



School of Curriculum

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Professional Learning 3

Addition and Subtraction

Booklet 3.4: Strategies for Algorithms for 2-Digit Numbers







his project is funded by an Australian Research Council Linkage Project grant with support from Education Queensland (EQ)

DEADLY MATH TUTORS PROGRAM Professional Learning 3: Addition and Subtraction

> **TRIAL PACKAGE 4** STRATEGIES FOR ALGORITHMS **FOR 2-DIGIT NUMBERS**

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YuMi Deadly Maths Past Project Resource

Acknowledgement

We acknowledge the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

YuMi Deadly Centre

The YuMi Deadly Centre is a Research Centre within the Faculty of Education at Queensland University of Technology which aims to improve the mathematics learning, employment and life chances of Aboriginal and Torres Strait Islander and low socio-economic status students at early childhood, primary and secondary levels, in vocational education and training courses, and through a focus on community within schools and neighbourhoods. It grew out of a group that, at the time of this booklet, was called "Deadly Maths".

"YuMi" is a Torres Strait Islander word meaning "you and me" but is used here with permission from the Torres Strait Island Regional Educational Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

More information about the YuMi Deadly Centre can be found at http://ydc.qut.edu.au and staff can be contacted at ydc@qut.edu.au.

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Queensland University of Technology, Australian Catholic University & Education Queensland

Deadly Maths Tutor Program

PROFESSIONAL LEARNING 3: ADDITION & SUBTRACTION

BOOKLET 3.4 STRATEGIES FOR ALGORITHMS FOR 2-DIGIT NUMBERS 2008

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OVERVIEW

PURPOSE

These materials were designed to be used in conjunction with a Professional Learning program for teacher aides. The objective of this Professional Learning was to empower teacher aides to enhance rural and remote Indigenous students' numeracy outcomes. This document contains the materials of the fourth of five different booklets on addition and subtraction.

If your school would like to receive a YuMi Deadly Maths Professional Learning program please contact the YuMi Deadly Centre (YDC) on: 07 3138 0035 or ydc@qut.edu.au.

DIRECTIONS

(1) Interviewing the students:

Pick one or more students who appear to be having trouble understanding addition. Interview these students using the interview schedule and the materials. Mark what they do and put their results on the Student Recording Sheet.

(2) Trialling the student activities:

Use the Recording Sheet to work out the activities the students need to do and trial these activities with the students (with each student one at a time or with a group of students). Keep a record of what happens and collect the students' work.

The activities SA1 and SA2 are designed to allow you to tutor students who are having difficulties with addition and subtraction algorithms for two-digit numbers. The activities are based on the belief that it is more important to use the algorithms to teach a variety of strategies than to get a correct answer. However, the activities also show how to get correct answers.

There are three strategies (or strategy groupings) associated with addition and subtraction of two-digit numbers. These are:

(1) Separation:

This strategy (separate, operate, combine) is to break both numbers into parts based on place value, add and subtract the numbers as separated place values, then recombine for the answer. It is widely used in mathematics, for example, adding and subtracting measures (m and cm), time (hrs and mins), mixed numbers (wholes and parts), and algebra (x's and y's). The strategy involves renaming or carrying. Materials to teach separation are place-value charts, bundling sticks, and MAB. Example:

ADDITION	SUBTRACTION
4 7	7 12
<u>+28</u>	8 2
$1\ 5\ 7+8$	<u>-45</u>
6 0 40 + 20	3 7
7 5	

(2) Sequencing:

This strategy is to leave one number as a whole and add or subtract parts of the second number until all parts have been added or subtracted. It is associated with 99 boards and number lines and is also useful for measures, time and mixed numbers. This is particularly so for a additive form of subtraction that is available with this strategy. Materials to teach sequencing are 99 boards and number lines. Example:

ADDITION	SUBTRACTION	ADDITIVE
4 7	8 2	SUBTRACTION
<u>+ 2 8</u>	<u>-45</u>	4 5 \nearrow 5
6747+20	4 2	$50 \frac{2}{4} 30$
+ 8	<u>- 5</u>	$80 \frac{2}{12} + 2$
$7 \ 5 \ 67 + 8$	3 7	8 2 2 3 7

(3) Compensation:

This strategy is to leave both numbers as they are (no parts) but to look for a change that will make the addition or subtraction easy, then compensate for that change. This is the basis of many of the methods used in algebra as well as being useful for adding or subtracting measures, time and mixed numbers. Materials to teach compensation are 99 boards and number lines. Example:

ADDITION	N SU	BTRACT	ΓΙΟΝ
4 7		8 2	
+ 2 8		<u>-45</u>	
4 7	- 47 + 20	8 2	
+ 3 0		<u>-50</u>	
7 7	- 67 + 8	3 2	
<u>- 2</u>	compensates for 28 to 30	<u>+ 5</u>	compensates for 45 to 50
7 5		3 7	

All the three strategies require knowledge of basic number facts (e.g., 6+8=14) and also higher-decade number facts (e.g., 26+8=34 and 60+80=140).

Thus, the five activities in this booklet are the separation strategy (Activities SA1 & SA2), sequencing strategy (Activities SA3 & SA4), and compensation strategy (Activity SA5).

INTERVIEW

INTERVIEW SCHEDULE

MATERIALS

Bundling sticks, MAB, tens-ones place-value chart, 99 boards, number lines, washable felt pen, pen, paper

Materials with this interview: interview cards

Student recording sheet

DIRECTIONS

- 1. Photocopy and laminate attached interview cards and card deck 1 pages.
- 2. Gather other material.
- 3. Place material in front of students. Give students pen and paper to write with.
- 4. Tell the students you are trying to find out what they know. Say they are not expected to know it and you will teach what is not known.
- 5. Give the student directions slowly read problems. Do not give hints. If student cannot do a question, pass on to the next question.
- 6. Allow students to use material and make drawings but only after they say they do not know how to do it with symbols alone.

Strategies for algorithms

$\left(C1\right)$

Show Card 1.

- Read the card. Ask: What is the answer?
- When student has answered, ask: *How did you work it out?*

Ask the same questions for Cards 2, 3 and 4.

 $\left(C2\right)$

Show Card 5.

Read the card. Ask: What is the answer?

When student has answered, ask: *How did you work this out?* If student cannot answer, ask: *Could you use the left hand sum to help?*

Ask the same questions for Card 6.

(C3)

Show and read Card 7.

Say: Calculate the answer by adding ones and tens separately!

 $\left(C4\right)$

Show and read Card 8.

Say: Calculate the answer by keeping the first number as it is and adding parts of the second number!

(C5)

Show and read Card 9.

Say: Calculate the answer by changing the addition to a sum with the same answer that is easier to add!

(C6)

Show and read Card 10.

