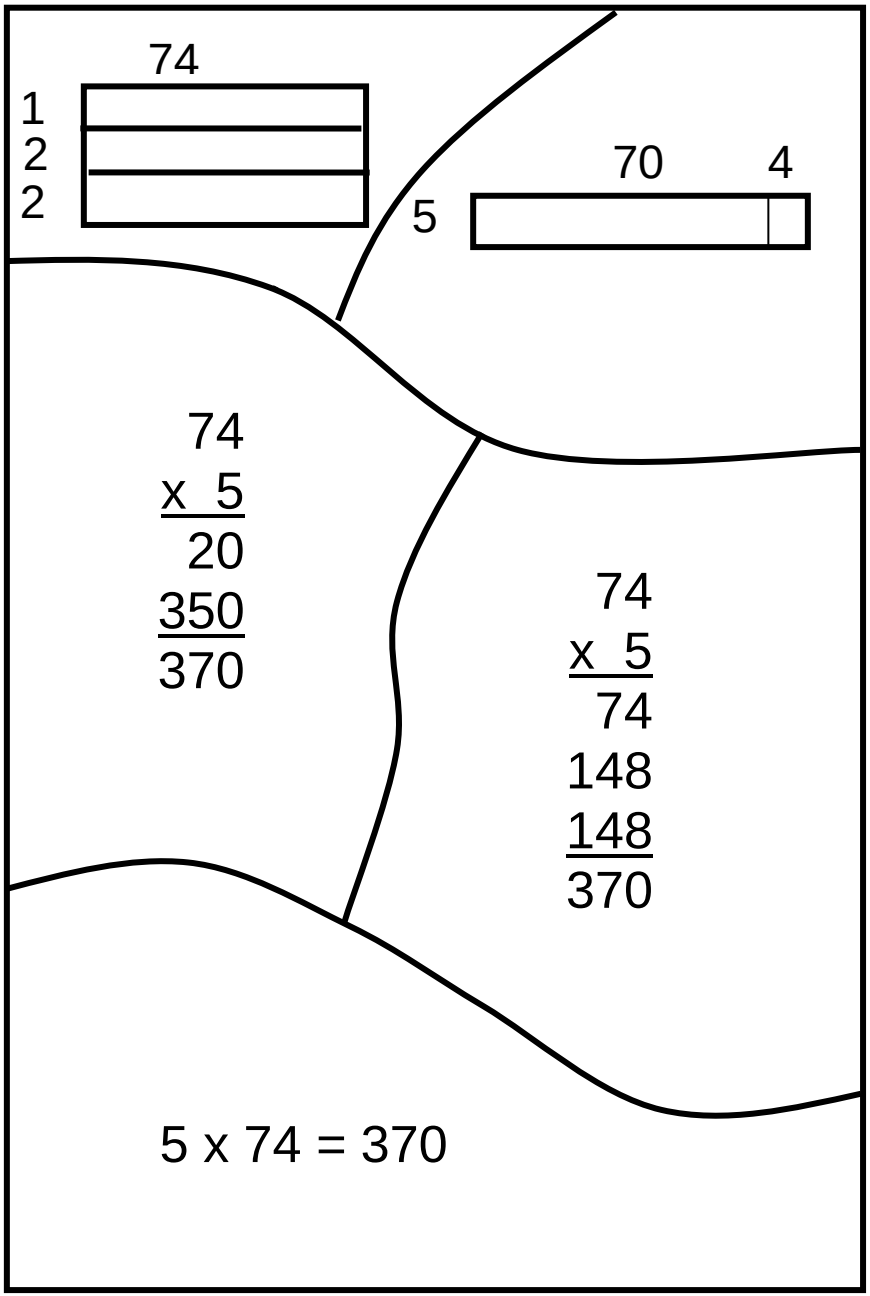


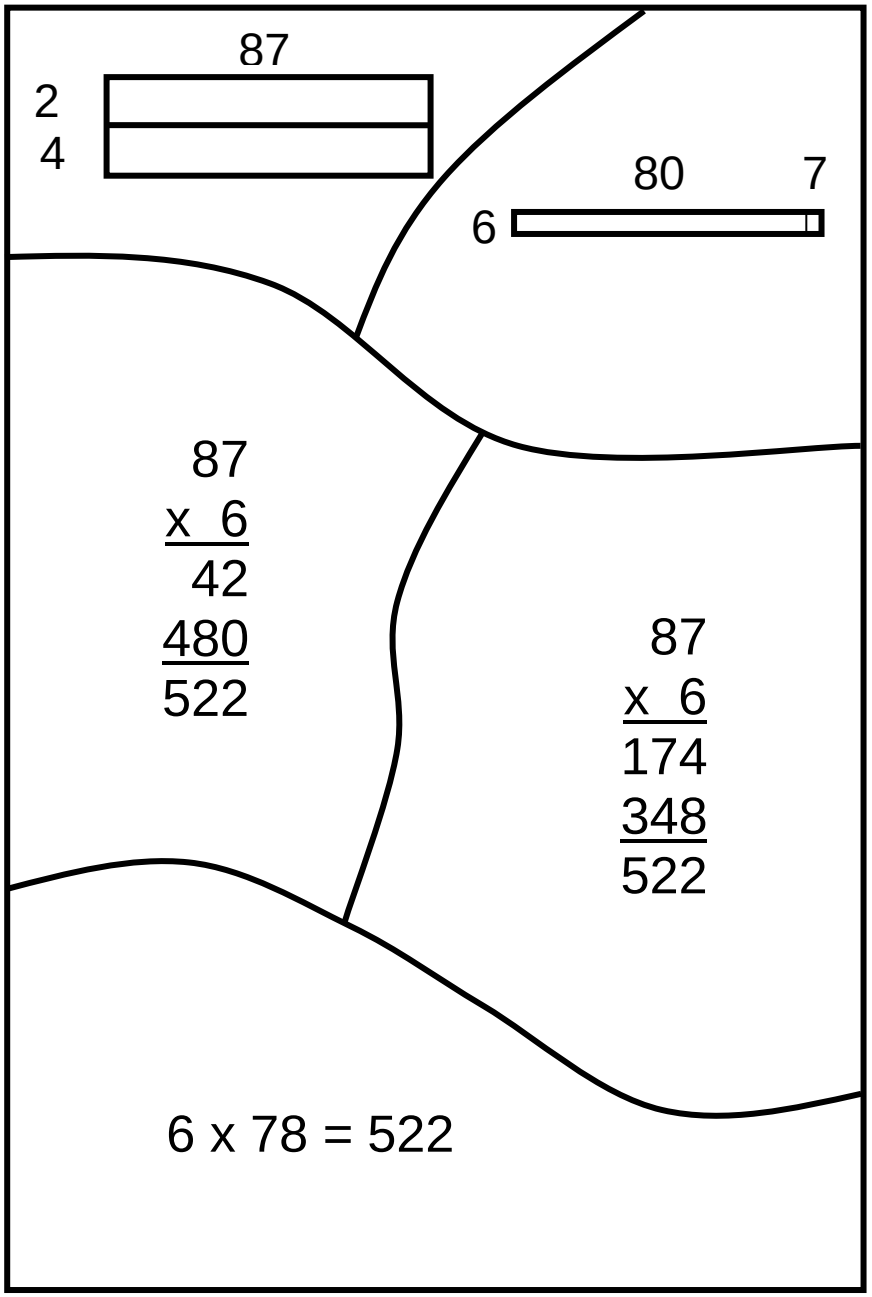
MS3 - Multiplication Computation Mix and Match Cards