Say: Calculate the answer by subtracting ones and tens separately

 $\left(C7\right)$

Show and read Card 11.

Say: Calculate the answer by keeping the first number as it is and subtracting parts of the second number!.

 $\left(C8\right)$

Show and read Card 12.

Say: Calculate the answer by changing the subtraction to a form with the same answer that is easier to subtract!

INTERVIEW MATERIAL

Interview Cards – Strategies for Algorithms

CARD 1	CARD 2	
6	13	
<u>+5</u>	<u>- 9</u>	
CARD 3	CARD 4	
8	15	
<u>+6</u>	<u>- 7</u>	
CARD 5	CARD 6	
4 40	9 90	
+3 +30	<u>- 5</u> <u>-50</u>	
7 :	4	

CARD 7 48	27
<u>+36</u>	<u>+44</u>
CARD 9	CARD 10
35	74
+28	<u>- 28</u>
CARD 11	CARD 12
82	91
<u>- 34</u>	<u>- 28</u>

STUDENT RECORDING SHEET

Strategies for algorithms

Name:	_ Date:
School/Class:	

	Interview item	Result (✓, ✗)	Comments	Activities to be completed if incorrect
C1:	Basic addition & subtraction number facts			Undertake practice of basic facts See book 3
C2:	Higher-decade addition & subtraction facts			Undertake practice of higher-decade facts
C3:	Separation strategy for addition algorithms			SA1
C4:	Sequencing strategy for addition algorithms			SA3
C5:	Compensation strategy for addition algorithms			SA5
C6:	Separation strategy for subtraction algorithms			SA2
C7:	Sequencing strategy for subtraction algorithms			SA4
C8:	Compensation strategy for subtraction algorithms			SA5

ACTIVITIES

ACTIVITY SA1

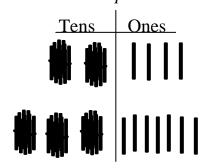
[Separation strategy for 2-digit addition]

Materials: Bundling sticks, calculators, 10s-1s place value chart, Shop cards,

materials for games (card decks, dice), worksheets A and B.

Directions:

1. Say: Suppose it costs \$24 for food and \$37 for clothes, how much do we pay? Ask: How many tens and ones in 24? [2 tens, 4 ones] Ask: How many tens and ones in 37? [3 tens, 7 ones] Say: Show these numbers with bundling sticks. Record the addition on a place value chart.



Tens	Ones
2	4
+ 3	7

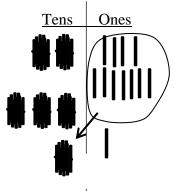
2. Ask: What do we add first? [Ones] Say: Join the ones. Ask: Do you have enough ones to make a 10? [yes] Say: Bundle ten sticks to make a 10. Ask: How many ones are left? Say: Record this on your place value chart.

Ask: What do we add next? [Tens] Then say: Join the tens. Say Calculate the final tens and ones. Write this on your place value chart.



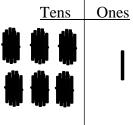
SHORT RECORDING

LONG RECORDING



Tens	Ones
1	4
2	7
+ 3	1

Tens	Ones
2	4
+ 3	7
1	1
	l



Tens	Ones
1	
2	4
+ 3	/ 1
U	1

Tens	Ones
2 + 3	4 7
1 + 5	1 0
6	1

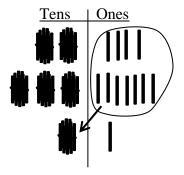
Note: It is possible to do tens first:

MATERIALS

Tens Ones

RECORDING

Tens	Ones
2	4
+ 3	7
5	0



Tens	Ones
2	4
+ 3	7
5	0
+ 1	1
6	1

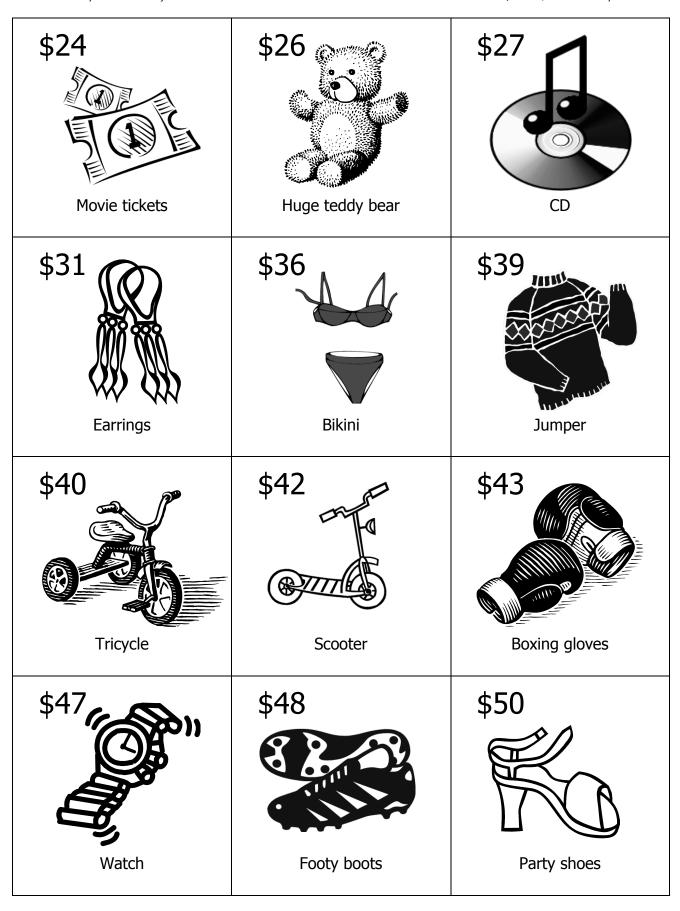
- 3. Repeat steps 1 and 2 for two cards from the Shop cards. Then repeat again for a second two cards. Always ask as the student works through the exercise: *How many tens and ones do I have? How many ones to the next ten? Do I have enough ones to make another ten? How many tens is this? How many ones left over?*
- 4. Play games A and B
- 5. Discuss what the students have done. Ask: *How would you do this without sticks? Can you think in the mind what to do?* Repeat steps 1 and 2 for two more Shop cards but try to encourage the students not to use bundling sticks and place value charts.
- 6. Complete worksheets A and B. Allow students to use sticks if they need to. Discuss the strategy used. State: *Can you describe the strategy you are using?* [Separate each number into place values and add parts separately] State: *This is called separation*.
- 7. Play games C and D.

SA1 – 10s/1s Place Value Chart

TENS	ONES

SA1 – Shop Cards





SA1 – Games A and B

Game A: Towards 100

Materials: Coins, dice, bundling sticks, 10/1s place value charts.

Number of players: 2-4.