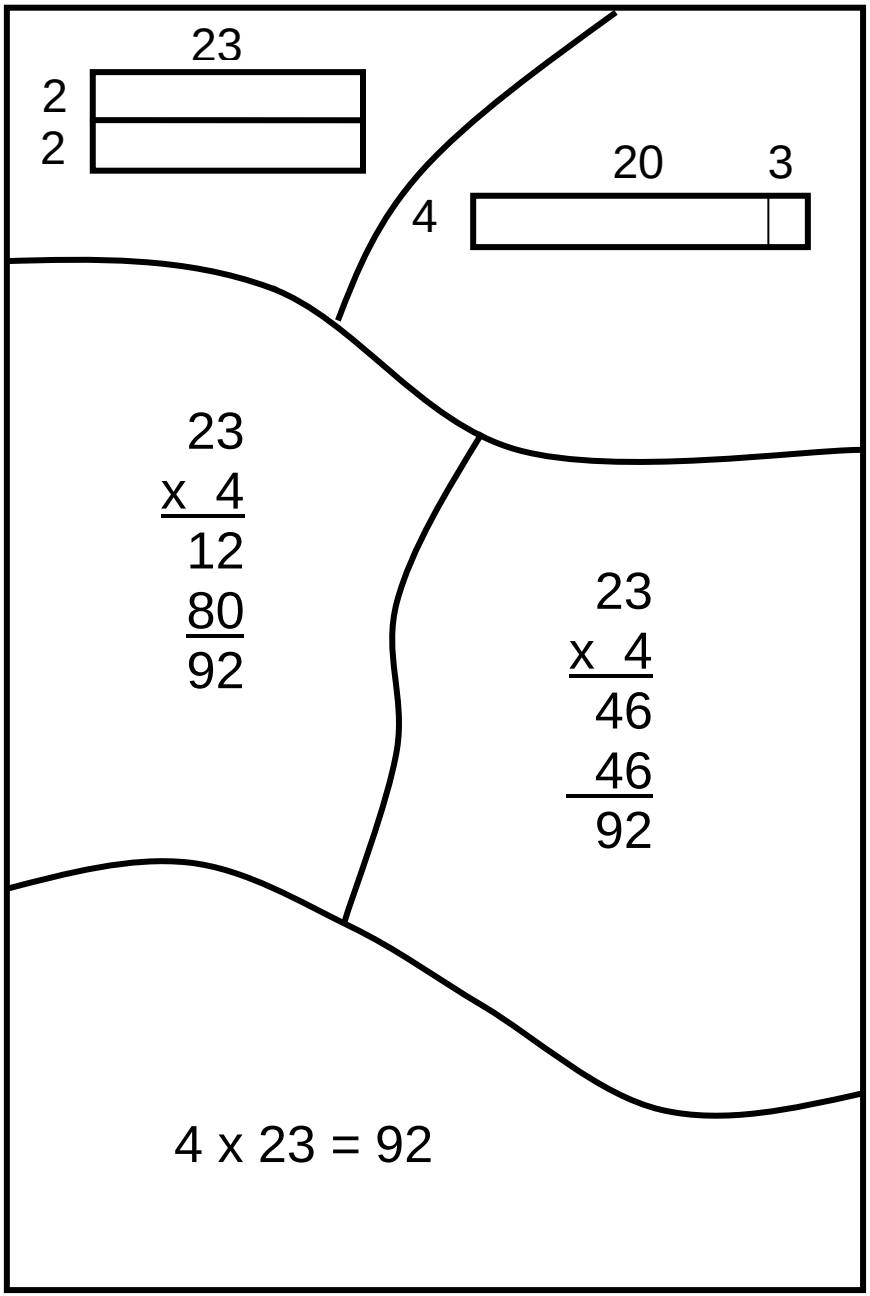


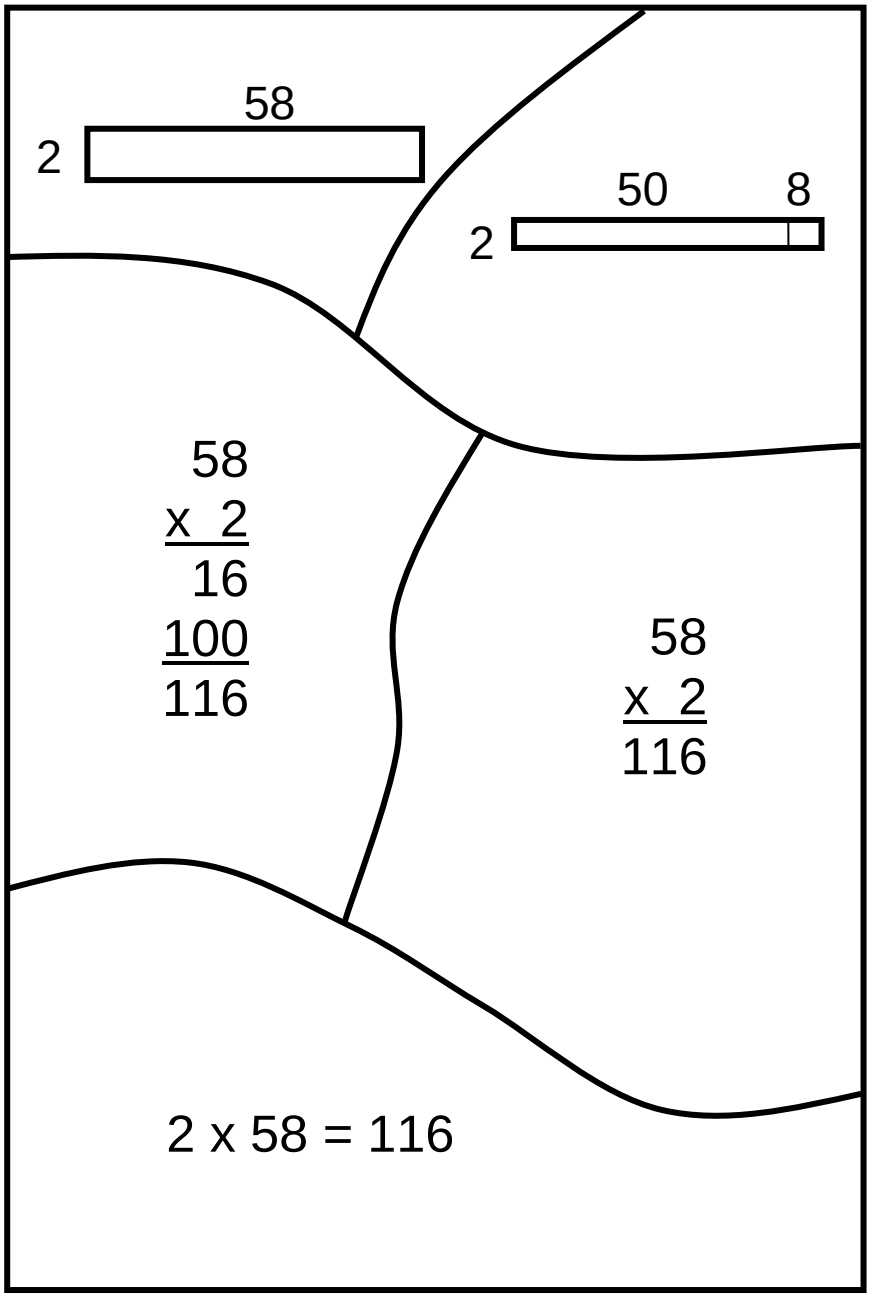
MS3 - Multiplication Computation Mix and Match Cards



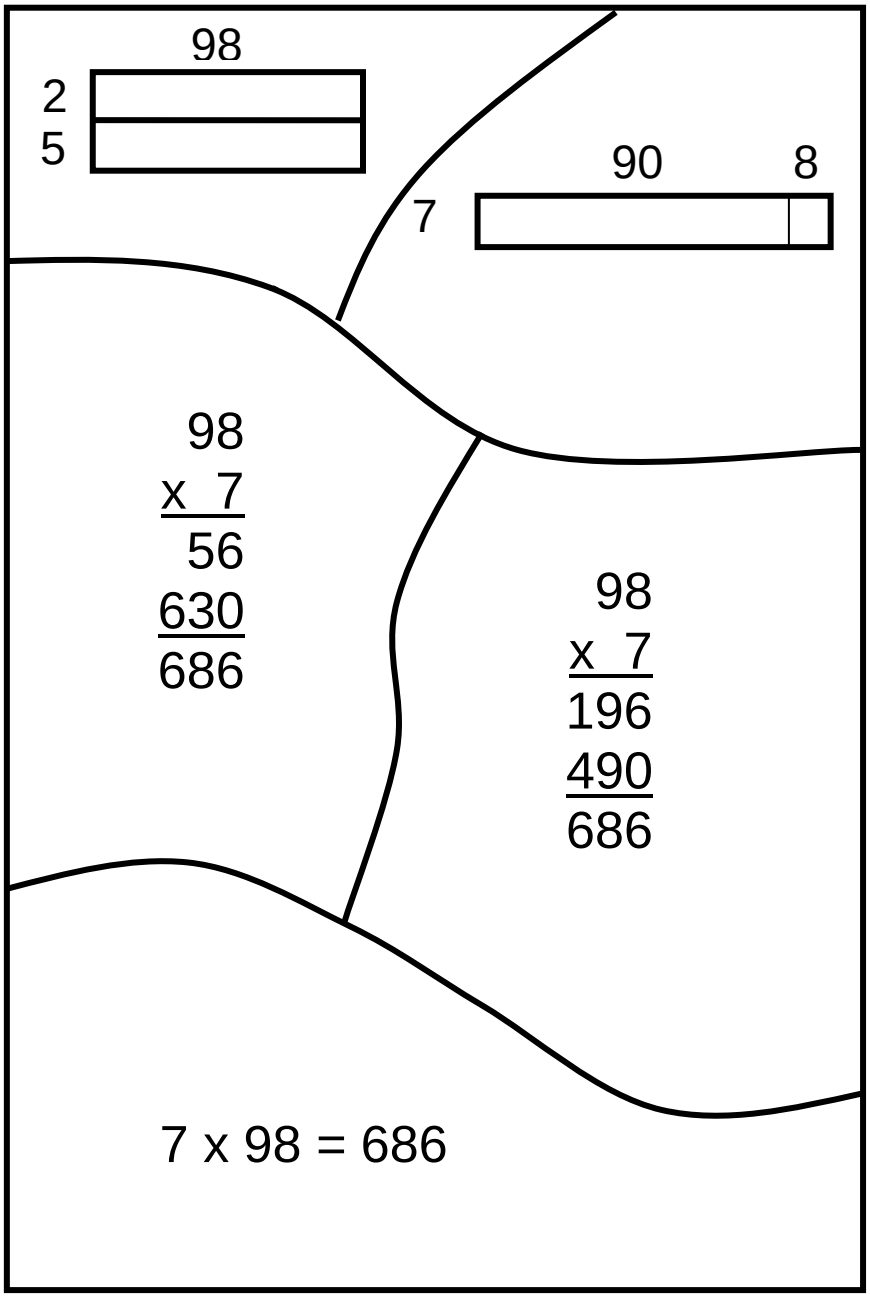


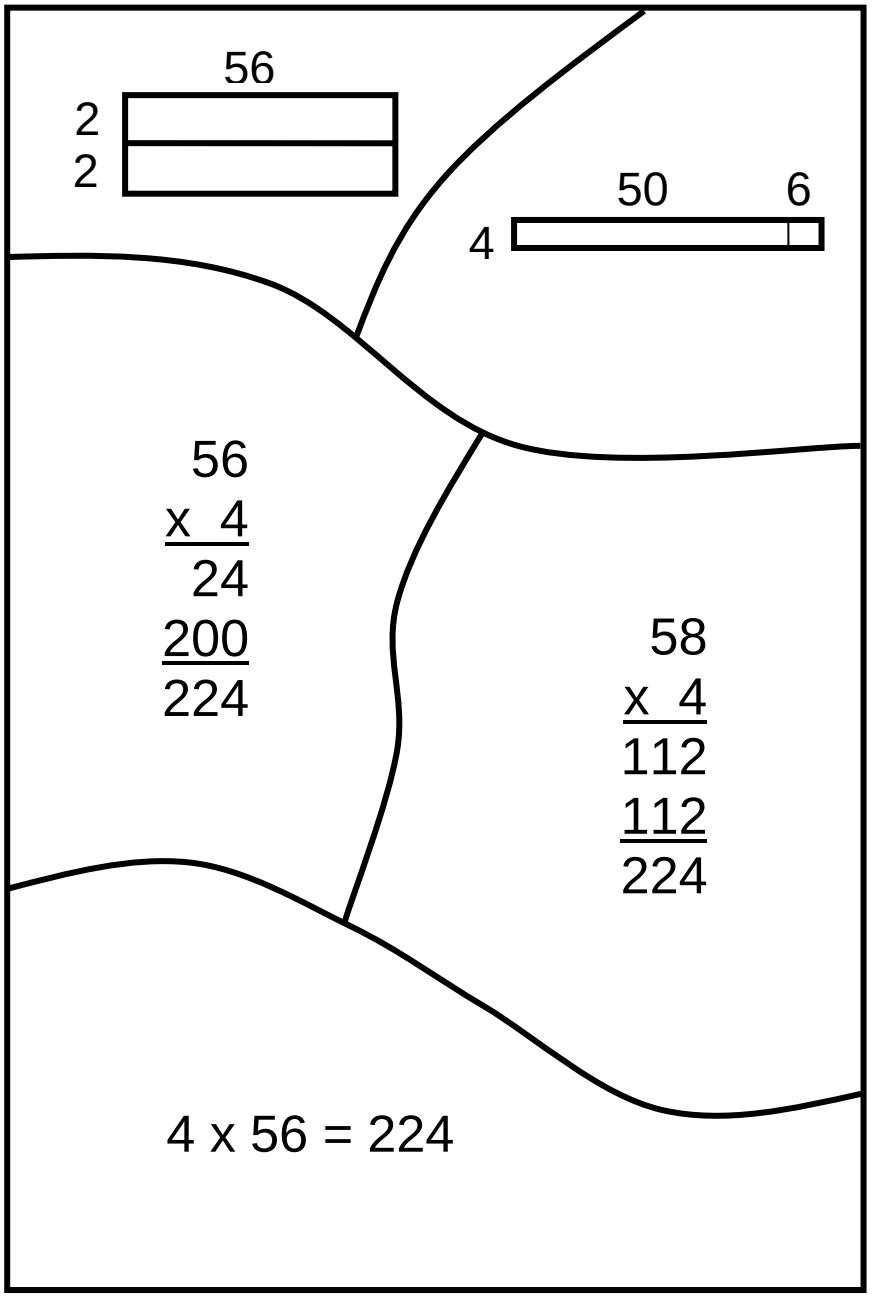
MS3 - Multiplication Computation Mix and Match Cards