Directions:

1. Place value charts placed in front of all players with no bundling sticks on it.

- 2. Players, in turn, throw a coin and a die where the coin gives tens (H-1 ten, T-2 tens) and die ones.
- 3. Then, players take bundling sticks to the value of the coin and die and add to the previous amount on the chart (this is zero for the first play).
- 4. Players bundle sticks to form new tens when appropriate.
- 5. Players type the amount into the calculator and write it on paper.
- 6. The first player to 100 wins.

Game B: Towards 200

Materials: 2 dice (2 colours - one for tens and one for ones), 10s/1s place value

charts, bundling sticks, calculators, pen and paper.

Number of players: 2-4

Directions:

- 1. Place value charts placed in front of all players with no bundling sticks on it.
- 2. Players, in turn, throw the two dice to give a number in terms of tens and ones.
- 3. Then, players take bundling sticks to the value of this number and add to the previous amount on the chart (this is zero for the first play).
- 4. Players bundle sticks to form new tens when appropriate.
- 5. Players type the amount into the calculator and write it on paper at each turn.
- 6. The first player to 200 wins.

SA1 – Worksheet A

Name:	Year:	School:

Complete the following to solve the puzzle

1. Kade bought lunch for \$12 and a drink for \$3. How much did Kade spend?

 $= \mathbf{K}$

2. Ellie gave \$4 to her cousin and \$15 to her aunty. How many dollars did Ellie give away?

 $= \mathbf{A}$

3. Michael bought a football for \$22 and a pair of footy socks for \$4. How much did Michael spend?

____ = I

4. Crystal spent \$11 on fruit and \$2 on a chocolate bar. How much did Crystal spend?

 $=\mathbf{E}$

5. Courtney bought a shirt for \$34 and a pair of shoes for \$60. How much did Courtney spend?

____ = N

6. Alex gave his brother \$55 and bought a CD for \$24. How much money did Alex give away and spend in total?

 $= \mathbf{B}$

7. Mona spent \$58 on a tennis racquet and \$11 on tennis balls. How many dollars did Mona spend?

 $= \mathbf{R}$

8. Reuben spent \$25 on dinner and \$32 on tickets on a date with Courtney. How much money did he spend?

= T



Where do fish keep their money?

SA1 – Worksheet B

Name: _____ Year: ____ School:_____

Complete the following to solve the puzzle

1. Jack bought drinks for \$17 and dinners for \$28. How much did Jack spend?

 $\mathbf{W} = \mathbf{W}$

2. Bob bought a coat for \$56 and pants for \$38. How much did Bob spend?

 \mathbf{L}

3. Del bought a DVD for \$27 and a CD for \$13. How much did Del spend?

____ = A

4. Wayne bought meat for \$36 and vegetables for \$18. How much did Wayne spend?

= T

5. Howie gave Joe \$26 and bought groceries for \$46. How much money did Howie give away and spend in total?

 $=\mathbf{E}$

6. Tania gave Sue and Jenn both \$37. How much did Tania give away?

= P

7. Pearl spent \$63 on a radio and \$8 on chocolates. How much did Pearl spend?

____ = K

8. Sue gave Jenn \$38 and spent \$29 on cosmetics. How much money did Sue give away and spend in total?

= \mathbf{S}

Н

What could the mad Australian skier not find?

\$40

\$94

440

\$71

\$72

\$45

0

\$54

\$72

\$40

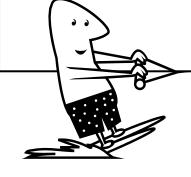
\$67

_

\$94

\$74

Ι



SA1 – Games C and D

Game C: Criss-cross 1

Materials: Deck of cards (without J, Q, and K and with Ace as 1), pen and paper.

Number of players: 4

Directions:

- 1. Deal 5 cards to each player.
- 2. Each player adds their cards to get a score.
- 3. Opposite players combine their scores using pen/paper to add the numbers.
- 4. Highest score gives partners 1 point.
- 5. First pair to 5 wins.

Game D: Criss-cross 2

Materials: Deck of cards (without J, Q, and K and with Ace as 1), pen and paper.

Number of players: 4

Directions:

- 1. All players are dealt 2 cards the first card is tens, and the second card is ones.
- 2. Opposite players combine their scores using pen/paper to add the numbers.
- 3. Highest score gives partners 1 point.
- 4. First pair to 5 wins.

ACTIVITY SA2

[Separation strategy for 2-digit subtraction]

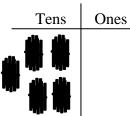
Materials: Bundling sticks, calculators, 10s-1s place value chart, materials for

games (card deck, dice), worksheets A and B.

Directions:

1. Pose a problem: You pay a \$37 bill with \$50; how much change? Ask: How many tens and ones in 50? [5 tens, 0 ones] Say: Show this on place value chart with bundling sticks. Say: Record 50–37 on paper.

CHART

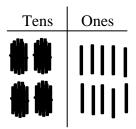


RECORDING

Tens	Ones
5 <u>- 3</u>	0 7

2. Ask: How many tens and ones in 37? [3 tens, 7 ones] Say: Can you subtract 7 ones with your 50 as it is? [no] What can you do? [change a 10 for ten ones] How many tens and ones now? [4 tens and 10 ones]. Say: Record this on paper.

CHART



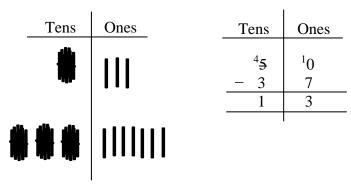
RECORDING

Tens	Ones
⁴ 5 - 3	¹ 0 7

3. Say: Subtract 7 ones by sliding 7 sticks down the chart. Subtract 3 tens by sliding 3 bundles down the chart. Ask: How many is left? [1 ten and 3 ones or 13] Say: Record this on paper.

CHART

RECORDING



Note there are other ways it could be done:

Tens	Ones
5 - 3	0 7
2	0 Down 7
1	3

- 4. Repeat steps 1 to 3 for 63-25. Always ask: Do you have enough ones to subtract the ones? If not, what do you do? How many ones does this give you? How many tens?
- 5. Play games A and B.
- 6. Discuss what the students have done. Ask: *How would you do this without sticks? Can you think in the mind what to do?* Repeat steps 1 to 3 for 54-27. Try to encourage the students not to use bundling sticks and place value charts.
- 7. Complete worksheets A and B. Allow students to use sticks if they need to. Discuss the strategy used. State: *Can you describe the strategy you are using?* [Separate each number into place values and subtract parts separately after necessary unbundling] State: *This is called separation strategy*.
- 8. Play shop activity. One student acts as shopkeeper. Other students given \$50. They approach shopkeepers select a Shop card and pay for this with \$50. Shopkeeper has to correctly calculate change?
- 9. Play games C, D and E.