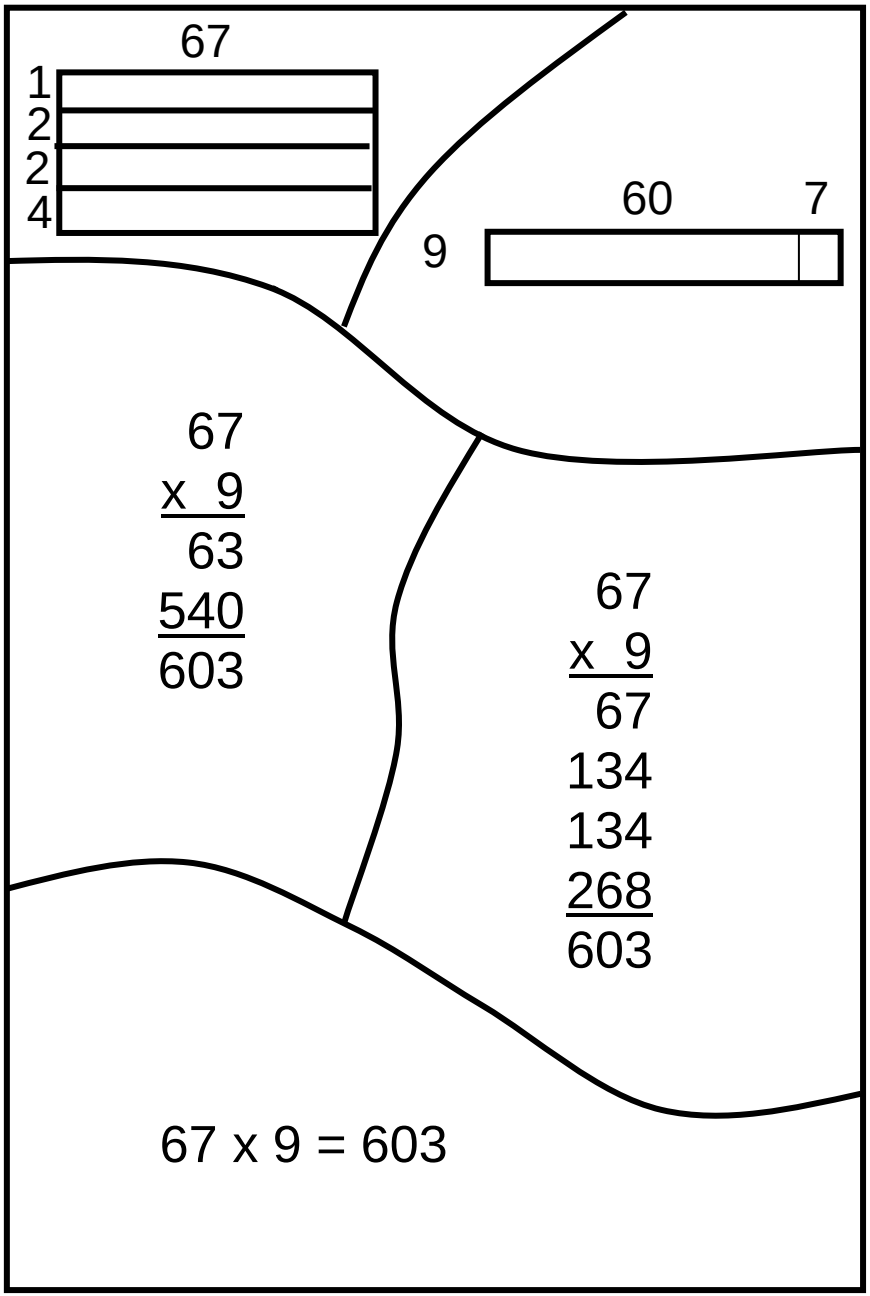


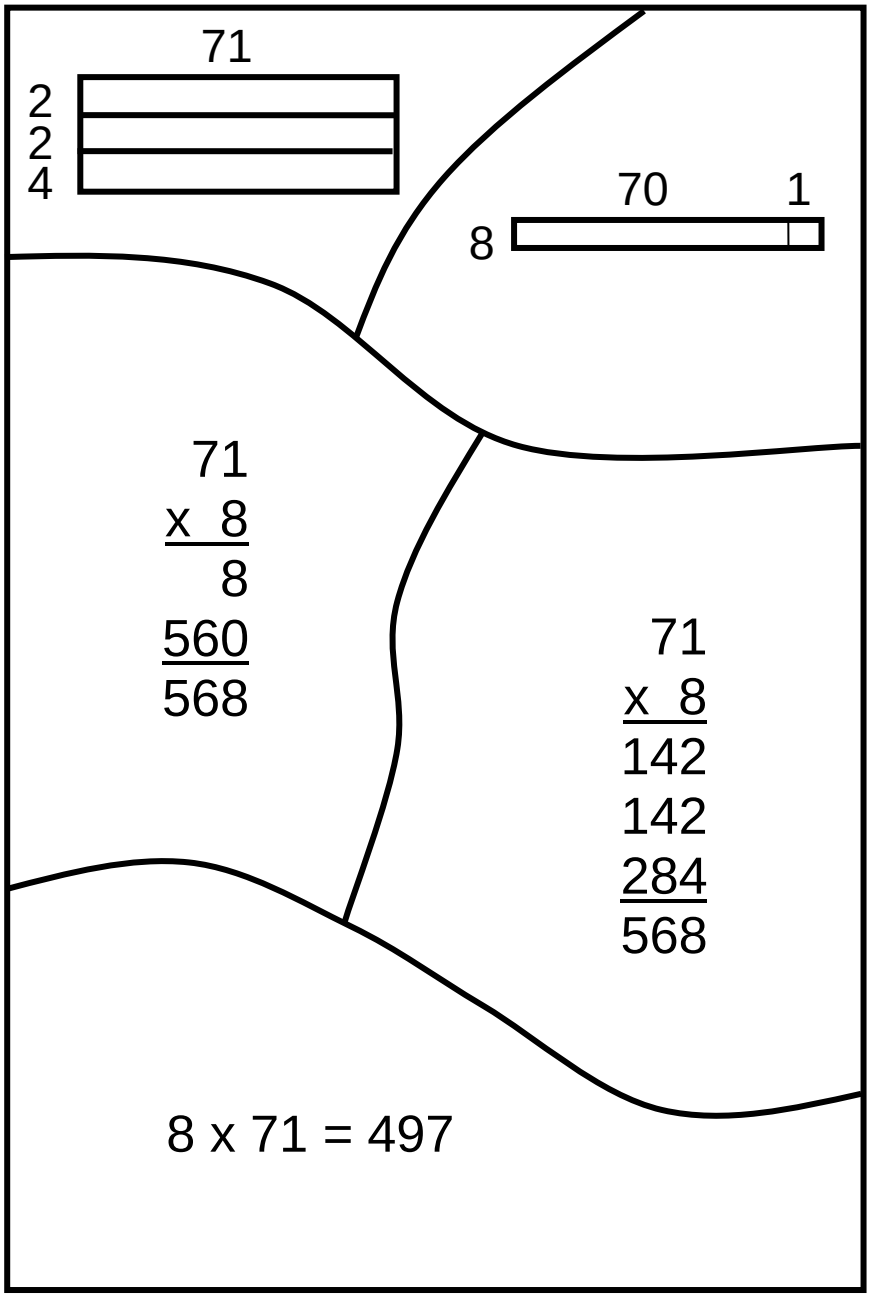
MS3 - Multiplication Computation Mix and Match Cards





MS3 - Multiplication Computation Mix and Match Cards

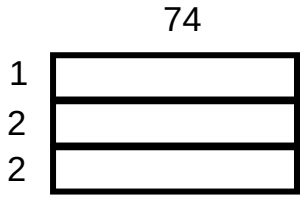
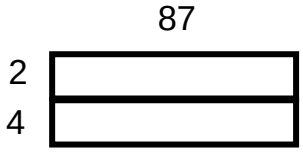
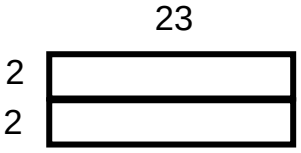
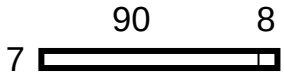
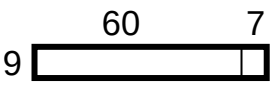
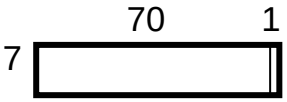
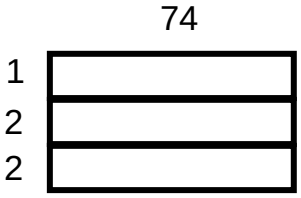
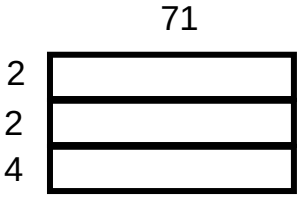
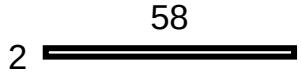
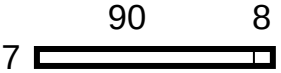
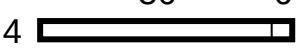
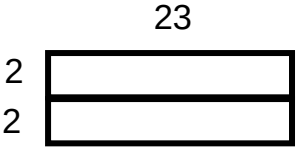




MS3 – Multiplication Computation Flash Cards (equation)

$7 \times 34 = 238$	$3 \times 42 = 126$	$4 \times 61 = 244$
$9 \times 26 = 234$	$5 \times 74 = 370$	$6 \times 87 = 522$
$4 \times 23 = 92$	$2 \times 58 = 116$	$7 \times 98 = 686$
$4 \times 56 = 224$	$9 \times 67 = 603$	$8 \times 71 = 568$

MS3 - Multiplication Computation Bingo Boards

34 x 7 28 210 238	42 x 3 42 84 126	61 x 4 4 240 244	26 x 9 26 52 52 102 234
			58 x 2 116
	56 x 4 24 200 224		
	42 x 3 6 120 126		26 x 9 54 180 234
	67 x 9 63 540 603		34 x 7 28 210 238
61 x 4 4 240 244		87 x 6 174 348 522	

MS3 - Multiplication Computation Bingo Boards

23 x 4 46 46 92	58 x 2 16 100 116	56 2 2	20 6 9
67 9	71 2 2 4	30 4 7	40 2 3
98 1 2 4	61 x 4 4 240 244	74 x 5 74 148 148 370	87 x 6 42 480 522

67 1 2 2 4	23 x 4 46 46 92	56 x 4 112 112 224	71 2 2 4
87 x 6 42 480 522	34 x 7 28 210 238	98 1 2 4	40 2 3
20 6 9	60 1 4	74 x 5 74 148 148 370	58 x 2 16 100 116

MS3 - Multiplication Computation Bingo Boards

61 2 2	42 3	34 2 5	56 x 4 24 200 224
67 x 9 63 540 603	26 1 2 2 4	98 x 7 196 490 686	87 x 6 42 480 522
20 3 4	71 x 8 142 142 284 568	70 4 5	50 8 2

98 x 7 56 630 686	50 8 2	67 x 9 67 134 134 268 603	74 x 5 20 350 370
20 3 4	56 x 4 112 112 224	80 7 6	71 x 8 8 560 568
26 1 2 2 4	61 2 2	42 1 2	34 x 7 34 68 136 238

MS3 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning

2. How did you feel about trialling the activity?

   Mark the line with an X: Unconfident ——— Very confident

3. Do you think the student was engaged in the activity? Explain.

   _____

4. What do you think the student learnt from the activity?

   _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

   _____

6. What do you think of the activity?

   _____

7. What are your suggestions for improving the activity?

   _____

   _____

8. What else do you suggest could be done to help students who have trouble with this activity?

   _____

   _____

ACTIVITY MS4

[Computation Strategies for Multiplication-Compensation]

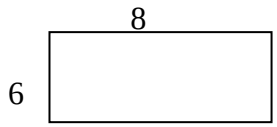
Materials: Pen, paper, attached resources (worksheets)

Directions:

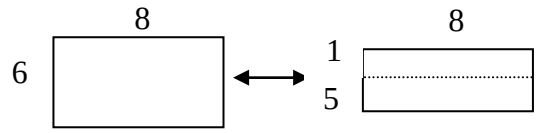
1. Special Multiples: There are numbers which are easy to multiply because of their relationship to each other and to 10 and 100. Discuss what these may be with students. Try to elicit the following :

   (a) 1, 2, 4 & 8 which come from doubles, i.e. 4×22=88
   (b) 10 and multiples of 10, e.g. 10×56 and 3×40.
   (c) 5 because it is ½ of 10: 5×23 = ½ (10×23) = ½ 230 = 115
   (d) 25 because it is ¼ of 100: ¼ (100×27) = ¼ 2700 = 625 or 6¼ 100's
   (e) 50 because it is ½ of 100: ½ (100×39) = ½ 3900 = 1950 or 19½ 100's
   (f) Examples like 35 because these are 3½ 10's: 6×35 = 6×30+ ½ 6×10 = 180+30 = 210

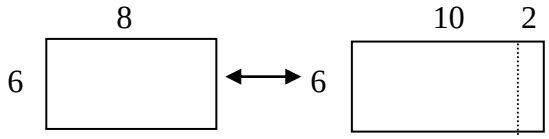
2. Effect of change. State: *Draw a diagram for 6×8*



Draw 5×10 and 6×10. Look at the changes. Assist students to see relationships when something is made larger/smaller.

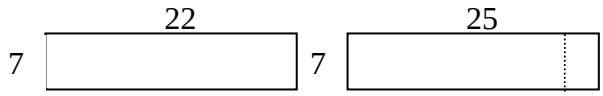


5×8 is smaller than 6×8 by 1 row. so the difference is 1×8 or one 8.
So 6 × 8 = 5 × 8 + 8

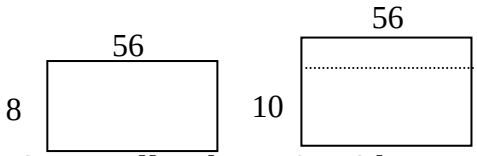


6×10 is larger than 6×8 by 2 columns so the difference is 6×2 or two 6's.
So 6 × 8 = 6 × 10 − 6 − 6.

Ask the students to look at larger numbers.



7×22 is smaller than 7×25 by 3 columns that is, three 7's.
So 7 × 22 = 7 × 25 − 7 − 7 − 7



8×56 is smaller than 10×56 by 2 rows, that is, two 56's.
So 8 × 56 = 10 × 56 − 56 − 56

3. Complete Worksheet 4.1

4. Compensation Strategy: This strategy relies on your good number sense to see if there is an easier way as examples below show. State: *There are 2 steps – finding an easy way to do it followed by compensation.*

   (a) 6×53:
   Step 1: easy example – $5 \times 53 = ½ (10 \times 53) = 265$
   Step 2: compensation – need to add another 53, i.e. $6 \times 53 = 265 + 53 = 318$.