SA2 – Games A and B

Game A: Away from 100

Materials: Coins, dice, bundling sticks, 10/1s place value charts.

Number of players: 2-4.

Directions:

1. Place value charts placed in front of all players with 100 in bundling sticks on it.

- 2. Players, in turn, throw a coin and a die where the coin gives tens (H-1 ten, T-2 tens) and die ones.
- 3. Then, players remove bundling sticks to the value of the coin and die from the previous amount on the chart (this is 100 for the first play).
- 4. Players un-bundle sticks to form extra ones when appropriate.
- 5. Players type the amount into the calculator and write it on paper.
- 6. The first player to zero wins.

Game B: Away from 200

Materials: 2 dice (2 colours - one for tens and one for ones), 10s/1s place value

charts, bundling sticks, calculators, pen and paper.

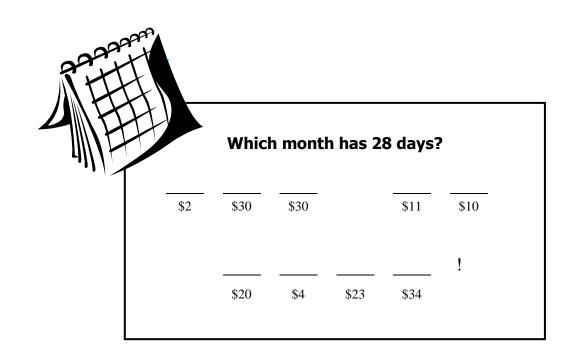
Number of players: 2-4

Directions:

- 1. Place value charts placed in front of all players with 100 in bundling sticks on it.
- 2. Players, in turn, throw the two dice to give a number in terms of tens and ones.
- 3. Then, players remove bundling sticks to the value of this number from the previous amount on the chart (this is 100 for the first play).
- 4. Players un-bundle sticks to form extra ones when appropriate.
- 5. Players type the amount into the calculator and write it on paper at each turn.
- 6. The first player to zero wins.

SA2 – Worksheet A

Na	me: Year: School:	
Co	omplete the following to solve the puzzle	
1.	Rebecca bought tray of mangoes for \$20. What change did she get from \$50?	= L
2.	Russell bought a CD for \$20. He gave the shopkeeper \$40. How much change did he get?	= T
3.	Jake bought a pair of jeans for \$73. How many dollars in change did the shopkeeper give Jake from \$75?	= A
4.	Louise paid \$42 for a plant as a housewarming gift for her friend. She gave the shopkeeper \$65. How much change did she get?	= E
5.	Ally ordered a pizza for \$11. How much change will she get from \$15?	= H
6.	Peter bought groceries for \$74. What was his change from \$85?	= O
7.	Madeleine paid \$31 for art materials. How much change did she get from \$65?	= M
8.	Philip bought a new tank for his goldfish. It cost \$80. How much change did he get from \$90?	= F



 $= \mathbf{E}$

SA2 – Worksheet B

Name:	Year:	School:

Complete the following to solve the puzzle

- 1. Sebastian paid \$39 for his new shoes. What change did he get from \$50? _____ = **O**
- 2. Hooper bought some fruit for \$16. He gave the shopkeeper \$45. How much change did he get?
- 3. Emma bought a skirt for \$26. How many dollars in change did the shopkeeper give her from \$50?
- 4. Zac paid \$86 for a new CD player. He gave the shopkeeper \$100. How much change did he get?
- 5. Kelly paid \$13 for her lunch at the snack bar. How much change will she get from \$40?
- 6. Mavis bought groceries for \$88. What was her change from \$94?
- 7. Ethan paid \$46 for paint. How much change did he get from \$64?
- 8. Barbara bought a new pair of thongs. They cost \$7. She had \$15 in her purse.

 How much change did she get?

 _____ = C



What gets wetter the more it dries?

\$11 \$6 \$8 \$11 \$24 \$29

SA2 – Games C, D and E

Game C: Subtraction criss-cross 1

Materials: Deck of cards (without J, Q, and K and with Ace as 1), pen and paper.

Number of players: 4

Directions:

- 1. Deal 5 cards to each player.
- 2. Each player adds their cards to get a score.
- 3. Opposite players subtract their scores using pen/paper to subtract the numbers.
- 4. Highest score gives partners 1 point.
- 5. First pair to 5 wins.

Game D: Subtract criss cross 2

Materials: Deck of cards (without J, Q, and K and with Ace as 1), pen and paper.

Number of players: 4

Directions:

- 1. All players are dealt 2 cards the first card is tens, and the second card is ones.
- 2. Opposite players subtract their scores using pen/paper to subtract the numbers.
- 3. Highest score gives partners 1 point.
- 4. First pair to 5 wins.

Game E: Biggest difference:

Materials: Deck of cards (without 10, J, Q and K and with Ace as 1), pen and

paper.

Number of players: 2-6

Directions:

- 1. All players are dealt 4 cards.
- 2. Players form 2 two-digit numbers that give the biggest possible subtraction for the four cards.
- 3. Player with largest difference scores 1.
- 4. First player to 5 wins.

ACTIVITY SA3

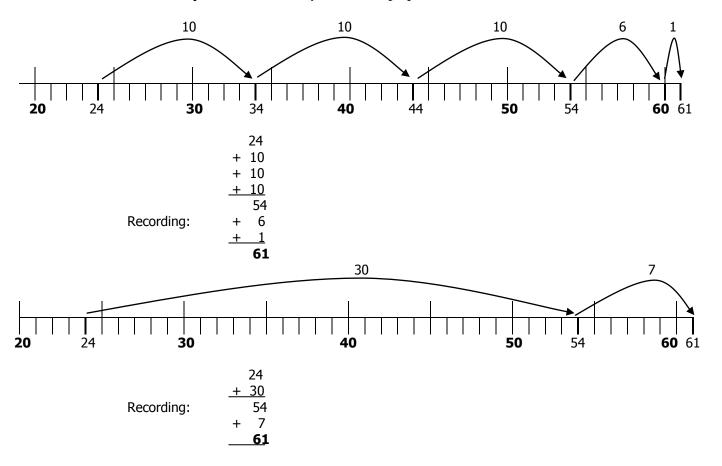
[Sequencing strategy for 2-digit addition]

Materials: 99 boards, number lines, pen, paper, calculators.

Directions:

- 1. Assist students to **get to know the 99 board**.
 - (a) <u>Setting up</u>: Hand students a 99 board each. Say: *Look at the board*. Ask: *What numbers do you know?*
 - (b) Get to know the board. Say: Put your finger on the board at 60. Move across the board reading the numbers. [60, 61, 62 ...] Ask: What is the pattern here? [Numbers go up by 1; tens stay the same]. Say: Put your finger on 4 and move down the board reading the numbers. [4, 14, 24, ...] Ask: What is the pattern here? [Numbers go up by 10; ones stay the same].
 - (c) Finding numbers. Say: Put your finger on the shaded area just before the 1. Say: Move 5 steps down. What number is this? [50] Then say: Move 4 steps across. What number is this? [54]. Ask: How many tens and ones has 54? [5 tens, 4 ones]. Repeat the process for 3 steps down and 7 steps across [37]. 8 steps down and 1 across [81] and 6 steps down and 8 across [68]. Ask: What is the pattern here? [Down gives the tens and across gives the ones].
 - (d) Solo finding. Say: Find 23, 58, and 91.
 - (e) Shut eye. Say: Shut your eyes and with a finger on imaginary board find 32, 57 and 73.
 - (f) Practice: Play 99 board game A
- 2. Assist students to use the sequencing strategy to add with the 99 board.
 - (a) <u>Adding numbers</u>. Consider the sum 42 + 34. Tell students to use their fingers to find 42 on the 99 board. Then direct students to move 3 steps down and 4 across. Ask: *What number are you at?* [76]. Ask to use calculator to find 42 + 34. Ask: *Is it the same number?*
 - (b) Getting pattern. Repeat direction 2(a) for 52 + 27, 37 + 34 and 19 + 48. Ask: What is the pattern here? [Find the first number, go down tens and across ones of second number]
 - (c) <u>Imagining</u>. Ask students to shut eyes, work out 62 + 25 with an imaginary board moving fingers along the board. Repeat direction 3 for 18 + 31, 37 + 25 and 48 + 36.

- Practice. Complete Worksheet A. Play 99 board games B, C, D and E.
- Extension. Ask: Can you do sums like these in your head? Show me?
- 3. Assist students to use the sequencing strategy to **add with a number line**.
 - Adding numbers. Say: Food costs \$24 and clothes \$37; we will now show how a number line can help you work out the total to pay? State: Mark where 24 is on number line, then hop on 37 - first as 10s and then as 1s. Ask: How many 10s and 1s in 37? [3 tens and 7 ones] Say: Hop 10s along the line and then 1s. Do it your own way. Ask: What 10s and 1s did you add? Say: Record what you do on paper.



- Getting pattern. Repeat step for 2 cards from Shop cards. Do this 3 times. Ask as students begin adding the ones: How much to the next \$10. Do we have enough? Just enough or some left over? How much left over? Try to use the lines with less information on them. Discuss being efficient.
- Imagining. Say: Think about how you would do these sums without the number line. What would you think of? Say: Try adding two Shop cards by imagining the line.
- (d) Strategy. Discuss the strategy being used. Ask: *How would you describe* the general way we are doing this? [Starting with one number – adding bits of other numbers]. State: This is called sequencing.
- Practice: Complete Worksheet B. Play number line games A, B and C.

SA3 - 99 Board

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100									

SA3 - 99 board games

99 board game A: Three together

Materials: 99 board (enlarged to A3 size), unifix (one colour for each player),

card deck (no 10, J, Q or K; Ace is 1)

Number of players: 2-3

Directions:

1. One 99 board is placed between the players who each have a pile of unifix of one colour (different colour for each player).

- 2. Each player, in turn, is dealt two cards. The player turns cards over and makes a two digit number with the two cards (if cards are 4 and 6, player can make 46 or 64 it is the player's choice).
- 3. Each player then places one of his/her unifix over the number. If number already covered by his/her colour, player misses his/her turn. If number covered by another colour, player places his/her unifix over the existing unifix ("takes over" that position).
- 4. First player who gets three of his/her colours together wins. The following is allowed (in any direction):

Three in a row Three in an "L" Three at an angle

99 board game B: First to 50

Materials: 99 board, 2 dice, different coloured counter for each player.

Number of players: 2-4

Directions:

- 1. Place one 99 board between players. Each player places their counter on zero (shaded area) at the start.
- 2. Each player, in turn, throws 2 dice and moves their counter along the board the sum of the two dice.
- 3. The first player to pass 50 wins.

99 board game C: Sweep the board

Materials: 99 board, deck of cards (no 10, J, Q, or K; Ace is 1), different coloured

counter for each player.

Number of players: 2-4.

Directions: Same as game "first to 50" but adding card numbers and finishing at

100.

SA3 – Worksheet A

Complete these addition examples with the 99 board.

$$M 64 + 27$$

$$L 29 + 46$$

$$R \ 4 \ 3 + 2 \ 8$$

$$A 2 8 + 57$$

Shut eyes and complete the sums with an imaginary board.

E
$$3.6 + 2.3$$

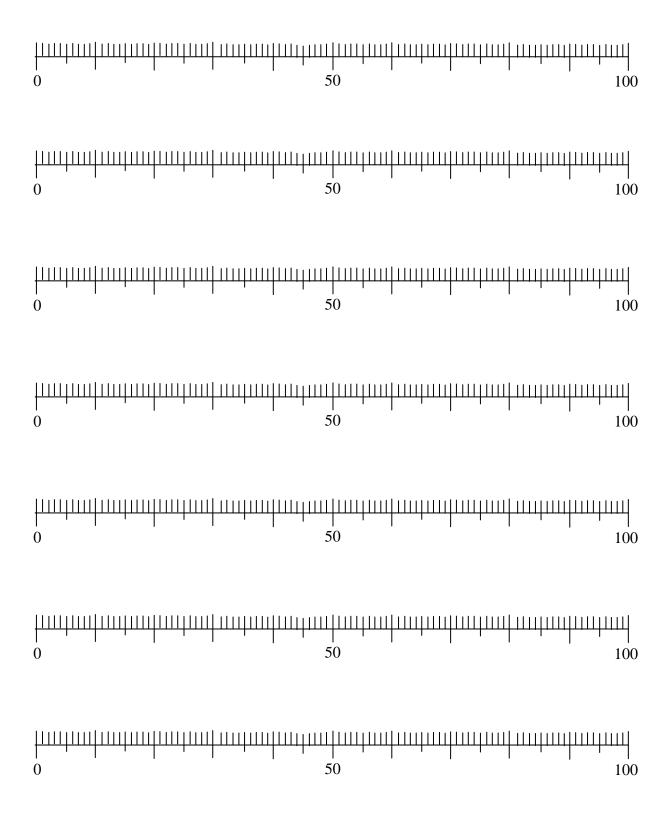
W 14
$$+ 34$$

What do you get when you cross a sheep with a kangaroo?