   (b) 9×67
   Step 1: easy example – $10 \times 67 = 670$
   Step 2: compensation – one too many 67's, i.e., $9 \times 67 = 670 – 67 = 603$

   (c) 4×72
   Step 1: easy example – $5 \times 72 = ½\ 720 = 360$
   Step 2: compensation – one too many 72's, i.e., $4 \times 72 = 360 – 72 = 288$

   (d) 8×39
   Step 1: easy example – $8 \times 40 = 320$
   Step 2: compensation – one too many 8's, i.e., $8 \times 39 = 320 – 8 = 312$

   (e) 7×24
   Step 1: easy example – $7 \times 25 = 175$
   Step 2: compensation – one too many 7's, i.e., $7 \times 24 = 175 – 7 = 168$

5. Deadly Thinking: Discuss with the students: *Compensation works on finding easier multiplication. We know the 1, 2, 4 and 8 doublings; the 10 and 5 relationships; the 100, 50 and 25 relationships. So we need to get close to these. This requires swift and deadly thinking.* Say: *Look at example, **6 × 38**. What is this close to?* Discuss. Elicit some of the following:

   (a) **5 × 38**: $5 \times 38 = ½ (10 \times 38) = ½\ 380 = 190$
   $6 \times 38 = 5 \times 38 + 38 = 190 + 38 = 228$
   (b) **6 × 40**: $6 \times 40 = 240$
   $6 \times 38 = 6 \times 40$ – two 6's $= 240 – 12 = 228$
   (c) **6 × 35**: $6 \times 35 = 6 \times 3\ ½$ 10's $= 180 + 30 = 210$
   $6 \times 38 = 6 \times 35 + 3$ 6's $= 210 + 18 = 228$

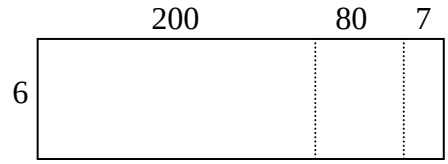
   Discuss different methods for 8×53.

6. Complete Worksheet 4.2.

7. Working out which method: State: *We now have 3 strategies: separation, sequencing and compensation. What we use is up to us. Let's look at example 6 × 287.* Go through the three methods.

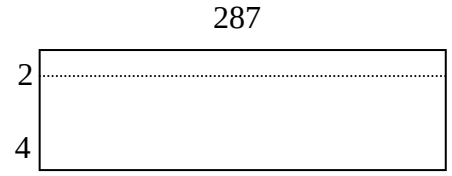
   (a) Separation:

```
 287
x 6
  42 (6x7)
 480 (6x80)
1200 (6x200)
1722
```



   (b) Sequencing:

```
 287
x 6
 574 (2x287)
1148 (4x287)
1722
```



   (c) Compensation: Try 6×300=1800. This is 13 6's too much.

```
  13
x 6
  60
  78
```

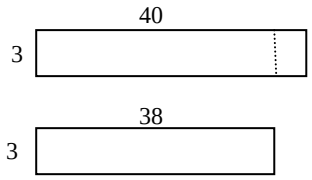
```
6 x 287 =  1800
         − 78
          1722
```

   Ask students to think of ways to multiply 8 × 68.

8. Complete Worksheets 4.3 and 4.4

MS4 – Worksheet 4.1

Find the following differences. The first has been done for you.

Example	Drawings	Differences
1. 3×38; 3×40		two 3's
2. 6×49; 6×50		
3. 8×67; 8×70		
4. 6×64; 5×64		
5. 10×73; 8×73		
6. 4×72; 4×70		
7. 9×28; 9×27		

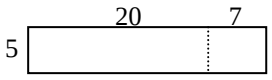
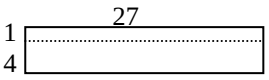
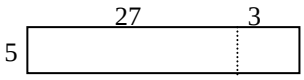
MS4 – Worksheet 4.2

Try to do some deadly thinking to find ways of *compensating*. The first has been done for you.

Example	Ways	Answer
1. 4×56	4×55 + 4, 4×60 −16, double 16, 5×56 −56	240 −16 226
2. 6×38		
3. 7×84		
4. 8×67		
5. 3×152		
6. 9×271		
7. 7×364		

MS4 – Worksheet 4.3

Complete the following. The first one is done for you.

Example	Separation	Sequencing	Compensation
1. 5×27	 27 × 5 35 (5×7) 100 (5×20) 135	 27 × 5 27 (1×27) 108 (4×27) 135	 5×30=150 5×27=150 -15 135
2. 6×38			
3. 7×84			
4. 8×67			
5. 3×152			
6. 9×271			
7. 7×364			

MS4 – Worksheet 4.4

Complete the following.

What did the waiter say when asked by the impatient customer if his pizza would be long?

1.	28 ×4	☐ D	2.	36 ×7	☐ W	3.	45 ×5	☐ R
4.	49 ×6	☐ N	5.	54 ×3	☐ E	6.	23 ×8	☐ I
7.	108 × 9	☐ S	8.	164 × 7	☐ L	9.	217 × 9	☐ T
10.	349 × 6	☐ O	11.	614 × 8	☐ U	12.	283 × 5	☐ B

___	___	___	___	___,	___	___	___	___	___	___	___	___	___	
294	2094	972	184	225	184	1953	252	184	1148	1148				
___	___	___	___	___	___	___								
1415	162	225	2094	4912	294	112								

MS4 Activity Feedback Sheet

1. How the student found the activity (put a cross on lines)

NAME	STUDENTS' REACTIONS
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning
	Boring ——— Interesting Difficult ——— Easy Not learning ——— Learning

2. How did you feel about trialling the activity?

   Mark the line with an X: Unconfident ——— Very confident

3. Do you think the student was engaged in the activity? Explain.

   _____

4. What do you think the student learnt from the activity?

   _____

5. Do you think the student has gained an understanding of the concept being taught? Explain.

   _____

6. What do you think of the activity?

   _____

7. What are your suggestions for improving the activity?

   _____

   _____

8. What else do you suggest could be done to help students who have trouble with this activity?

   _____

   _____