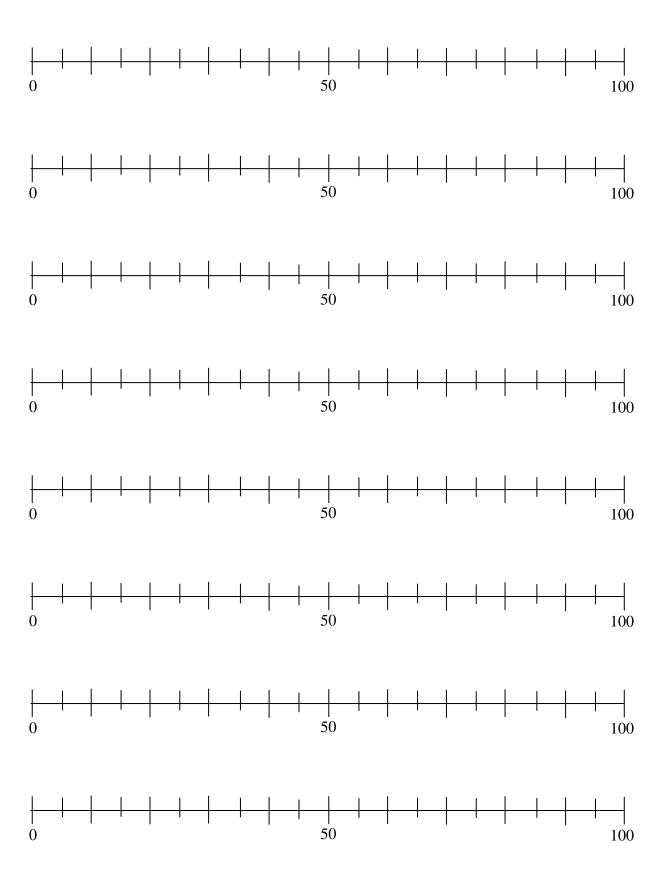
85 48 91 91 75 75 82

51 76 91 83 59 71

SA3 – Number lines – all markings



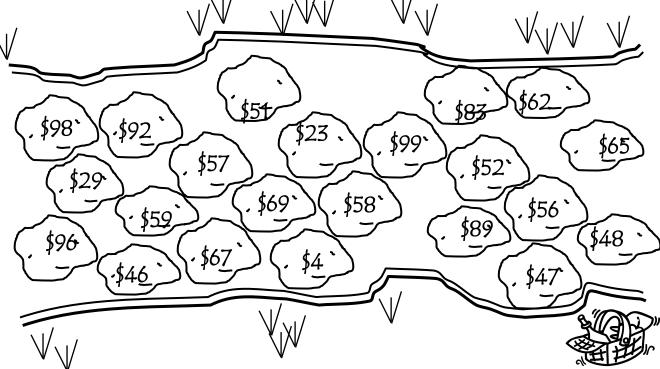
SA3 – Number lines – 5 and 10 markings



SA3 - Number lines - no markings

SA3 – Worksheet B

Na	me: Year: School:
	e a number line to do these word problems Crocodiles are lurking under some of the cks. Can you make it across the rocks to the picnic without being eaten by a crocodile?
1.	Georgia has \$40 in notes and \$12 in coins. How much money does she have altogether?
2.	Billy bought a CD for \$27 and some speakers for \$32. How much did Billy spend?
3.	Charlotte gave \$35 to her brother and \$23 to her sister. How many dollars did she give away?
4.	Rose bought a box of chocolates for \$12 and a bunch of flowers for \$45 for her sick mother. How much money did she spend?
5.	Eddie bought a pair of board shorts for \$21 and a beach towel for \$48. How much did Eddie pay?
6.	Jordan paid \$34 for vegetables and \$22 for steak. How much did the food cost?
7.	Shania spent \$33 on skin care products and \$65 on a haircut. How much did Shania spend?
8.	Andrew bought a cricket bat for \$87 and a cricket ball for \$12. How many dollars did Andrew spend?
9.	Elvis and Lucy went to lunch. They paid \$26 for food and \$21 for drinks. How much did Elvis and Lucy pay?
10.	Rachael paid \$16 for a teddy bear and \$13 for a pair of booties for her sister's new baby. How much did Rachael spend?



SA3 - Number line games

Number line game A: Race to 100

Materials: No line (0-100), 2 dice, pen and paper.

Number of players: 2-4.

Directions:

- 1. All players have their own 0-100 number line. All start at zero.
- 2. Each player, in turn, throws 2 dice, adds the two numbers, moves this distance along the line (marking the move with a jump arrow), and records the position reached on paper.
- 3. Player who is first to 100 wins.

Number line game A: Race to 200

Materials: Two number lines (the first from 0-100 and the second 101-200), 1 die

(4, 5 and 6 replaced with 1, 2 and 3), card deck (without 10, J, Q and

K; with Ace as 1), pen and paper.

Number of players: 2-4.

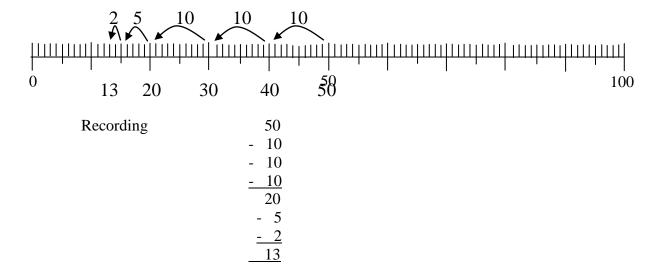
- 1. All players have 2 number lines going from 0-200 number lines. All start at zero on first line.
- 2. Each player, in turn, throws the die and selects a card to get a number (die is tens and card gives ones).
- 3. Players move this number along the line (marking the move with a jump arrow) and recording the position reached on paper.
- 4. Player who is first to 200 wins.

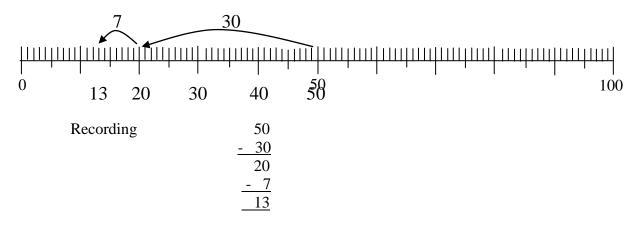
ACTIVITY SA4

[Sequencing strategy for 2-digit subtraction]

Materials: 99 board, number lines, calculators, pen and paper

- 1. Assist students to use the sequencing strategy to **subtract with the 99 board**.
 - (a) <u>Subtracting numbers</u>. Say: Consider 56-23. Direct: Use fingers to find 56 on the 99 board. Say: To subtract 23, move 2 steps up and 3 back. Ask: what numbers are you at? [33] Say: Use calculators to find 56-23. Ask: Is it the same number?
 - (b) <u>Finding pattern</u>. Repeat direction 1(a) for 87 32, 72-28, and 56-17. Ask: *What is the pattern here?* [Find the first number, go up tens and back ones of second number]
 - (c) <u>Imagining subtraction</u>. Say: *Shut your eyes, imagine the board and work out* 58 26 *on this imaginary board*. Repeat this for 47 15, 64-15, and 80-25. Ask: *Can you do these subtractions in your head? Show me?*
 - (d) <u>Practice</u>. Complete worksheet A. Play 99-board games A, B and C.
- 2. Assist students to use the sequencing strategy to **subtract with a number line**.
 - (a) Ask: How much change when you give \$50 for a CD that costs \$37? Say: Let's look how to do this on a number line. Direct: Find 50 on the number line. Say: To subtract 37, hop back 37 first 10s and then as 1s. Ask: How many 10s and 1s in 37? [3 tens and 7 ones] Say: Hop back the 3 tens and then the 7 ones. Record what you do on paper. Ask: Can you do this different ways?





- (b) Getting pattern. Repeat step 2(a) for 54-27, 82-64 and 75-37. Discuss how to be efficient. Ask questions about going backwards? For sequencing always ask about going backwards: *How much are the ones below 10?* When we subtract, what will this leave us with? Questions should always focus on thinking that is, on the method or strategy being used.
- (c) <u>Imagining</u>. Say: Think about how you would do these sums without the number line. What would you think of? Say: Try adding two Shop cards by imagining the line.
- (d) <u>Strategy</u>. Discuss the strategy being used. Ask: *How would you describe the general way we are doing this?* [Starting with one number subtracting bits of other numbers]. State: *This is called sequencing*. Discuss how it is similar to sequencing in addition.
- (e) <u>Practice</u>: Complete Worksheet B and C. Play number-line games A, B and C.

SA4 – 99-board games

99 board game A: First to 50

Materials: 99 board, 2 dice, different coloured counter for each player.

Number of players: 2-4

Directions:

4. Place one 99 board between players. Each player places their counter on 50 at the start.

- 5. Each player, in turn, throws 2 dice and moves their counter backwards along the board the sum of the two dice.
- 6. The first player to reach 0 wins.

99 board game C: Sweep the board 1

Materials: 99 board, deck of cards (no 10, J, Q, or K; Ace is 1), different

coloured counter for each player.

Number of players: 2-4.

Directions: Same as game "first to 50" but starting from 100, subtracting the sum

of the card numbers and finishing at 0.

99 board game C: Sweep the board 2

Materials: 99 board, deck of cards (no 10, J, Q, or K; Ace is 1), different

coloured counter for each player.

Number of players: 2-4.

Directions: Same as game "sweep the board1" but subtracting the sum of three

cards.

SA4 – Worksheet A

Complete these sums with the 99 board.

N 57 - <u>3 3</u> D 71 - <u>2 6</u> E 54 - <u>3 7</u>

S 87 - <u>2 6</u> Y 43 - 28

A 60 - <u>2 8</u>

Shut eyes and complete these sums with an imaginary board.

P 58 - <u>4 4</u> I 84 - <u>7 1</u> W 77 - <u>19</u>

R 62 - <u>2 6</u>

L 48 - <u>19</u>

What's black and white and red all over.

45 32 13 29 15 24 17 58 61 14 32 14 17 26

SA4 – Worksheet B

Name:	Year:	School:

Use the number line to solve these word problems. Try to use an imaginary line.

Help Josh finish his homework so he can ride his dirt bike. Colour in the squares with the correct answers.

- 1. Jackie and Brad paid \$32 for a gift for their uncle. How much change did they get from \$55?
- 2. Rosie bought seeds for her vegetable garden for \$14. What was the change from \$25?
- 3. Alexis bought her dog a new collar. It cost her \$12. How much change was she given from \$46?
- 4. Warwick had \$96 in his wallet. He gave \$82 to his brother. How much money did he have left?
- 5. Suzanne paid \$67 for her groceries. How much change should she get from \$100?
- 6. Brad had \$44 in his holey pants pocket. \$26 fell through the hole. How much money did he have left?
- 7. Jarred bought sausages for \$17. How much change should he get from \$35?
- 8. Josh owed the mechanic \$88 for repairing his dirt bike. How much change should he expect from \$110?



\$14	\$15	\$98	\$37	\$43	\$45
\$83	\$18	\$23	\$19	\$39	\$16
\$29	\$14	\$35	\$34	\$32	\$81
\$31	\$10	\$17	\$56	\$11	\$12
\$24	\$91	\$10	\$48	\$22	\$33



SA4 – Worksheet C

Name: _____ Year: ____ School: ____

Find the change to break the code



T-shirt \$26 Paid \$40

How much change? ____ = W



Cap \$17 Paid \$50

How much change? ____ = T

Earrings \$31 Paid \$40

How much change? ____ = **E**



2 movie tickets \$24 Paid \$30

How much change? ____ = **S**





Football \$20 Paid \$50

How much change? ____ = A



Par of thongs \$14 Paid \$50

How much change? ____ = I

A kilogram of prawns \$22

Paid \$25

How much change? ____ = R



Super supreme pizza \$17

Paid \$40

How much change? ____ = **O**





Restaurant meal \$19

Paid \$60

How much change? ____ = **C**



Watch \$47

Paid \$100

How much change? $\underline{\hspace{1cm}}$ = \mathbf{Y}

Why couldn't the bicycle stand up?

\$36 \$33

\$14 \$30 \$6



\$33 \$14 \$23 \$33

\$33 \$53

\$41

SA4 – Number-line games

Number line game A: Race from 100

Materials: No line (0-100), 2 dice, pen and paper.

Number of players: 2-4.

Directions:

1. All players have their own 0-100 number line. All start at 100.

- 2. Each player, in turn, throws 2 dice, adds the two numbers, moves this distance backwards along the line (marking the move with a jump arrow), and records the position reached on paper.
- 3. Player who is first to zero wins.

Number line game A: Race from 200

Materials: Two number lines (the first from 0-100 and the second 101-200), 1 die

(4, 5 and 6 replaced with 1, 2 and 3), card deck (without 10, J, Q and

K; with Ace as 1), pen and paper.

Number of players: 2-4.

- 1. All players have 2 number lines going from 0-200 number lines. All start at 200 on second line.
- 2. Each player, in turn, throws the die and selects a card to get a number (die is tens and card gives ones).
- 3. Players move this number backwards along the line (marking the move with a jump arrow) and recording the position reached on paper.
- 4. Player who is first to zero wins.

ACTIVITY SA5

[Compensation strategies for 2-digit addition and subtraction]

Materials: 99 boards, number lines, pen, paper, calculators.

- 1. Assist students to add and subtract with compensation on the 99 board.
 - (a) Setting up: Hand students a 99 board each. Say: Look at the board. Ask: How do we add and subtract on the board? [Add move down and to right; subtract move up and to left] Ask: Could we do both together? [Yes] State: By doing just that, we will make addition and subtraction easy.
 - (b) Looking at adding. Say: Let's consider 24+39? Ask: How did we do this on 99 board? [Find 24, move down 3 tens and across 9 ones] Ask: Is there another quicker way we could get to the same answer? [Encourage students to see that we could by moving down 4 tens and back 1 one] Direct: Find 24, move down 4 tens and back 1 one. Ask: Why do we get the same answer? [The last ten we added is 1 too many (should only have added 9) so we go back 1].
 - (c) Practice. Repeat this method for 45+28 and 68+27.
 - (d) Looking at subtracting. Say: Let's consider 82-59. Ask: How did we do this on the 99 board? [Find 82, go up 5 tens, and back to the left 9 ones] Ask: Is there another quicker way to the same answer? [Encourage students to see that we could by moving up 6 tens and forward 1 one] Direct: Find 82, move up 6 tens and forward 1 one. Ask: Why do we get the same answer? [The last ten is 1 too many (should only have subtracted 9) so we have to add 1].
 - (e) <u>Practice</u>. Repeat this method for 61-28 and 54-27.
- 2. Assist students to add and subtract with compensation on a number line.
 - (a) Setting up: Organise students to work on number lines. Say: Look at the line. Ask: How do we add and subtract on the board? [Add move across to right; subtract move back to left] Ask: Could we do both together? [Yes] State: By doing just that, we will make addition and subtraction easy.
 - (b) Looking at adding. Say: Let's consider 36+28? Ask: How did we do this on a number line? [Mark 36 and move right 2 ten jumps and 8 one jumps] Ask: Is there another quicker way we could get to the same answer? [Encourage students to see that we could by moving right 3 ten jumps and back 2 one jumps] Direct: Find 36, move 3 tens and back 2 ones. Ask: Why do we get the same answer? [The last ten we added is 2 too many

(should only have added 8) so we go back 2]. The following diagram may assist:

36	36	Is this answer too low
<u>+28</u>	<u>+30</u>	or too high?
	66	What is the correct answer?

- (c) Practice. Repeat this method for 24+59 and 38+47.
- (d) Looking at subtracting. Say: Let's consider 63-29. Ask: How did we do this on the 99 board? [Find 63, go back 2 ten jumps and 9 one jumps] Ask: Is there another quicker way to the same answer? [Encourage students to see that we could by moving back 3 tens and forward 1 one] Direct: Find 63, move back 3 tens and forward 1 one. Ask: Why do we get the same answer? [The last ten is 1 too many (should only have subtracted 9) so we have to add 1]. The following diagram may assist:

63	63	Is this answer too low
<u>-29</u>	<u>-30</u>	or too high?
	33	What is the correct answer?

- (e) <u>Practice</u>. Repeat this method for 42-27 and 95-38.
- 3. Complete worksheet A. Ask as students do the exercises: Would it be easier to go to the 10 above? How much do we increase to go to next \$10? How do we compensate?
- 4. Complete Worksheet B. Ask as students do the exercises: Would it be easier to go down another 10? If we subtracted a whole extra 10, how much are we out? If we were paying, how much less change are we getting? How do we compensate?

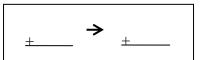
SA5 – Worksheet A

1. Use the left hand side to do the exercises on the right hand side. The ones marked ** are harder.

(a)
$$\begin{array}{c} \$35 \\ + \ \$40 \\ \hline \$75 \end{array} \rightarrow \begin{array}{c} \$35 \\ + \ \$39 \\ \hline \end{array} + \begin{array}{c} \$35 \\ \$40 \\ \hline \$75 \end{array} \rightarrow \begin{array}{c} \$35 \\ + \ \$39 \\ \hline \end{array} + \begin{array}{c} \$40 \\ \hline \$93 \end{array} \rightarrow \begin{array}{c} \$53 \\ + \ \$38 \\ \hline \end{array} + \begin{array}{c} \$23 \\ + \ \$50 \\ \hline \$73 \end{array} \rightarrow \begin{array}{c} \$23 \\ + \ \$48 \\ \hline \end{array} + \begin{array}{c} \$48 \\ + \ \$40 \\ \hline \$69 \end{array} \rightarrow \begin{array}{c} \$48 \\ + \ \$40 \\ \hline \$88 \end{array} \rightarrow \begin{array}{c} \$48 \\ + \ \$39 \\ \hline \end{array} + \begin{array}{c} \$50 \\ + \ \$50 \\ \hline \$67 \end{array} \rightarrow \begin{array}{c} \$17 \\ + \ \$47 \\ \hline \end{array}$$

2. Do these 4 problems using the same method.

(a) Albert had \$28 in his wallet. Gina gave him \$19. How much money does Albert have altogether?



(b) Felix bought his father a pair of shoes for \$54 and a pair of socks for \$19. How much did he spend?



(c) Jessma paid \$58 for her share of the phone bill. She paid \$38 for electricity. How much money did she pay?



(d) Karl's jeans cost \$49 and his shirt cost \$47. How much did he spend on new clothes?

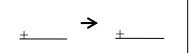


SA5 – Worksheet B

1. Use the left hand side to do the exercises on the right hand side. The ones marked ** are harder.

(a)	\$50		\$50	(b) \$6	0	\$60	(c)	70	\$70
_	\$20	→	- \$19	- \$3		- \$28		10 →	- \$17
	\$30			\$3	0		\$	60	
(d)				(e)			(f)		
	\$82		\$82	\$6	4	\$64	\$	85	\$85
-	\$40	\rightarrow	- \$38	- \$4	<u>0</u> →	- \$36	- \$	50 →	- \$48
	\$42			\$2	4		\$3	30	
(g)				(h)			(i)*		
(5)	\$70		\$70	\$6	0	\$60	` ´	84	\$84
_	\$40	→	- \$34	- \$4		- 36		20 →	- \$27
	\$30			\$2	0		\$	64	
(j)				(k)			(1)		
**	\$82		\$82	** \$7	0	\$72	** \$	60	\$61
-	\$40	→	- \$33	- \$4	<u>0</u> →	- \$38	- \$1	20 →	- \$17
	\$42			\$3	0		\$4	40	

- 2. Do these 4 problems using the same method.
 - (a) What change would I get if I had \$50 and spent \$28 on shoes?



- (b) Heather paid \$41 for gardening equipment. What was her change from \$100?
- <u></u> → ±
- (c) Simon had \$31 in his wallet. He bought icecreams for his family for \$1. How much money did he have left?
- *** * ***
- (d) Tiffany paid \$27 for a CD. She gave the shopkeeper \$40. What was her change?

<u>+</u> → <u>+</u>
